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Research paper

# A comparative study of Adomain decomposition method and the new integral transform "Elzaki transform"

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## Abstract

In this article, we present a comparative study between Adomain decomposition method and the new integral transform "Elzaki Transform". We use the methods to solve the linear Partial differential equations with constant coefficients.

***Keywords:*** *Adomain Decomposition Method; Elzaki Transform; Linear Partial Differential Equation.*

## 1. Introduction

There are many linear and Nonlinear Partial differential equations which are quite useful and applicable in engineering and physics. Such as the well-known, heat equations, wave equations and Laplace equations [1-5], etc. Linear and Nonlinear Partial differential equations are generally difficult to be solved and their exact solution are difficult to be obtained. The exact solution and numerical solutions of this kind of equations play an important role in physical science and in engineering fields; therefore, there have been attempts to develop new techniques for obtaining analytical solutions which reasonably approximate the exact solutions. In recent years, several such techniques have been drawn particular attention, such as Hirtoa's bilinear method [6], the homogeneous balance method [7], [8] the inverse scattering method [9], the Adomain decomposition method, the variational iterational method [10], Fourier Transform, Laplace Transform, etc. the Adomain decomposition method (ADM) developed by Adomain in [11], [12], and used heavily in the Literature in [13-20] and the reference therein. The New Integral transform "Elzaki Transform" was first introduce by Tarig Elzaki in [21] and used heavily in the literature in [22-26] and the reference therein .the main objective is to introduce a comparative study to solve linear partial differential equations by using Adomain decomposition method and Elzaki transform. The plane of the paper is as follows: In section 2, we introduce the basic idea of Adomain decomposition method, then, Elzaki Transform in 3, Application in 4 and conclusion in 5, respectively.

## 2. Adomain decomposition method

Adomain decomposition method [13], [14] define the unknown function $u(x)$ by an infinite series

$$u(x) = \sum_{n=0}^{\infty} u_n(x), \tag{1}$$

Where the components $u_n(x)$, are usually determined recurrently. The nonlinear operator $F(u)$ can be decomposed into an infinite series of polynomials given by

$$F(u) = \sum_{n=0}^{\infty} A_n \tag{2}$$

Where $A_n$ are the so called Adomain polynomial of $u_0, u_1 u_2, \dots, u_n$ defined by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F(\lambda^i u_i)\right]_{\lambda=0}, n = 0,1,2, \dots. \tag{3}$$

Or equivalently

$A_0 = F(u_0),$

$A_1 = u_1 F'(u_0),$
$A_2 = u_2 F'(u_0) + \frac{1}{2} u_1^2 F''(u_0),$
$A_3 = u_3 F'(u_0) + u_1 u_2 F''(u_0) + \frac{1}{3} u_1^3 F'''(u_0),$
$$A_4 = u_4 F'(u_0) + \left(u_1 u_3 + \frac{1}{2} u_2^2\right) F''(u_0) + \frac{1}{2} u_1^2 u_2 F'''(u_0) + \frac{1}{24} u_1^4 F^{(iv)}(u_0) \quad (4)$$
$\vdots$

It is now well known that these polynomials can be generated for all classes of nonlinear according to specific algorithms defined by (3). Recently, an alternative algorithm for constructing Adomain polynomials has been developed by Wazwaz [16].

This powerful technique handles both linear and nonlinear equations in unified manner without any need for the so called Adomain polynomials .however, Adomin decomposition method provides the component of the exact solution, where these components should follow the summation given in (1), whereas ADM requires the evaluation of the Adomain polynomials that mostly require tedious algebraic work.

# 3. Elzaki transform

A new transform called the Elzaki transform defined for function of exponential order we consider functions in the set **A,** defined by: [14]

$$\mathbf{A} = \left\{ f(t) : \exists M, k_1, k_2 > 0, |f(t)| < M e^{\frac{|t|}{k_j}}, \text{if } t \in (-1)^j \times [0, \infty) \right\} \quad (5)$$

For a given function in the set M must be finite number, $k_1, k_2$ may be finite or infinite.

