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Research paper

Preliminary test shrinkage estimators for the shape parameter of generalized exponential distribution

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Abstract

The present paper deals with the estimation of the shape parameter α of Generalized Exponential GE (α, λ) distribution when the scale parameter λ is known, by using preliminary test single stage shrinkage (SSS) estimator when a prior knowledge $\alpha_0$available about the shape parameter as initial value due past experiences as well as optimal region R for accepting this prior knowledge.
The Expressions for the Bias [B (.)], Mean Squared Error [MSE] and Relative Efficiency [R.Eff (.)] for the proposed estimator is derived. Numerical results about conduct of the considered estimator are discussed include study the mentioned expressions. The numerical results exhibit and put it in tables.
Comparisons between the proposed estimator $\tilde{\alpha}_{ss}$ withe classical estimator $\hat{\alpha}_{mle}$ as well as with some earlier studies were made to show the effect and usefulness of the considered estimator.

Keywords: *Generalized Exponential Distribution; Maximum Likelihood Estimator; Single Stage Shrinkage Estimator; Mean Squared Error and Relative Efficiency.*

1. Introduction

Generalized Exponential (GE) distribution is one of the most widely used models for failure time in both lifetime and reliability analysis.
The origin of two parameter generalized exponential (GE) distribution goes back to Gompertz (1825), VAerhulst (1838, 1845, and 1847) (see Ahuja and Nash (1967)) [1], [2] and [14].Also known as the Exponentiaed exponential (EE) distribution. Many of anthers introduced the (GE) distribution.
This model was introduced and studied in the most original paper by Gupta and Kundu (1999, 2000, 2001, 2004, and 2007).
The Generalized Exponential (GE) distribution has a right skewed unimodel density function and monotone hazard function similar to the density function and hazard function of Gamma and Weibull distribution. It can be used as an alternative to Gamma or Weibull distribution in many situations sees [8], [9], [10], [11], [12] and [16]. The applications of (GE) distribution have been widespread. We mention models to determine but criteria for analysis of animal behavior (Yeates et. al. 2001).
Desing rainfall estimation in coast of Chiapas (Escalant_Sandoval 2007); analysis of Los Angeles rainfall data (Madi and Raqab 2007); software reliability growth models for vital quality metrics (Subburaj et. at. 2007); estimating mean life of power system equipment with limited end-of-life failure data (Cota-Felix et. al. 2009) and cure rate modeling (Kannan et. al. 2010).[15]
A continuous random variable X is said to have the (GE) distribution if its cumulative distribution function is defined as:

$$F(x) = [1-\exp(-\lambda x)]^{\alpha} \;; x>0, \alpha, \lambda > 0 \tag{1}$$

And the probability density function is given by:

$$f(x) = \begin{cases} \alpha\lambda \exp(-\lambda x)[1-\exp(-\lambda x)]^{\alpha-1} & ; x>0, \alpha,\lambda>0 \\ 0 & \text{o.w.} \end{cases} \tag{2}$$

Where α and λ are respectively shape and scale parameter.
We denoted by GE (α, λ) to Generalized Exponential distribution with shape parameter α and scale parameter λ.
In this article we introduce the problem for estimating the unknown shape parameter α of (GE) distribution with known scale parameter λ when some prior knowledge α_0 regarding true value α is available using preliminary test single stage shrinkage procedure.
Noted that, the prior knowledge regarding due reasons introduced by "Thompson 1968" as well as the classical estimator of $\hat{\alpha}_{mle}$ and using shrinkage weight factor $[\Psi_1(\hat{\alpha})]$;
$0 \le \Psi_1(\hat{\alpha}) \le 1$ Leads to what it is known as "Shrinkage estimator", which though perhaps biased has smaller mean squared error [MSE] than that of $\hat{\alpha}$.
Thus "Thompson – type" shrinkage estimator will be:

$$\Psi_1(\hat{\alpha})\hat{\alpha} + [1-\Psi_1(\hat{\alpha})]\alpha_0 \tag{3}$$

Now, the preliminary test single stage shrinkage estimator (SSSE) introduced in this article as an estimator of level of significance (Δ) for test the hypotheses $H_0: \alpha = \alpha_0$ vs. $H_1: \alpha \ne \alpha_0$ if H_0 accepted we use the shrinkage estimator defined in (3).
However; if H_0 rejected, we chose another shrinkage weight tor $\Psi_2(.)$; $0 \le \Psi_2(.) \le 1$ and then using the following shrinkage estimator;

