Optimum Droplet Motion in Fire Plumes

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Abstract

The present paper introduces analytical model applicable for analysis of motion of water droplets injected into fire-generated plumes. The model is derived from Lagrangian equation of droplet motion. Application of the developed model to the practically important problem that is fire suppression by water sprays is discussed. A criterion for optimum spray dynamics is proposed. An analytical expression is provided for the optimum droplet size in the spray as a function of Heat Release Rate (HRR) of fire. The present approach provides a quick estimation of optimum spray parameters for a particular fire suppression application.

Keywords: *Droplet Dynamics, Fire Suppression, Lagrangian Model, Optimum Fire Fighting*

1 Introduction

Droplet motion in flows of various configurations is of great theoretical and practical interest. Very little research is done, however, on this topic in terms of developing approximate analytical solutions. Morsi and Alexander [1] considered the problem of spherical particles movement in a one-dimensional and in some two-dimensional flows (around a circular cylinder and around a lifting aerofoil section). They were only able to solve equations analytically in the case of one-dimensional flow. Recently, Novozhilov [2] presented analytical solutions for both non-evaporating and evaporating particles in the two-dimensional axisymmetrical case, with the flow parallel to the axis of symmetry and changing in the radial direction only.

The present paper concerns with the specific case of spherical droplet movement in a flow generated by fire plume. Flow is approximated as one-

dimensional but its velocity profile behaves in a peculiar manner, specific for convective plumes.

The problem considered is of great practical interest for fire fighting which still mostly relies on fire suppression by different forms of water sprays. In particular, the effect of particle sixe is quite important. Droplet size distribution has a strong influence on spray/fire plume interaction; it controls the rate of spray evaporation in the flame and eventually the efficiency of flame suppression. These features have been extensively observed in fire suppression studies [3-5]. Fire suppression by water spray can occur in the two distinctive regimes [3]: via fuel cooling (surface suppression) or via flame cooling (gaseous suppression). First corresponds to relatively coarse sprinkler sprays, the second to fine water mists. The most efficient way of fire suppression is associated with the latter mode. These regimes are demonstrated qualitatively in Figure 1.
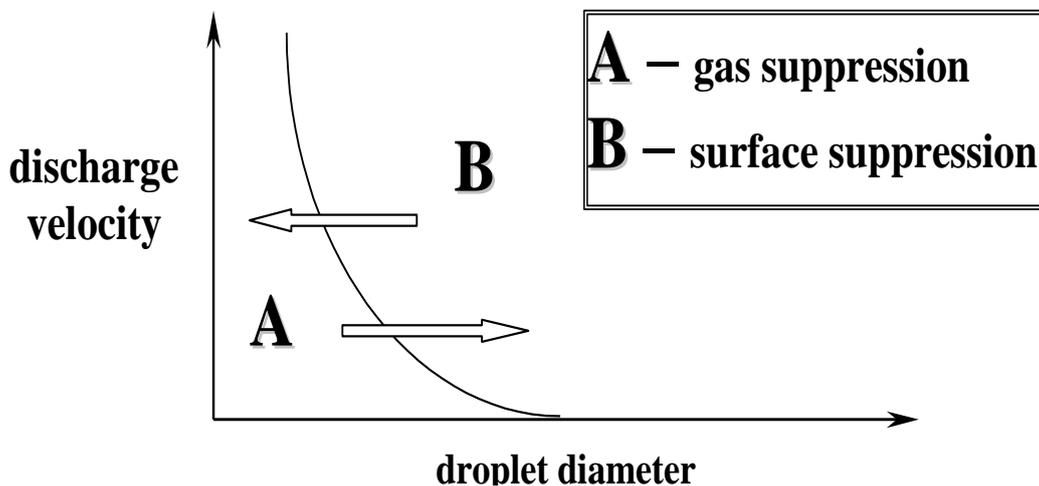
Figure 1   Schematic of suppression regimes

An efficient application of either of these strategies requires that the spray parameters are optimized to achieve maximum heat extraction rate. As explained above, the primary parameter that can be varied for such an optimization is droplet size.