Elzaki transform which is defined by the integral equation

$$E(f(t)) = \breve{T}(v) = v \int_0^\infty f(t) e^{-\frac{v}{t}} dt, t \geq 0, k_1 \leq v \leq k_2 \quad (6)$$

## 3.1. Elzaki transform of some functions

$$E(1) = \breve{T}(v) = v \int_0^\infty 1. e^{-\frac{v}{t}} dt = v \left[-ve^{-\frac{v}{t}}\right]_0^\infty = v^2.$$

$$E(e^{at}) = \breve{T}(v) = v \int_0^\infty e^{at} e^{\frac{-v}{t}} dt = \frac{v^2}{1 - av}, \quad E(e^{-at}) = v \int_0^\infty e^{-at} e^{\frac{-v}{t}} dt = \frac{v^2}{1 + av}.$$

$E(sin(at)) = \breve{T}(v) = \frac{av^3}{1+a^2v^2}, E(sinh(at)) = \frac{av^3}{1-a^2v^2}$ .
$E(cos(at)) = \breve{T}(v) = \frac{v^2}{1+a^2v^2}, E(cosh(at)) = \frac{v^2}{1-av^2}.$

**Theorem:** [21] *if* $E[u(t)] = \breve{T}(v)$ *then*

1) $E\left(\frac{du}{dt}\right) = E[u'(t)] = \frac{\breve{T}(v)}{v} - vu(0).$
2) $E\left(\frac{d^2u}{dt^2}\right) = E[u''(t)] = \frac{\breve{T}(v)}{v^2} - u(0) - vu'(0).$
3) $E\left(\frac{d^nu}{dt^n}\right) = E[u^n(t)] = \frac{\breve{T}(v)}{v^n} - \sum_{k=0}^{n-1} v^{2-n+k} \ u^k(0).$

# 4. Applications

## 4.1. Example 1

Considers the following the first order initial value problem:

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \quad (7)$$
$u(x, 0) = 6e^{-3x}$ And u is bounded for $x > 0$ , $t > 0$

### 4.1.1. Use Adomain decomposition method

We first rewrite Eq. (7) in an operator L is

$$L_t u = \frac{1}{2} L_x u - u \quad (8)$$

Where the differential operators $L_t$& $L_x$ are

$$L_t(\cdot) = \frac{\partial}{\partial t}(\cdot)\,, L_x(\cdot) = \frac{\partial}{\partial x}(\cdot) \quad (9)$$

The inverse ${L_t}^{-1}$ are assumed as an integral operator given by

$$L_t^{-1}(\cdot) = \int_0^t (\cdot)dt \quad (10)$$

Appling the inverse operator ${L_t}^{-1}$ on both sides of (8) and using initial condition we find

$$u(x,t) = 6e^{-3x} + \frac{1}{2}L_t^{-1}[L_x u] - L_t^{-1}[u] \quad (11)$$

Substituting (1) into the function equation (11) give

$$\sum_{n=0}^{\infty} u_n(x,t) = 6e^{-3x} + \frac{1}{2}L_t^{-1}\left[L_x[\sum_{n=0}^{\infty} u_n(x,t)]\right] - L_t^{-1}[\sum_{n=0}^{\infty} u_n(x,t)] \quad (12)$$

This can be rewrite at the form

$$u_0 + u_1 + u_2 + \cdots = 6e^{-3x} + \frac{1}{2}L_t^{-1}[L_x(u_0 + u_1 + u_2 + \cdots)] - L_t^{-1}[u_0 + u_1 + u_2 + \cdots] \quad (13)$$

In view of (12), the following recursive relation

$$u_0 = 6e^{-3x}$$

$$u_{k+1}(x,t) = \frac{1}{2}L_t^{-1}[L_x(u_k)] - L_t^{-1}[u_k],\ \ k \ge 0 \quad (14)$$

