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Research paper

Availability evaluation of repairable system requiring two types of supporting device for operations

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Abstract

With advancement of modern science and technology, complex systems connected to an external supporting device for their operations have been manufactured to meet the demand of industries, economic growth and populace in general. Companies and organizations heavily rely on these systems to conduct their business. This study presents the availability assessment of a single unit system connected to two types of an external supporting device for its operation. Each type of supporting device has two copies I and II. First order differential equations method is used to obtain the explicit expression for the steady-state availability. Based on assumed numerical values given to system parameters, graphical illustrations are given to highlight important results. Comparisons are performed to highlight the impact of unit failure and repair rates.

Keywords: *Availability; Supporting Device; Probabilistic; Single Unit.*

1. Introduction

High system reliability and availability play a vital role towards industrial growth as the profit is directly dependent on production volume which depends upon system performance. Thus the reliability and availability of a system may be enhanced by proper design, optimization at the design stage and by maintaining the same during its service life. Because of their prevalence in power plants, manufacturing systems, and industrial systems, many researchers have studied reliability and availability problem of different systems. Hajeeh [1] deals with availability of a system with different repair options. Hu et al. [2] presents availability analysis and design optimisation for a repairable series-parallel system with failure dependencies. Jain and Rani [3] studied the availability analysis for repairable system with warm standby, switching failure and reboot delay. Wang and Chen [5] performed comparative analysis of availability between three systems with general repair times, reboot delay and switching failures. Wang et al. [7] presents analysis of a repairable system with warm standbys plus balking and reneging .Wang et al. [8] performed comparative analysis of availability between two systems with warm standby units and different imperfect coverage.

In real-life situations we often encounter cases where the systems that cannot work without the help of external supporting devices connect to such systems. These external supporting devices are systems themselves that are bound to fail. Such systems are found in power plants, manufacturing systems, and industrial systems. Large volumes of literature exist on the issue relating to prediction of various systems performance connected to an external supporting device for their operations. Yusuf and Bala [9] analyzed the reliability characteristic of parallel system with external supporting devices for operation. Yusuf [10] performed comparative analysis of profit between three dissimilar repairable redundant systems using supporting external device for operation. Yusuf et al [11] present mathematical modelling approach to analysis of mean time to system failure of two unit cold standby system with a supporting device. Yusuf et al. [12] performed comparative analysis of MTSF between systems connected to supporting device for operation. Yusuf et al. [13] performed reliability computation of a linear consecutive 2-out-of-3 system in the presence of supporting device. Yusuf [14] presents reliability evaluation of a parallel system with a supporting device and two types of preventive maintenance.

The problem considered in this paper is different from the work of discussed authors above. In this paper, a single unit system connected to two types of dissimilar supporting device is considered and derived its corresponding mathematical models. The focus of our analysis is primarily to capture the effect of both type I and II failure and repair rates on availability for different values of unit failure and repair rates.

2. Description and states of the system

In this paper, a single unit system is considered. It is assumed that the system most work with one copy of both type I and II supporting devices. It is also assumed that each type of supporting has a copy on standby and the switching is perfect. Both the unit and supporting devices are assumed to be repairable. Each of the primary supporting devices fails independently of the state of the other and has an exponential failure distribution with parameter λ_1 and λ_2 for type I and II supporting devices respectively. Whenever a primary supporting device fails, it is immediately sent to repair with parameter μ_1 and μ_2 and the standby supporting device is switch to operation. System failure occur when the unit has failed with parameter λ and it is sent for repair with parameter with parameter μ or the failure of all copies of type I or type II supporting devices.

Following Trivedi (2007), Wang et al. (2000), and Wang et al. (2006) the state transition diagram of the proposed system is shown in Figure 1 below;

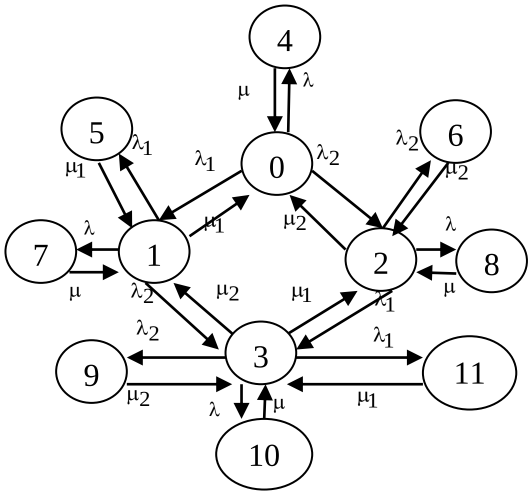

Fig. 1: Transition Diagram of System.