Several studies [6-9] attempted to define optimum droplet size in a different manner, and obtained numerical estimations corresponding to their definitions. Criteria for optimality and predicted optimum droplet diameters vary wildly.

In the present study an entirely novel concept of optimum spray diameter is proposed. It is based on the strategy of flame suppression in the gaseous phase, and applies therefore, in general, to fire suppression regimes delivered by water mist systems. A distinguishing feature of this approach is that the estimation for

the optimum diameter is derived from exact equation of droplet motion and exact flow velocity profile in fire plume and flame. Starting from exact equations, the model however provides surprisingly simple formulas for optimum droplet diameter in fire fighting sprays.

It is argued that a proper scientific definition of water mist sprays need be based on consideration of spray dynamics under specific fire conditions.

2 Mathematical Model

Consider the basics of water droplet interaction with flame and fire plume (Figure 2). Fire-induced flow structure consists of three distinctive zones: continuous flame zone, intermittent flame zone (where luminous flame is appearing only for fraction of time), and fire plume zone (convective flow of inert combustion products mixed with surrounding air).
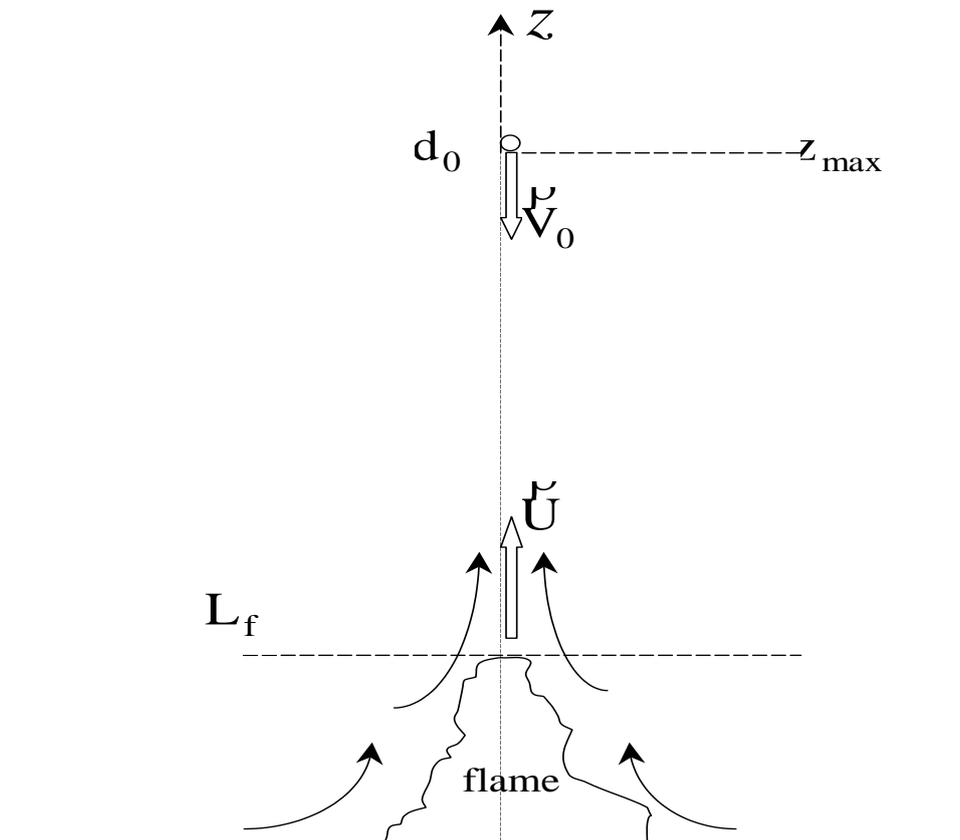
Figure 2 Schematic of droplet injection into fire plume

The equation of droplet motion is taken in the form [10]

$$m\frac{d\vec{V}}{dt}=-\frac{\pi}{8}d^2\rho C_D\left|\vec{V}-\vec{U}(z)\right|\left(\vec{V}-\vec{U}(z)\right)+\vec{F}_g \tag{1}$$

or

$$\frac{d\vec{V}}{dt}=-\left(\frac{3}{4}\frac{\rho}{\rho_p}\right)\frac{C_D}{d}\left(\vec{V}-\vec{U}(z)\right)\left|\vec{V}-\vec{U}(z)\right|+\vec{g} \tag{2}$$

According to arrangements of Figure 2, components of the velocities obey $V<0$; $U>0$; $V-U<0$.