Follows immediately. Consequently, we obtain

$$\begin{aligned} u_0 &= 6e^{-3x}, \\ u_1 &= \frac{1}{2}L_t^{-1}[L_x(6e^{-3x})\,] - L_t^{-1}[6e^{-3x}] = -15te^{-3x}, \\ u_2 &= \frac{1}{2}L_t^{-1}[L_x(-15te^{-3x})\,] - L_t^{-1}[-15te^{-3x}] = \frac{15}{4}t^2e^{-3x} \\ u_3 &= \frac{1}{2}L_t^{-1}\left[L_x(-\frac{15}{4}t^2e^{-3x})\,\right] - L_t^{-1}\left[-\frac{15}{4}t^2e^{-3x}\right] = -\frac{25}{8}\,t^3e^{-3x} \\ &\vdots \end{aligned} \quad (15)$$

Finally, using (1) we obtain the solution in series form:

$$u(x,t) = u_0 + u_1 + u_2 + \cdots$$

That is

$$u(x,t) = 6e^{-3x} - 15te^{-3x} + \frac{15}{4}t^2e^{-3x} - \frac{25}{8}\,t^3e^{-3x} + \cdots \quad (16)$$

$$u(x,t) = 6e^{-3x}(1 - \frac{15}{6}t + \frac{15}{24}t^2 - \frac{25}{48}t^3 + \cdots)$$

The exact solution is given by

$$u(x,t) = 6e^{-3x}.\,e^{-2t}.$$

### 4.1.2. Elzaki transform

Let U be the ELzaki transform of u .then, taking the ELzaki transform of (7) we have

$$\frac{dU(x,v)}{dx} - \left(\frac{2}{v} + 1\right)U(x,v) = -12ve^{-3x} \quad (17)$$

This is the linear ordinary differential equation

The integration factor is

$$p = \exp\left(\int -\left(\frac{2}{v} + 1\right)dx\right) = e^{-\left(\frac{2}{v}+1\right)x} \quad (18)$$

Therefore

$$U(x,v) = \frac{12v^2}{2+4v}e^{-3x} + ce^{\left(\frac{2}{v}+1\right)x} \quad (19)$$

Since U is bounded, C should be zero .Taking the inverse ELzaki transform we have:

$$u(x,t) = 6e^{-2t}\,.\,e^{-3x} \quad (20)$$

## 4.2. Example 2

Consider the Laplace equation

$$u_{xx} + u_{tt} = 0\,, u(x,0) = 0\,, u_t(x,0) = \cos x,\ \ x,t > 0 \quad (21)$$

### 4.2.1. Use Adomain decomposition method

We first rewrite Eq. (21) in an operator L is

$$L_t u = -L_x u \quad (22)$$

Where the differential operators $L_t$& $L_x$ are

$$L_t(\cdot) = \frac{\partial^2}{\partial t^2}(\cdot), L_X(\cdot) = \frac{\partial^2}{\partial x^2}(\cdot), \quad (23)$$

The inverse ${L_t}^{-1}$ are assumed as an integral operator given by

$${L_t}^{-1}(\cdot) = \int_0^t \int_0^t (\cdot)dtdt, \quad (24)$$

Appling the inverse operator $L_t^{-1}$ on both sides of (22) and using initial condition we find

$$u(x,t) = t\cos x - L_t^{-1}[L_x u] \tag{25}$$

Substituting (1) into the function equation (25) give

$$\sum_{t=0}^{\infty} u_n(x,t) = t\cos x - L_t^{-1}[L_x(u_n(x,t))] \tag{26}$$

This can be rewrite at the form

$$u_0 + u_1 + u_2 + u_3 + \cdots = t\cos x - L_t^{-1}[L_x(u_0 + u_1 + u_2 + u_3 + \cdots)] \tag{27}$$

In view of (26), the following recursive relation

$$u_0(x,t) = t\cos x$$

$$u_{k+1}(x,t) = -L_t^{-1}[L_x(u_k(x,t))], k \geq 0 \tag{28}$$

Follows immediately. Consequently, we obtain

$$u_0 = t\cos x,$$

$$u_1 = -L_x^{-1}[L_x(t\cos x)] = -\frac{t^3}{3!}\text{Cos } x,$$

$$u_2 = -L_x^{-1}\left[L_x\left(-\frac{t^3}{3!}\text{Cos } x\right)\right] = \frac{t^5}{5!}\text{Cos } x,$$

$$\vdots \tag{29}$$

Finally, using (1) we obtain the solution in series form:

$$u(x,t) = u_0 + u_1 + u_2 + \cdots$$

$$= \text{Cos } x\left(t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \frac{t^9}{9!} - \cdots\right)$$