S_0: Initial state, main unit, type I copy I, and type II copy I supporting devices are working, type I copy II and type II copy II are on standby. The system is operative.

S_1: Type I copy I supporting device has failed and is under repair, main unit, type I copy II, and type II copy I supporting devices are working, type II copy II is on standby. The system is operative.

S_2: Type II copy I supporting device has failed and is under repair, main unit, type I copy I and type II copy II supporting devices are working, type I copy II is on standby. The system is operative.

S_3: Type I copy I and type II copy I supporting devices have failed and are under repair, main unit, type I copy II, and type II copy II supporting devices are working. The system is operative.

S_4: Main unit has failed, type I copy I and type II copy I supporting devices are idle, I copy II and type II copy II are on standby. The system is inoperative.

S_5: Type I copy I and II supporting devices have failed and are under repair, main unit and type II copy I supporting device are idle, type II copy II is on standby. The system is inoperative.

S_6: Type II copy I and II supporting devices have failed and are under repair, main unit and type I copy I supporting device are idle, type I copy II is on standby. The system is inoperative.

S_7: Main unit and type I copy I supporting device have failed and are under repair, type I copy II and type II copy I supporting devices are idle, type II copy II is on standby. The system is inoperative.

S_8: Main unit and type II copy I supporting device have failed and are under repair, type I copy I and type II copy II supporting device are idle, type I copy II is on standby. The system is inoperative.

S_9: Type I copy I, type II copy I and II supporting devices have failed and are under repair, main unit and type I copy II supporting device are idle. The system is inoperative.

S_{10}: Type I copy I and type II copy I supporting devices and main unit have failed and are under repair, type I copy II and type II copy II supporting devices are idle. The system is inoperative.

S_{11}: Type I copy I and type II copy I supporting devices and main unit have failed and are under repair, type I copy II, and type II copy II supporting devices are idle. The system is inoperative.

3. Formulation of the model

In order to analyze the system availability of the system, we define $P_i(t)$ to be the probability that the system at $t \geq 0$ is in state $S_i, i = 0, 1, 2, 3, \ldots, 11$. Also let $P(t)$ be the row vector of these probabilities at time t. The initial condition for this problem is:

$$P(0) = [p_0(0), p_1(0), p_2(0), \ldots, p_{11}(0)]$$
$$= [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

Following Trivedi [4], Wang and Kuo [6], we obtain the following differential equations from Figure 1:

$$p_0'(t) = -(\lambda + \lambda_1 + \lambda_2) p_0(t) + \mu_1 p_1(t) + \mu_2 p_2(t) + \mu p_4(t)$$
$$p_1'(t) = -(\lambda + \lambda_1 + \lambda_2 + \mu_1) p_1(t) + \lambda_1 p_0(t) + \mu_2 p_3(t) + \mu_1 p_5(t) + \mu p_7(t)$$
$$p_2'(t) = -(\lambda + \lambda_1 + \lambda_2 + \mu_2) p_2(t) + \lambda_2 p_0(t) + \mu_1 p_3(t) + \mu_2 p_6(t) + \mu p_8(t)$$
$$p_3'(t) = -(\lambda + \lambda_1 + \lambda_2 + \mu_1 + \mu_2) p_2(t) + \lambda_2 p_1(t) + \lambda_1 p_2(t) + \mu_2 p_9(t) + \mu p_{10}(t) + \mu_1 p_{11}(t)$$
$$p_4'(t) = -\mu p_4(t) + \lambda p_0(t) \quad (1)$$
$$p_5'(t) = -\mu_1 p_5(t) + \lambda_1 p_1(t)$$
$$p_6'(t) = -\mu_2 p_6(t) + \lambda_2 p_2(t)$$
$$p_7'(t) = -\mu p_7(t) + \lambda p_1(t)$$
$$p_8'(t) = -\mu p_8(t) + \lambda p_2(t)$$
$$p_9'(t) = -\mu_2 p_9(t) + \lambda_2 p_3(t)$$
$$p_{10}'(t) = -\mu p_{10}(t) + \lambda p_3(t)$$
$$p_{11}'(t) = -\mu_1 p_{11}(t) + \lambda_1 p_3(t)$$