Taking this into account, the equation (2) in components becomes

$$\frac{dV}{dt}=\left(\frac{3}{4}\frac{\rho}{\rho_p}\right)\frac{C_D}{d}\left(V-U(z)\right)^2-g \tag{3}$$

At this point, an approximation regarding the form of dependence of the drag coefficient on the particle Reynolds number is made. It is customary to characterize particle drag coefficient, for example, by the following correlation [11]

$$C_D=\begin{cases}27/\mathrm{Re}_p^{0.84}, & \mathrm{Re}_p<80\\ 0.271\mathrm{Re}_p^{0.217}, & 80<\mathrm{Re}_p<10^4\end{cases} \tag{4}$$

In the present analysis, this dependence is replaced by the following simplifying one

$$C_D=27/\mathrm{Re}_p, \quad \mathrm{Re}_p<10^4 \tag{5}$$

It has been demonstrated [2] that the correlation (5) is quite reasonable, and certainly provides good practical results in the context of the present analysis.
The use of a simple universal correlation (5) greatly simplifies the equation (3). Indeed, on the RHS of (3) one has

$$\frac{C_D}{d}\left(V-U(z)\right)^2=\frac{27\nu(U(z)-V)^2}{d\left(U(z)-V\right)d}=27\nu\frac{1}{d^2}\left(U(z)-V\right) \tag{6}$$

Finally, the basic approximate equation of the droplet motion used throughout the present paper is

$$\frac{dV}{dt}=\frac{\alpha}{d^2}\left(U(z)-V\right)-g \tag{7}$$

with $\alpha = \dfrac{81}{4} \nu \dfrac{\rho}{\rho_p}$. The profile $U(z)$ in the above equation is assumed to be known, and is determined by a particular fire under consideration.

Even in the case of constant droplet diameter no analytical solutions for equation (7) are known, although the equation form itself is rather familiar. In the latter case (constant diameter) the equation (7) belongs to the class of Abel Equations of the Second Kind. Existence of closed-form solutions for these equations depend on the form of a known function on the RHS, in our case, essentially, on the form of the profile $U(z)$. Unfortunately, the case of velocity profiles in fire plumes $U(z) \sim z^{-1/3}$ does not fall into the classified cases which admit integration [12].

3 Results and Discussion

The basic idea of the analysis is that for the particular present application (optimum droplet movement), only the lowest point of the droplet trajectory is most important. This is explained in more detail below. This consideration allows the problem of non-integrable case of equation (7) to be circumvented. It will be demonstrated the closed model predicting the lowest point of the droplet trajectory can be obtained.

3.1 Non-evaporating droplet

To illustrate the method for estimating optimum diameter, consider movement of a non-evaporating particle first. This case illustrates essential properties of solution of the equation (7).

In general, the optimum should be defined in the following way: droplet posses the optimum diameter if upon injection it evaporates exactly in the flame region (i.e. neither penetrates to the surface nor evaporates in inert fire plume). In application to a non-evaporating particle, it is sensible to impose the following criterion: zero-velocity point of the droplet's trajectory is inside the flame. This point will also be a reversing point for a droplet trajectory, since upon achieving this point the droplet will still experience acceleration by fire plume, and will start to move upwards. The rationale for such a criterion is that if the droplet had a capability to evaporate, complete evaporation is likely to happen in the vicinity of a reversing point where the temperature is high (flame temperature) and droplet motion is slow (residence time large).

It will be demonstrated below that a similar, slightly modified criterion is suitable for a general case of evaporating droplet.