The exact solution is given by

$$u(x,t) = \text{Cos}(x)\text{Sinh}(t). \tag{30}$$

### 4.2.2. Elzaki transform

Let T (v ) be the ELzaki transform of u .Then, taking the ELzaki transform of equation (21)we have:

$$\frac{T(v)}{v^2} - u(x,0) - v\, u_t(x,0) + T''(x,v) = \tag{31}$$

$$v^2 T''(x,v) + T(x,v) = v^3\cos x \quad (32)$$

This is the second order differential equation have the particular, solution in the Form

$$T(x,v) = \frac{v^3\cos x}{v^2D^2+1} = \frac{v^3\cos x}{1-v^2} \text{ Where } D^2 \equiv \frac{d^2}{dx^2} \tag{33}$$

If we take the inverse ELzaki transform for Eq. (33), we obtain solution of Eq (21)
In the form

$$u(x,t) = \text{Cos}(x)\,\text{Sinh}(t) \tag{34}$$

## 4.3. Example 3

Solve the wave equation

$$u_{tt} - 4u_{xx} = 0, u(x,0) = \sin(\pi x), u_t(x,0) = 0, x,t > 0. \tag{35}$$

### 4.3.1. Use Adomain decomposition method

We first rewrite Eq. (35) in an operator L is

$$L_t u = 4L_x u \tag{36}$$

Where the differential operators $L_x$& $L_t$ are

$$L_X(\cdot) = \frac{\partial^2}{\partial x^2}(\cdot), L_t(\cdot) = \frac{\partial^2}{\partial t^2}(\cdot) \tag{37}$$

The inverse $L_t^{-1}$ are assumed as an integral operator given by

$$L_t^{-1}(\cdot) = \int_0^t\int_0^t(\cdot)dtdt, \tag{38}$$

Appling the inverse operator $L_t^{-1}$ on both sides of (36) and using initial condition we find

$$u(x,t) = \sin(\pi x) + 4L_t^{-1}[L_x[u]] \tag{39}$$

Substituting (1) into the function equation (39) give

$$\sum_{t=0}^{\infty} u_n(x,t) = \sin(\pi x) + 4\,L_t^{-1}[L_x(u_n(x,t))] \tag{40}$$

This can be rewrite at the form

$$u_0 + u_1 + u_2 + u_3 + \cdots = \sin(\pi x) + 4L_t^{-1}[L_x(u_0 + u_1 + u_2 + u_3 + \cdots)] \tag{41}$$

In view of (40), the following recursive relation

$$u_0(x,t) = \sin(\pi x),$$

$$u_{k+1}(x,t) = 4L_t^{-1}[L_x(u_k(x,t))], k \geq 0 \tag{42}$$

Follows immediately. Consequently, we obtain

$u_0 = \sin(\pi x),$

$u_1 = 4L_t^{-1}[L_x(\sin(\pi x))] = -4\pi^2 \frac{t^2}{2!}\mathrm{Sin}(\pi x),$

$$u_2(x,t) = 4L_t^{-1}[L_x(-2\pi^2 t^2 \sin(\pi x))] = 4\pi^4 \frac{t^4}{4!}\sin(\pi x), \quad (43)$$

$u_2(x,t) = 4L_t^{-1}\left[L_x\left(4\pi^4 \frac{t^4}{4!}\sin(\pi x)\right)\right] = -4\pi^6 \frac{t^6}{6!}\sin(\pi x),$

$\vdots$

Finally, using (1) we obtain the solution in series form:

$u(x,t) = u_0 + u_1 + u_2 + \cdots$

$$=\sin(\pi x) - 4\pi^2 \frac{t^2}{2!}\mathrm{Sin}(\pi x) + 4\pi^4 \frac{t^4}{4!}\sin(\pi x) - 4\pi^6 \frac{t^6}{6!}\sin(\pi x) \ldots \quad (44)$$