This can be written in the matrix form as

$$\dot{P} = MP, \quad (2)$$

Where

$$M = \begin{bmatrix} -h_1 & \mu_1 & \mu_2 & 0 & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_1 & -h_2 & 0 & \mu_2 & 0 & \mu_1 & 0 & \mu & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & -h_3 & \mu_1 & 0 & 0 & \mu_2 & 0 & \mu & 0 & 0 & 0 \\ 0 & \lambda_2 & \lambda_1 & -h_4 & 0 & 0 & 0 & 0 & 0 & \mu_2 & \mu & \mu_1 \\ \lambda & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & -\mu_2 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_1 \end{bmatrix}$$

$$h_1 = (\lambda + \lambda_1 + \lambda_2), h_2 = (\lambda + \lambda_1 + \lambda_2 + \mu_1),$$
$$h_3 = (\lambda + \lambda_1 + \lambda_2 + \mu_2), h_4 = (\lambda + \lambda_1 + \lambda_2 + \mu_1 + \mu_2)$$

Equation (2) is expressed explicitly in the form

$$\begin{bmatrix} p_0'(t) \\ p_1'(t) \\ p_2'(t) \\ p_3'(t) \\ p_4'(t) \\ p_5'(t) \\ p_6'(t) \\ p_7'(t) \\ p_8'(t) \\ p_9'(t) \\ p_{10}'(t) \\ p_{11}'(t) \end{bmatrix} = \begin{bmatrix} -h_1 & \mu_1 & \mu_2 & 0 & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_1 & -h_2 & 0 & \mu_2 & 0 & \mu_1 & 0 & \mu & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & -h_3 & \mu_1 & 0 & 0 & \mu_2 & 0 & \mu & 0 & 0 & 0 \\ 0 & \lambda_2 & \lambda_1 & -h_4 & 0 & 0 & 0 & 0 & 0 & \mu_2 & \mu & \mu_1 \\ \lambda & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & -\mu_2 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_1 \end{bmatrix} \begin{bmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \\ p_5(t) \\ p_6(t) \\ p_7(t) \\ p_8(t) \\ p_9(t) \\ p_{10}(t) \\ p_{11}(t) \end{bmatrix}$$

The steady-state availability (the proportion of time the system is in a functioning condition or equivalently, the sum of the probabilities of operational states) is given by

$$A_V(\infty) = p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty) \tag{3}$$

In the steady state, the derivatives of the state probabilities become zero and therefore equation (2) become

$$MP = 0 \tag{4}$$

This is in matrix form

$$\begin{bmatrix} -h_1 & \mu_1 & \mu_2 & 0 & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_1 & -h_2 & 0 & \mu_2 & 0 & \mu_1 & 0 & \mu & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & -h_3 & \mu_1 & 0 & 0 & \mu_2 & 0 & \mu & 0 & 0 & 0 \\ 0 & \lambda_2 & \lambda_1 & -h_4 & 0 & 0 & 0 & 0 & 0 & \mu_2 & \mu & \mu_1 \\ \lambda & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & -\mu_2 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_1 \end{bmatrix} \begin{bmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \\ p_5(t) \\ p_6(t) \\ p_7(t) \\ p_8(t) \\ p_9(t) \\ p_{10}(t) \\ p_{11}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Subject to following normalizing conditions:

$$p_0(\infty) + p_1(\infty) + p_2(\infty) + \ldots + p_{11}(\infty) = 1 \tag{5}$$

Following Wang and Kuo (2000) and Wang et al (2006) we substitute (5) in the last row of (4) to compute the steady-state probabilities.