$$\Psi_2(\hat{\alpha})\hat{\alpha} + [1-\Psi_2(\hat{\alpha})]\alpha_0 \tag{4}$$

Thus, the general form of preliminary test single stage shrinkage estimator (SSSE) will be:

$$\tilde{\alpha} = \begin{cases} \Psi_1(\hat{\alpha})\hat{\alpha} + [1 - \Psi_1(\hat{\alpha})]\alpha_0 & \text{if } \hat{\alpha} \in R \\ \Psi_2(\hat{\alpha})\hat{\alpha} + [1 - \Psi_2(\hat{\alpha})]\alpha_0 & \text{if } \hat{\alpha} \notin R \end{cases} \quad (5)$$

Where $\Psi_i(\hat{\alpha})$, $0 \leq \Psi_i(\hat{\alpha}) \leq 1$; $i = 1,2$ is a shrinkage weight factor specifying the belief of $\hat{\alpha}$ and $[1 - \Psi_2(\hat{\alpha})]$ specifying the belief of α_0 and $\Psi_i(\hat{\alpha})$ may be a function of $\hat{\alpha}$ or may be a constant (ad hoc basis), while (R) is a pretest region for acceptance the prior knowledge with level of significance Δ.

Many authors have been studied preliminary test single stage shrinkage estimator (SSSE) defined in (5), see for example; [3], [4], [7] and [17].

The main aim of this article is to estimate the shape parameter (α) of two parameter Generalized Exponential (GE) distribution with known scale parameter ($\lambda = 1$) using proposed preliminary test (SSSE) defined in (5) due study the expressions of Bias, Mean Squared error and Relative Efficiency of this estimator and display the numerical results for mentioned expressions in annexed tables. Also, study the performance of the consider estimator and make comparisons with the classical estimator as well as with some studies introduced by some authors.

2. Maximum likelihood estimator of α.

In this section, we consider the maximum likelihood estimator (MLE) of GE (α, λ).

Let $x_1, \dots, x_n$ is a random sample of size n from GE (α, λ) then the log-likelihood function $L(\alpha, \lambda)$ can be written as:

$$L(\alpha,\lambda) = n \ln \alpha + n \ln(\lambda) + (\alpha - 1)\sum_{i=1}^{n} \ln\left(1 - e^{-(\lambda x_i)}\right) - \lambda \sum_{i=1}^{n} x_i \quad (6)$$

As we mention above, in this treatise, we assume that λ is known (λ=1), then the normal equation become:

$$\frac{\partial L}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} \ln\left(1 - e^{-(x_i)}\right) = 0 \quad (7)$$

Thus, we obtain the (MLE) of α, say $\hat{\alpha}_{mle}$ as below

$$\hat{\alpha}_{mle} = -\frac{n}{\sum_{i=1}^{n} \ln(1 - e^{-x_i})} \quad (8)$$

The distribution of $\hat{\alpha}_{mle}$ is same as the distribution of $(n\alpha/y)$, where y follows $\text{Gamma}(n, 1)$; see[13].

3. Preliminary teat single stage shrinkage estimator (SSSE) $\tilde{\alpha}_{ssi}$.

In this subsection we consider two preliminary test (SSSE) $\tilde{\alpha}_{ssi}$; $i = 1,2$, which is defined in (5) when $\Psi_{1i}(\hat{\alpha}) = 0$; $i = 1,2$ and $\Psi_{2i}(\hat{\alpha}) = k$. Also we take two choices of k, the first one is equal to $n\Delta^4$ and the second equal to $\frac{n\alpha_0}{\hat{\alpha}}\Delta^4$ for estimate the shape parameter α of GE (α, 1) distribution.

Thus, (SSSE) $\tilde{\alpha}_{ssi}$; $i = 1,2$ will be:

$$\tilde{\alpha}_{ssi} = \begin{cases} \Psi_{1i}(\hat{\alpha}) & \text{if } \hat{\alpha} \in R \\ \Psi_{2i}(\hat{\alpha})\hat{\alpha} + [1 - \Psi_{2i}(\hat{\alpha})]\alpha_0 & \text{if } \hat{\alpha} \notin R \end{cases} \quad (9)$$

Where, $\Psi_{1i}(\hat{\alpha}) = 0 \ \forall i = 1,2$, $\Psi_{21}(\hat{\alpha}) = n\Delta^4$ and

$\Psi_{22}(\hat{\alpha}) = \frac{n\alpha_0}{\hat{\alpha}}\Delta^4$.