Using the identity

$$\frac{dV}{dt} = \frac{dV}{dz} \cdot \frac{dz}{dt} = V \frac{dV}{dz}, \tag{8}$$

the equation (7) is transformed into

$$V \frac{dV}{dz} = \frac{\alpha}{d^2} \left(U(z) - V \right) - g. \tag{9}$$

This equation can now be integrated between the initial (z_{max}) and final (z_{min}) points of the trajectory (we remind that $d = const$ at the moment) to get

$$-\frac{1}{2}V_0^2 = \frac{\alpha}{d^2} \left[\int_{z_{max}}^{z_{min}} U(z)\,dz - \int_{z_{max}}^{z_{min}} V\,dz \right] - g \left(z_{min} - z_{max} \right) \tag{10}$$

The integral term containing plume velocity profile $U(z)$ is a known function, obtained upon specification of the latter profile.

The term involving unknown profile of the particle velocity $V(z)$ needs, however, be estimated.

This integral term can be written (using (8) again) as

$$\int_{z_{max}}^{z_{min}} V\,dz = \int_{z_{max}}^{z_{min}} V^2\,dt, \tag{11}$$

and the simplest estimation (mean value theorem) yields

$$\int_{z_{max}}^{z_{min}} V^2\,dt \approx V(z_*) \int_{z_{max}}^{z_{min}} V\,dt = V(z_*)[z_{min} - z_{max}] \approx -\frac{|V_0|}{2}[z_{min} - z_{max}] = \frac{|V_0|}{2}[z_{max} - z_{min}]. \tag{12}$$

Therefore, the equation (10) gives

$$-\frac{1}{2}V_0^2 = \frac{\alpha}{d^2} \left[\int_{z_{max}}^{z_{min}} U(z)\,dt - \frac{|V_0|}{2}(z_{max} - z_{min}) \right] - g \left(z_{min} - z_{max} \right) \tag{13}$$

$$\frac{1}{d^2} \left[\int_{z_{max}}^{z_{min}} U(z)\,dz - \frac{|V_0|}{2}(z_{max} - z_{min}) \right] = \frac{1}{\alpha} \left[g \left(z_{min} - z_{max} \right) - \frac{1}{2}V_0^2 \right] \tag{14}$$

and

$$d_{opt}^2 = \alpha \frac{\left[\int_{z_{max}}^{z_{min}} U(z)\,dz - \frac{|V_0|}{2}(z_{max} - z_{min}) \right]}{\left[g(z_{min} - z_{max}) - \frac{1}{2}V_0^2 \right]} \tag{15}$$

Finally, one arrives at the estimation:

$$d_{opt} = \left[\alpha \frac{\int_{z_{max}}^{z_{min}} U(z)\,dz - \frac{|V_0|}{2}(z_{max} - z_{min})}{g(z_{min} - z_{max}) - \frac{1}{2}V_0^2} \right]^{1/2} \tag{16}$$

At this point, plume velocity profile must be specified. The reference fire used for illustration is a methanol pool fire, which exhibits nearly constant burning rate over a wide range of pool diameters.

For a particular fire HRR, a corresponding pool diameter can be found from the burning rate assuming complete combustion of the fuel. Then, flame heights, virtual origin position, and plume velocity profile can be predicted by well-known correlations of Heskestad [13].

The plume velocity profile is given as

$$U(z) = \gamma \left(\frac{g}{c_p T_\infty \rho_\infty} \right)^{1/3} \dot{Q}_c^{1/3} (z - z_0)^{-1/3} \tag{17}$$

$(\gamma \approx \frac{34}{10})$

Velocity distribution in the flame zone is different. Velocity is nearly constant in the intermittent zone, and falls to zero following square-root dependence in the continuous zone [13]. Intermittent zone typically covers about 60% of the total flame height [13]. Based on these estimations, velocity profile (17) can be extended into the flame zone as

$$U(z) = \sqrt{\frac{5}{2}} U(L_f) \left(\frac{z}{L_f} \right)^{1/2}; \qquad z \le \frac{2}{5} L_f \tag{18}$$

$$U(z) = U(L_f); \qquad \frac{2}{5}L_f \leq z \leq L_f \tag{19}$$

This procedure uniquely relates fire HRR to the plume geometry and thermal properties.