$$\begin{bmatrix} -h_1 & \mu_1 & \mu_2 & 0 & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_1 & -h_2 & 0 & \mu_2 & 0 & \mu_1 & 0 & \mu & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & -h_3 & \mu_1 & 0 & 0 & \mu_2 & 0 & \mu & 0 & 0 & 0 \\ 0 & \lambda_2 & \lambda_1 & -h_4 & 0 & 0 & 0 & 0 & 0 & \mu_2 & \mu & \mu_1 \\ \lambda & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & -\mu_2 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \\ p_5(t) \\ p_6(t) \\ p_7(t) \\ p_8(t) \\ p_9(t) \\ p_{10}(t) \\ p_{11}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Solving (4), we obtain the steady-state probabilities

$$p_0(\infty), p_1(\infty), p_2(\infty), p_3(\infty), \ldots, p_{11}(\infty)$$

The expressions for the steady-state availability involving minor and major maintenance and replacement given in equations (3) above is given by

$$A_V(\infty) = \frac{\mu\mu_1^2\mu_2^2 + \mu\mu_1\mu_2^2\lambda_1 + \mu\mu_1^2\mu_2\lambda_2 + \mu\mu_1\mu_2\lambda_1\lambda_2}{D}$$

$$D = \mu\mu_1^2\mu_2\lambda_2 + \mu_1^2\mu_2\lambda\lambda_2 + \mu_1^2\mu_2^2\lambda + \mu\mu_1^2\lambda_2^2 + \mu\mu_1^2\mu_2^2 + \mu\mu_2^2\lambda_1^2 + \mu_1\mu_2\lambda\lambda_1\lambda_2 + \mu\mu_1\lambda_1\lambda_2^2 + \mu\mu_1\mu_2\lambda_1\lambda_2 + \mu\mu_1\mu_2^2\lambda_1 + \mu_1\mu_2^2\lambda\lambda_1 + \mu\mu_2\lambda_1^2\lambda_2$$

4. Numerical examples

Numerical examples are presented to demonstrate the impact of failure and repair rates on steady-state availability based on given values of the parameters. For the purpose of numerical example, the following sets of parameter values are used: $\mu_1 = 0.6$, $\mu_2 = 0.5$, $\mu = 0.5$, $\lambda_1 = 0.2$, $\lambda_2 = 0.3$, $\lambda(0.4, 0.6, 0.8)$ for Figures 2 – 5 and $\mu_1 = 0.6$, $\mu_2 = 0.5$, $\lambda = 0.6$, $\lambda_1 = 0.2$, $\lambda_2 = 0.3$, $\mu(0.4, 0.6, 0.8)$ for Figures 6 – 9 respectively. The MATLAB package was used to program the simulations in this study. The results are presented below.

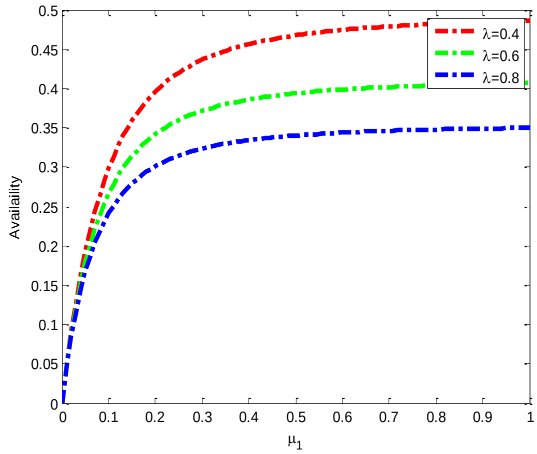

Fig. 2: Availability against Type I Supporting Device Repair Rate μ_1 for Different Values of $\lambda(0.4, 0.6, 0.8)$

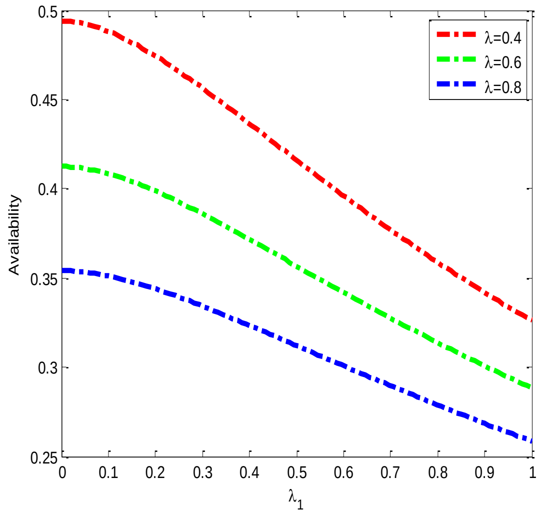

Fig. 3: Availability against Type I Supporting Device Failure Rate λ_1 for Different Values of $\lambda(0.4, 0.6, 0.8)$