And R is a pretest region for testing the hypothesis $H_0: \alpha = \alpha_0$ vs. $H_1: \alpha \neq \alpha_0$ with the level of significance (Δ) using test statistic $T = \hat{\alpha}/\alpha_0 = \frac{n\alpha_0}{\hat{\alpha}}$ i.e.

$$R = \left[\mathcal{X}^2_{\frac{\Delta}{2},2n} \leq \frac{2N\alpha_0}{\hat{\alpha}} \leq \mathcal{X}^2_{1-\frac{\Delta}{2},2n}\right] \quad (10)$$

Assume R = [a, b]

$$\text{Where } a = \mathcal{X}^2_{\frac{\Delta}{2},2n} \text{ and } b = \mathcal{X}^2_{1-\frac{\Delta}{2},2n} \quad (11)$$

are respectively the lower and upper $100(\Delta/2)$ percentile point of chi-square distribution with degree of freedom 2n.

The expression for Bias of $\tilde{\alpha}$ is

$$\text{Bias}(\tilde{\alpha}_{ssi}|\alpha; R) = E(\tilde{\alpha} - \alpha)$$

$$= \int_R (\alpha_0 - \alpha) f(\hat{\alpha}_{mle}) d\hat{\alpha}_{mle} + \int_{\bar{R}} \left(k(\hat{\alpha}_{mle} - \alpha_0) + (\alpha_0 - \alpha)\right) f(\hat{\alpha}_{mle}) d\hat{\alpha}_{mle}$$

Where, $\bar{R}$ is the complement region of R in real space and $f(\hat{\alpha})$ is a p.d.f. of $\hat{\alpha}$ with the following form

$$f(\hat{\alpha}_{mle}, \alpha) = \begin{cases} \dfrac{\left(\frac{n\alpha}{\hat{\alpha}_{mle}}\right)^{n+1} e^{-\frac{n\alpha}{\hat{\alpha}_{mle}}}}{\Gamma(n).n\alpha} & ; \hat{\alpha}_{mle} > 0, \alpha > 0 \\ 0 & \text{o.w.} \end{cases} \quad (12)$$

We, conclude,

$$Bias(\tilde{\alpha}_{ssi}|\alpha; R) = \alpha\left\{k\frac{1}{n-1} + k(\zeta - 1) + (\zeta - 1) - nkJ_1(a^*, b^*) - kJ_0(a^*, b^*) - k(\zeta - 1)J_0(a^*, b^*)\right\} \quad (13)$$

Where,

$$J_l(a^*, b^*) = \int_{a^*}^{b^*} \frac{1}{y^l} \frac{y^{n-1}e^{-y}}{\Gamma(n)} dy \ ; l = 0,1,2, \dots \quad (14)$$

$$\text{Also, } \zeta = \frac{\alpha_0}{\alpha}, y = \frac{n\alpha}{\hat{\alpha}_{mle}}, a^* = \frac{\mathcal{X}^2\left(\frac{\Delta}{2},2n\right)}{2\zeta} \ \& \ b^* = \frac{\mathcal{X}^2\left(1-\frac{\Delta}{2},2n\right)}{2\zeta}$$

The Bias ratio [B (.)] of $\tilde{\alpha}$ is defined as below:

$$B(\tilde{\alpha}_i) = \frac{Bias(\tilde{\alpha}_{ssi}|\alpha; R)}{\alpha} \ ; i = 1,2 \quad (15)$$

The expression of Mean Squared error (MSE) of $\tilde{\alpha}_{ssi}$ given as:

$$MSE(\tilde{\alpha}_{ssi}|\alpha; R) = E(\tilde{\alpha} - \alpha)^2$$

$$= \alpha^2\left\{k^2\frac{(n+2)}{(n-1)(n-2)} + 2k^2\frac{(\zeta-1)}{n-1} + k^2(\zeta - 1)^2 + 2k(\zeta - 1)\left[\frac{1}{n-1} - (\zeta - 1)\right] + (\zeta - 1)^2 - k^2n^2\left[J_2(a^*, b^*) - \frac{2}{n}\zeta J_1(.) + \frac{\zeta^2}{n^2}J_0(a^*, b^*)\right] - 2kn(\zeta - 1)\left[J_1(a^*, b^*) - \frac{\zeta}{n}J_0(a^*, b^*)\right]\right\} \quad (16)$$