In the present example, half height of the flame is taken as a trajectory reverse point, i.e. $z_{\min} = L_f / 2$. Upon integration of the profile (17-19), the following expression is obtained:

$$\int_{z_{\max}}^{L_f/2} U(z)\,dz = \gamma_1 \left(\frac{g}{c_p T_\infty \rho_\infty}\right)^{1/3} \dot{Q}_c^{1/3} \left[\left(L_f - z_0\right)^{2/3} - \left(z_{\max} - z_0\right)^{2/3}\right] - \frac{1}{2}U(L_f)L_f \approx$$

$$\approx \left(\frac{g}{c_p T_\infty \rho_\infty}\right)^{1/3} \dot{Q}_c^{1/3} \left[\gamma_1 \left[\left(L_f - z_0\right)^{2/3} - \left(z_{\max} - z_0\right)^{2/3}\right] - \gamma_2 \left(L_f - z_0\right)^{-1/3} L_f\right]$$

$$\tag{20}$$

($\gamma_1 = 3/2\,\gamma$; $\gamma_2 = 1/2\,\gamma$)

The equations (16,20) provide required estimation for the optimum droplet diameter.

Quality of prediction is tested against numerical solution of the equation of droplet motion, coupled to the mass transfer equation. Description of the Lagrangian method of particle (droplet) tracking is widely available in the literature, e.g. [3,4].

Droplet initial velocity is kept constant at V_0=20 m/s. Initial droplet height is 3 m for the present case, and 5 m for the evaporating droplet analysis (below).

Prediction results for a non-evaporating droplet are presented in Figure 3. There is an excellent agreement between the analytical estimation (16,20) and the solution obtained by numerical integration.

3.2   Evaporating droplet

Similar ideas are applicable for the droplet experiencing evaporation in fire plume and flame.

The optimum is redefined here in the following way: droplet trajectory ends at the surface of the material due to complete evaporation. This ensures, first of all, that droplet is delivered into the flame zone and completely evaporates in this zone, i.e. it is not being carried away by the plume. Secondly, major part of evaporation

process is expected to occur in the flaming zone, since the droplet will significantly decelerate before it disappears at the surface. (Note that the point of complete evaporation is always also a point of zero velocity for the droplet trajectory). Therefore, the droplet will have large residence time in the flame. Strict mathematical proof of the above criterion is yet to follow in subsequent publications.
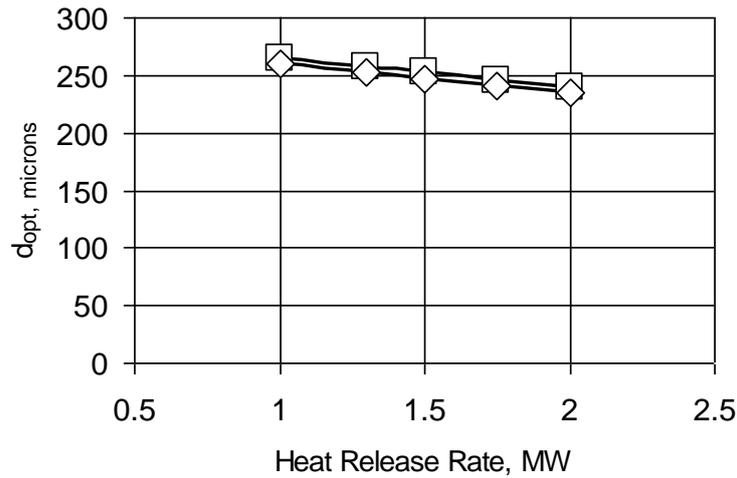
Figure 3 Comparison between optimum droplet diameters predicted by analytical formulae (16, 20) and by numerical analysis
□ - analytical; ◇ - numerical

The equation (9) is modified slightly to give

$$d^2 V \frac{dV}{dz} = \alpha(U(z) - V) - d^2 g \qquad (21)$$

A d^2-law of evaporation [14] is a good approximation for the droplet diameter history. Relevance of this approximation for droplets moving in fire plumes can be confirmed, for example, by numerical analysis of the droplet motion.