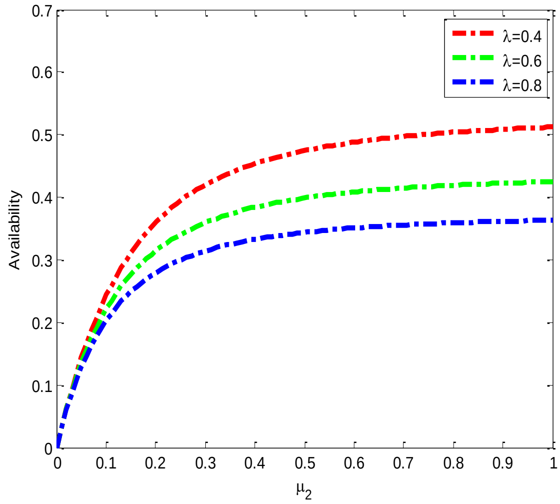

Fig. 4: Availability against Type II Supporting Device Repair Rate μ_2 for Different Values of $\lambda(0.4, 0.6, 0.8)$

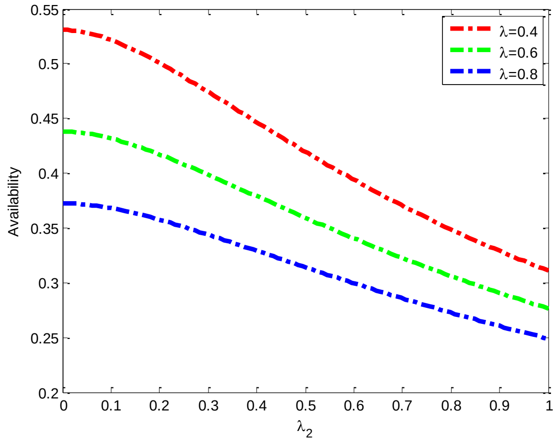

Fig. 5: Availability against Type II Supporting Device Failure Rate λ_2 for Different Values of $\lambda(0.4, 0.6, 0.8)$

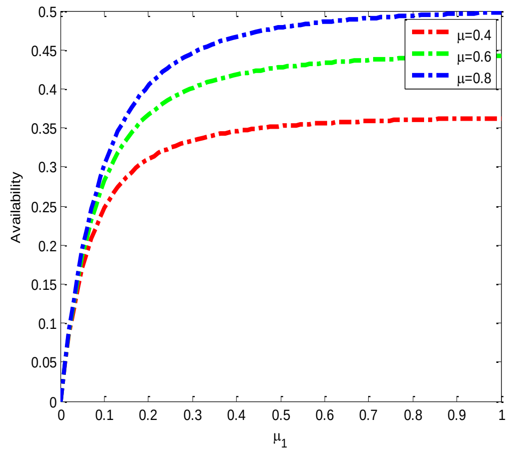

Fig. 6: Availability against Type I Supporting Device Repair Rate μ_1 for Different Values of $\mu(0.4, 0.6, 0.8)$.

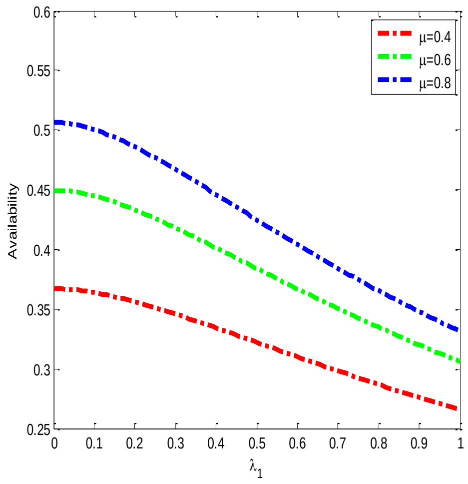

Fig. 7: Availability against Type I Supporting Device Failure Rate λ_1 for Different Values of $\mu(0.4, 0.6, 0.8)$.