The Efficiency of $\tilde{\alpha}_{ssi}$ relative to the $\hat{\alpha}$ denoted by R. Eff$(\tilde{\alpha}_{ssi}|\alpha; R)$ defined as

$$R.Eff(\tilde{\alpha}_{ssi}|\alpha; R) = \frac{MSE(\hat{\alpha}_{mle})}{MSE(\tilde{\alpha}_i|\alpha; R)} \quad (17)$$

4. Conclusions and numerical results

The computations of Relative Efficiency [R. Eff (.)] and Bias Ratio [B(.)] expression used for the considered estimator $\tilde{\alpha}_{ssi}$ (using MathCAD program). These computations were performed for the constants

$\Delta = 0.01 , 0.05 , 0.1$ & $n = 12 ,20 , 30 , 50 ,100$

$\zeta = 0.25 , 0.50 , 0.75 , 1 , 1.25 , 1.50 , 1.75 ,2$

Some of these computations are displayed in tables (1) and (2) for some samples of these constants. The observation mentioned in the tables leads to the following results:

i) The Relative Efficiency [R. Eff (·)] of $\tilde{\alpha}_{ssi}$ are adversely proportional with small value of Δ especially when $\alpha_0 = \alpha\ (\zeta = 1)$ i.e. Δ=0.01 yield highest efficiency.

ii) The Relative Efficiency [R. Eff (·)] are increasing function with increasing value of $\zeta \leq 1$ and decreasing wise $(\zeta \geq 1)$.

iii) The estimator $\tilde{\alpha}_{ss2}$ is better than the estimator $\tilde{\alpha}_{ss1}$ in the sense of R.Eff. Especially when $\alpha_0 = \alpha$.

iv) Bias ratio [B (·)] of $\tilde{\alpha}_{ssi}$ increases when ζ increases.

v) Bias ratio [B (·)] of $\tilde{\alpha}_{ssi}$ are reasonably small when $\alpha_0 = \alpha$ for each n, Δ, $\tilde{\alpha}$, $\Psi_i(\hat{\alpha})$ and decreases otherwise.

vi) The Relative Efficiency [R.Eff (·)] of $\tilde{\alpha}_{ssi}$ decreasing function when n increases.

vii) The Effective Interval [the value of □ that makes R.Eff. (·) greater than one] using proposed estimator $\tilde{\alpha}_{ssi}$ is at most [0.75, 1.25] for $n \geq 20$ and the interval will be reduced otherwise.

viii) Here the pretest criterion is very closely to the actual value and prevents it far away from it, which get optimal effect of the considered estimator to obtain high efficiency.

ix) The considered estimator $\tilde{\alpha}_{ssi}$ is better than the classical estimator especially when $\alpha_0 \approx \alpha$ which is given the effective of $\tilde{\alpha}$ when given an important weight of prior knowledge. And the augmentation of efficiency may be reach to ten times.

x) The proposed estimator $\tilde{\alpha}_{ssi}$ has smaller MSE than some existing estimators introduced by authors, see for example [5], [6].

Table 1: Shown Bias Ratio [B (·)] and R.E.FF of $\tilde{\alpha}_{ss1}$ w.r.t. Δ, n and ζ