Specifically, the d^2-law implies that the droplet surface area diminishes at a constant rate:

$$d_0^2 - d^2(t) = k_{ev} \cdot t \qquad (22)$$

which is equivalent to

$$\frac{d}{dz}\left(d^2\right) = -\frac{k_{ev}}{V} \tag{23}$$

The evaporation constant k_{ev} is estimated as follows [14]

$$k_{ev} = \frac{8k_g}{\rho_p c_p}\ln\left(1 + \frac{c_p}{L}\left(T_g - T_s\right)\right) \tag{24}$$

Integration of the equation (21) from the initial point down to the fuel surface gives an approximate relation

$$-\frac{1}{2}V_0^2 d_0^2 + \frac{k_{ev}V_0}{4}z_{max} \approx \alpha\left[\int_{z_{max}}^0 U(z)dz - \int_{z_{max}}^0 V(z)dz\right] - g\int_{z_{max}}^0 d^2(z)dz \tag{25}$$

(Integration by parts is performed on the LHS, with the relation (23) being taken into account. Further, we made an approximation $\int_{z_{max}}^0 V(z)dz \approx \frac{V_0}{2}z_{max}$).

Estimating the last two integrals on the RHS of (25) gives

$$-\frac{1}{2}V_0^2 d_0^2 + \frac{k_{ev}V_0}{4}z_{max} \approx \alpha\int_{z_{max}}^0 U(z)dz - \alpha\frac{V_0}{2}z_{max} - gz_{max}\frac{d_0^2}{2} \tag{26}$$

and, finally

$$d_{opt} \approx \left[\frac{2}{\left(V_0^2 - gz_{max}\right)}\left\{\alpha\left(\frac{V_0 z_{max}}{2} + \int_0^{z_{max}} U(z)dz\right) + \frac{k_{ev}V_0}{4}z_{max}\right\}\right]^{1/2} \tag{27}$$

The integrated gas velocity profile for the present case takes the following form

$$\int_{z_{max}}^0 U(z)dz = \gamma_1\left(\frac{g}{c_p T_\infty \rho_\infty}\right)^{1/3}\dot{Q}_c^{1/3}\left[\left(L_f - z_0\right)^{2/3} - \left(z_{max} - z_0\right)^{2/3}\right] - \frac{13}{15}U(L_f)L_f \approx$$

$$\approx \left(\frac{g}{c_p T_\infty \rho_\infty}\right)^{1/3}\dot{Q}_c^{1/3}\left[\gamma_1\left[\left(L_f - z_0\right)^{2/3} - \left(z_{max} - z_0\right)^{2/3}\right] - \gamma_3\left(L_f - z_0\right)^{-1/3}L_f\right]$$

$$\tag{28}$$

($\gamma_3 = 13/15\,\gamma$)

Comparison with the numerical integration results are presented in Figure 4 for a range of fire sizes spanning nearly order of magnitude in HRR. The agreement is particularly good for smaller fires (~*1 MW*) with the optimum diameter starting to be slightly under-predicted for larger fires. Overall, deviation of the analytical estimation from "exact" (numerical) solution does not exceed 13%.

Of the particular interest is a dependence of an optimum diameter on the fire HRR. The rate of the diameter increase with fire size is easily deducible from (27) in the form

$$\frac{d}{d\dot{Q}}\left(d^2\right) \propto \dot{Q}^{-9/30} = \dot{Q}^{-0.3} \tag{29}$$