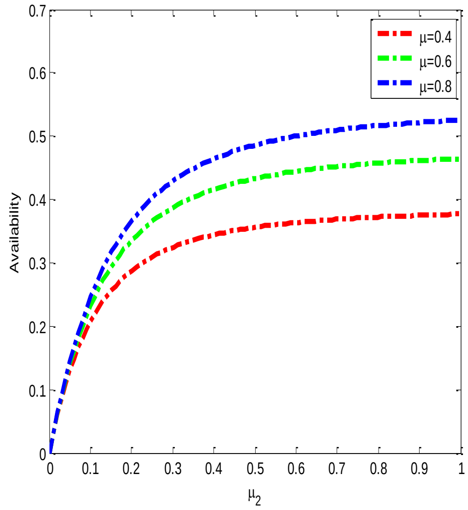

Fig. 8: Availability against Type II Supporting Device Repair Rate μ_2 for Different Values of $\mu(0.4, 0.6, 0.8)$.

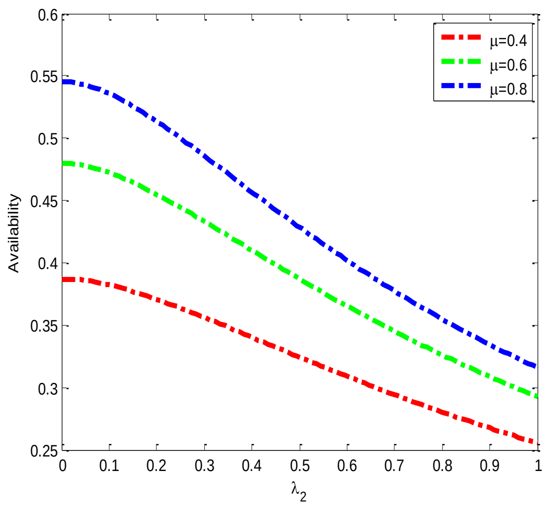

Fig. 9: Availability against Type II Supporting Device Failure Rate λ_2 for Different Values of $\mu(0.4, 0.6, 0.8)$.

Numerical results of availability with respect to type $k, k = \text{I}, \text{II}$ repair μ_i and failure rates $\lambda_i, i = 1, 2$ for different values of $\lambda(0.4, 0.6, 0.8)$ are depicted in Figures 2 - 5 respectively. In Figures 2 and 4, the availability increases as μ_1 and μ_2 for different values of unit failure rate λ. This sensitivity analysis implies that major maintenance to the unit and supporting devices should be invoked to improve and maximize the system availability, production output as well as the profit. On the other hand, Figures 3 and 5 show that the availability decreases as λ_1 and λ_2 increases for different values of unit failure rate λ. This sensitivity analysis implies that major maintenance should be invoked to the unit and supporting devices to minimize the failure of the system in order to improve and maximize the system availability, production output as well as the profit. Simulations in Figures 6 and 8 display the increasing pattern of system availability against the μ_1 and μ_2 for different values of unit repair rate. $\dot{\mu}$. It is evident from these Figures that system availability display increasing pattern with respect to μ_1 and μ_2 for different values of hardware repair rate μ. Thus, the system availability is sensitive to repair of supporting devices. Similar pattern can be observed in Figures 5 and 9 between system availability against λ_1 and λ_2 for different values of hardware repair rate μ. This sensitivity analysis implies that major maintenance to the unit and supporting devices should be invoked to improve and maximize the system availability, production output as well as the profit.

5. Conclusion

This paper studied a single system connected to two types of supporting device type I and II for its operation. Explicit expression for the steady-state availability was derived. The numerical simulations presented in Figures 2 – 9 provide a description of the effect of failure rate and repair rate on steady-state availability. On the basis of the numerical results obtained for particular cases, it is suggested that the system availability can be improved significantly by:

i) Adding more cold standby units.
ii) Increasing the repair rate.
iii) Reducing the failure rate of the system by hot or cold duplication method.
iv) Incorporating preventive to system at minor and medium deterioration stages.
v) Exchange the system when old with new one before failure.

The system can further be developed into system with multiple standbys in solving reliability and availability problems.

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