						ζ				
Δ	n	R.Eff(-) B(-)	0.25	0.50	0.75	1	1.25	1.50	1.75	2
0.01	12	R.Eff(-)	0.2262626	0.509091	2.0363643	5.5547121E+13	2.0363647	0.5090912	0.2262628	0.1272728
		B(-)	-0.75	-0.5	-0.25	2.4550324E-10	0.2499999	0.4999999	0.7499998	0.9999997
	20	R.Eff(-)	0.11436	0.25731	1.0292403	1.6823623E+13	1.0292407	0.2573102	0.1143601	0.0643275
		B(-)	-0.75	-0.499999	-0.249999	2.4663571E-10	0.2499999	0.4999998	0.7499997	0.9999996
	30	R.Eff(-)	0.0700602	0.1576355	0.6305424	6.8666881E+12	0.6305427	0.1576357	0.0700603	0.0394089
		B(-)	-0.749999	-0.499999	-0.249999	2.4719598E-10	0.2499998	0.4999997	0.7499996	0.9999994
	50	R.Eff(-)	0.0393046	0.0884354	0.3537419	2.310189E+12	0.3537422	0.0884355	0.0393047	0.0221089
		B(-)	-0.749999	-0.499999	-0.249998	2.4764198E-10	0.2499997	0.4999996	0.7499993	0.999999
	100	R.Eff(-)	0.0186903	0.0420532	0.1682131	5.4906371E+11	0.1682133	0.0420533	0.0186904	0.0105133
		B(-)	-0.749998	-0.499997	-0.249998	2.4797458E-10	0.2499996	0.4999992	0.7499987	0.999998
0.05	12	R.Eff(-)	0.2262711	0.5091407	2.0367545	1.5870183E+8	2.0369438	0.5092437	0.2263333	0.1273148
		B(-)	-0.749986	-0.499976	-0.24998	4.7314558E-7	0.2499644	0.499925	0.7498829	0.9998347
	20	R.Eff(-)	0.1143671	0.2573454	1.029551	4.8015181E+7	1.0297175	0.25743	0.114415	0.0643608
		B(-)	-0.749977	-0.499966	-0.249962	4.7482603E-7	0.249942	0.4998834	0.7498195	0.9997408
	30	R.Eff(-)	0.0700668	0.1576656	0.6308096	1.9587364E+7	0.6309667	0.1577385	0.0701082	0.0394387
		B(-)	-0.749965	-0.499952	-0.249947	4.7565549E-7	0.2499158	0.4998367	0.7497434	0.9996214
	50	R.Eff(-)	0.0393108	0.088463	0.3539641	6.5870659E+6	0.3541139	0.0885238	0.0393479	0.0221365
		B(-)	-0.749941	-0.499922	-0.249921	4.7631398E-7	0.2498685	0.4997501	0.7495869	0.9993746
	100	R.Eff(-)	0.0186961	0.0420795	0.168388	1.5650536E+6	0.1685245	0.0421326	0.0187313	0.0105396
		B(-)	-0.749883	-0.499844	-0.249870	4.7680469E-7	0.2497686	0.4995284	0.7491796	0.99875
0.1	12	R.Eff(-)	0.2263985	0.509808	2.042058	6.8375769E+5	2.0446752	0.5113227	0.2273238	0.1279206
		B(-)	-0.749775	-0.499648	-0.249651	1.1223269E-5	0.2494908	0.4989073	0.748247	0.9974638
	20	R.Eff(-)	0.1144744	0.2578483	1.0337351	2.068458E+5	1.036129	0.2590933	0.1152088	0.0648546
		B(-)	-0.749625	-0.499478	-0.249456	1.1258268E-5	0.2491667	0.4982757	0.7472311	0.9959266
	30	R.Eff(-)	0.0701654	0.1581127	0.6343975	8.4375919E+4	0.636705	0.1591965	0.0708173	0.0398879
		B(-)	-0.749438	-0.499245	-0.249239	1.1275522E-5	0.2487863	0.4975418	0.7459787	0.9939745
	50	R.Eff(-)	0.0394031	0.0888793	0.3569794	2.8373589E+4	0.3592126	0.0898224	0.0400034	0.0225577
		B(-)	-0.749063	-0.498750	-0.248864	1.1289208E-5	0.248088	0.4961229	0.7434179	0.989998
	100	R.Eff(-)	0.0187841	0.0424769	0.1709037	6.7411875E+3	0.1729621	0.043346	0.0193617	0.0109466
		B(-)	-0.748125	-0.4975	-0.248024	1.1299402E-5	0.2465421	0.492482	0.7368748	0.98

Table 2: Shown Bias Ratio [B (·)] and R.E.FF of $\tilde{\alpha}_{ss2}$ w.r.t. Δ, n and ζ