In reality, the rate is slightly bigger, as suggested by Figure 4. It is intended to perform more sophisticated analysis of the equation (21) in order to improve predictions.
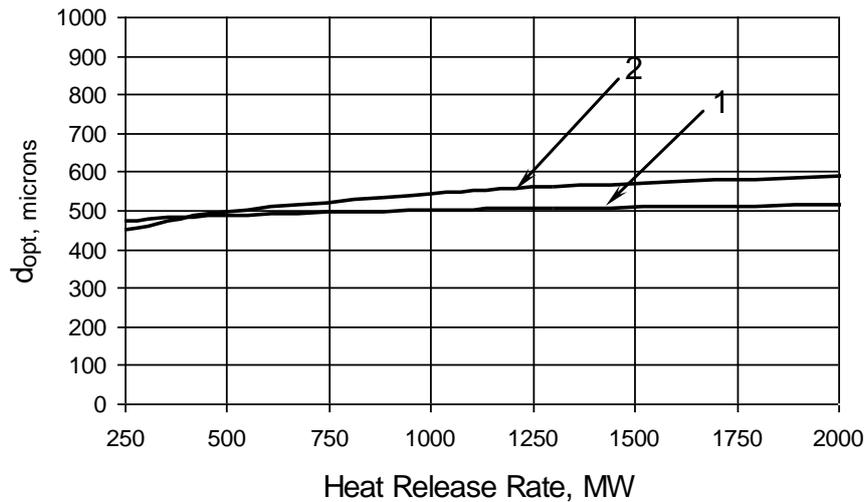
Figure 4   Comparison between optimum droplet diameters predicted by
analytical formulae (27, 28) and by numerical analysis
1 - analytical; 2 - numerical

The proposed concept of optimum droplet diameter allows scientifically clear definition of water mist to be proposed. At the present, definitions used in the literature are rather arbitrary. One can define water mist spray as such that aims at gaseous flame suppression, rather than surface suppression. In this mode, droplets will evaporate above the surface of the burning material. If such a definition is adopted then it becomes clear that the actual droplet size in water mist spray

cannot be specified without referring to a particular fire. Indeed, minimum diameter of droplets penetrating through the flame towards surface is a function of fire Heat Release rate (Figure 4). With the help of the estimation of optimum droplet diameter proposed in the present paper, one can quantitatively define water mist spray for a particular fire with a specific HRR as such that droplet diameters in the spray satisfy

$$d \leq d_{opt}\left(\dot{Q}\right)$$ (30)

4 Conclusion

Equations of droplet motion and evaporation in fire plumes do not admit exact solutions. Despite of this fact, it is possible to derive a range of useful quantitative results in an analytical manner. One possible application of the analysis, demonstrated in the present paper, is the concept of an optimum droplet diameter for fire fighting.

A new concept of an optimum diameter has been proposed, based on the strategy of complete droplet evaporation in the flame. Such definition would be of particular importance for water mist suppression systems, relying on gaseous flame suppression.

The proposed definition of the optimum droplet requires analysis of the droplet motion and evaporation within fire plume and flame. The analysis has been performed using Lagrangian equation of droplet dynamics. Despite the absence of exact solutions, the nature of applied optimum criterion allowed quantitative results to be derived.

An optimum diameter has been predicted analytically for fires in the range from 250 to 2000 kW to within 13% of the accurate numerical solution. An optimum diameter has turned out to be a weak function of the fire Heat Release Rate. The form of the latter dependence has been derived.
Results are encouraging in a sense that a rather complicated problem has been solved with a good accuracy by fairly simple considerations. It is planned to perform more sophisticated analysis in order to improve results further.

5 Notation

C_D	drag coefficient
c_p	gas specific heat

d	droplet diameter
d_0	initial droplet diameter
d_{opt}	optimum droplet diameter
$\vec{F}_g$	gravity force
g	gravity acceleration
k_{ev}	evaporation constant
k_g	gas thermal conductivity
L	latent heat of vaporization
L_f	mean flame height
m	droplet mass
$\dot{Q}$	fire Heat Release Rate
$\dot{Q}_c$	fire convective Heat Release Rate
Re_p	particle (droplet) Reynolds number
T_∞	ambient temperature
T_g	gas temperature
T_s	droplet surface temperature
t	time
U	fire plume velocity
V	droplet velocity
V_0	initial droplet velocity
z	height
z_{max}	initial droplet height
z_{min}	bottom point of droplet trajectory
z_0	fire plume virtual origin

Greek symbols

ν	gas kinematic viscosity
ρ	gas density
ρ_p	particle (droplet) density
ρ_∞	ambient density

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