ζ Δ	n	R.Eff(-) B(-)	0.25	0.50	0.75	1	1.25	1.50	1.75	2
0.01	12	R.Eff(-) B(-)	0.2262627 -0.749999	0.509091 -0.499998	2.0363644 -0.2499998	3.730609E+13 2.5866165E-7	2.0363637 0.2500003	0.509091 0.5000002	0.2262627 0.7500002	0.1272727 1.0000001
	20	R.Eff(-) B(-)	0.11436 -0.749998	0.25731 -0.499998	1.0292403 -0.4999998	1.3263478E+13 4.1636656E-7	1.02924 0.2500004	0.25731 0.5000003	0.11436 0.7500001	0.0643275 1
	30	R.Eff(-) B(-)	0.0700603 -0.749998	0.1576356 -0.499998	0.6305423 -0.2499995	5.8614713E+12 6.140354E-7	0.6305421 0.2500006	0.1576355 0.5000003	0.0700602 0.75	0.0394089 0.9999998
	50	R.Eff(-) B(-)	0.0393047 -0.749996	0.0884355 -0.499997	0.3537418 -0.2499992	2.1010475E+12 1.0097779E-6	0.3537417 0.2500008	0.0884354 0.5000002	0.0393046 0.7499997	0.0221089 0.9999995
	100	R.Eff(-) B(-)	0.0186903 -0.749992	0.0420533 -0.499995	0.168213 -0.249999	5.2363022E+11 1.9995889E-6	0.168213 0.2500011	0.0420533 0.4999997	0.0186903 0.7499993	0.0105133 0.999999
0.05	12	R.Eff(-) B(-)	0.2263006 -0.749937	0.5091686 -0.499915	2.0367713 -0.2498612	1.0873115E+8 1.5462682E-4	2.036402 0.2501419	0.5091281 0.5001051	0.2262848 0.7500561	0.1272873 1.0000071
	20	R.Eff(-) B(-)	0.1143906 -0.749900	0.2573757 -0.499906	1.0295146 -0.2497935	3.8321147E+7 2.4925299E-4	1.0293432 0.2502133	0.257352 0.5001185	0.1143828 0.7500145	0.0643417 0.9999328
	30	R.Eff(-) B(-)	0.0700877 -0.749853	0.1576973 -0.499892	0.6307665 -0.2497277	1.6858696E+7 3.6782445E-4	0.6306689 0.2502867	0.1576803 0.5000979	0.0700836 0.7499386	0.0394229 0.9998419
	50	R.Eff(-) B(-)	0.0393299 -0.749759	0.0884929 -0.499837	0.3539387 -0.2496511	6.0207796E+6 6.0518541E-4	0.3538862 0.2503836	0.0884836 0.4999947	0.0393282 0.749796	0.0221224 0.9996966
	100	R.Eff(-) B(-)	0.018714 -0.749525	0.0421069 -0.499681	0.1684111 -0.2496392	1.49632E+6 1.1988333E-3	0.1683786 0.2503988	0.0421041 0.4997269	0.0187134 0.7495381	0.0105263 0.9993813
0.1	12	R.Eff(-) B(-)	0.2268725 -0.748991	0.5103651 -0.498884	2.0422692 -0.2480044	4.70642E+5 2.3402726E-3	2.0368266 0.2520587	0.50968 0.5013641	0.2266258 0.7505508	0.1275121 0.9998055
	20	R.Eff(-) B(-)	0.114851 -0.748395	0.2583988 -0.498701	1.0333182 -0.2471292	1.6561784E+5 3.774882E-3	1.0307555 0.2530074	0.2579939 0.5013405	0.1147361 0.7497749	0.06456 0.9986414
	30	R.Eff(-) B(-)	0.0705019 -0.747647	0.1586417 -0.498346	0.6339942 -0.2463545	7.2788168E+4 5.5723072E-3	0.6324779 0.2538999	0.1583741 0.5008081	0.0704426 0.7486024	0.0396352 0.9973122
	50	R.Eff(-) B(-)	0.0397114 -0.746148	0.0893627 -0.497396	0.3569361 -0.2455987	2.5971676E+4 9.1703019E-3	0.3560413 0.2548447	0.089231 0.4991115	0.0396864 0.7465564	0.0223268 0.9951221
	100	R.Eff(-) B(-)	0.0190749 -0.742399	0.0429244 -0.494899	0.1715282 -0.2455724	6.4499247E+3 0.0181688	0.1709631 0.253987	0.0428839 0.4953653	0.0190645 0.742604	0.0107246 0.990101

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