The exact solution is

$$u(x,t) = \sin(\pi x)\cos(2\pi t) \quad (45)$$

### 4.3.2. Elzaki transform

Taking the ELzaki transform for Eq (35) and making use of Conditions we obtain

$$4vT''(x,v) - T(x,v) = -v^2 \sin \pi x \quad (46)$$

This is the second order differential equation we have the particular, solution in the form:

$$T(x,v) = -\frac{v^2 \sin \pi x}{4\,v^2 D^2 - 1} = \frac{v^2 \sin \pi x}{1 + (2\pi)^2 v^2} \quad (47)$$

Now we take the inverse ELzaki transform to find the particular solution of (35) in
The form

$$u(x,t) = \mathrm{Sin}(\pi x)\mathrm{Cos}(2\pi t). \quad (48)$$

## 4.4. Example 4

Consider the homogeneous heat equation in one dimension in a normalized form

$$4u_t = u_{xx}, u(x,0) = \sin\frac{\pi}{2}x, x, t > 0 \quad (49)$$

### 4.4.1. Use Adomain decomposition method

We first rewrite Eq. (49) in an operator L is

$$L_t u = \frac{1}{4}L_{xx}u \quad (50)$$

Where the differential operators $L_t$& $L_{xx}$ are

$$L_t(\cdot) = \frac{\partial}{\partial t}(\cdot), L_{xx}(\cdot) = \frac{\partial^2}{\partial x^2}(\cdot) \quad (51)$$

The inverse $L_t^{-1}$ are assumed as an integral operator given by

$$L_t^{-1}(\cdot) = \int_0^t (\cdot)\,dt \quad (52)$$

Appling the inverse operator $L_t^{-1}$ on both sides of (50) and using initial condition we find

$$u(x,t) = \mathrm{Sin}\frac{\pi}{2}x + \frac{1}{4}L_t^{-1}[L_{xx}[u(x,t)]] \quad (53)$$

Substituting (1) into the function equation (53) give

$$\sum_{t=0}^{\infty} u_n(x,t) = \sin\frac{\pi}{2}x + \frac{1}{4}L_t^{-1}[L_{xx}(u_n(x,t))] \quad (54)$$

This can be rewrite at the form

$$u_0 + u_1 + u_2 + u_3 + \cdots = \sin\frac{\pi}{2}x + \frac{1}{4}L_t^{-1}[L_{xx}(u_0 + u_1 + u_2 + u_3 + \cdots)] \quad (55)$$

In view of (54), the following recursive relation

$u_0(x,t) = \sin\frac{\pi}{2}x$

$$u_{k+1}(x,t) = \frac{1}{4}L_t^{-1}[L_{xx}(u_k(x,t))], k \geq 0 \quad (56)$$

Follows immediately. Consequently, we obtain

$u_0(x,t) = \sin\frac{\pi}{2}x,$

$u_1(x,t) = \frac{1}{4}L_t^{-1}\left[L_{xx}\left(\sin\frac{\pi}{2}x\right)\right] = -\frac{\pi^2}{16}t\,\mathrm{Sin}\left(\frac{\pi}{2}x\right)$

$$u_2(x,t) = \frac{1}{4}L_t^{-1}\left[L_{xx}\left(\frac{\pi^2}{16}t\sin\frac{\pi}{2}x\right)\right] = (\frac{\pi^2}{16})^2\ \frac{t^2}{2!}\ \mathrm{Sin}\left(\frac{\pi}{2}x\right) \quad (57)$$

$u_3(x,t) = \frac{1}{4}L_t^{-1}\left[L_{xx}\left(\left(\frac{\pi^2}{16}\right)^2\ t^2 \sin\frac{\pi}{2}x\right)\right] = -\left(\frac{\pi^2}{16}\right)^3\ \frac{t^3}{3!}\ \mathrm{Sin}\left(\frac{\pi}{2}x\right)$

$\vdots$

Finally, using (1) we obtain the solution in series form:

$$u(x,t) = u_0 + u_1 + u_2 + \cdots$$

$$=\sin\frac{\pi}{2}x - \frac{\pi^2}{16}t\,\mathrm{Sin}\left(\frac{\pi}{2}x\right) + \left(\frac{\pi^2}{16}\right)^2 \frac{t^2}{2!}\,\mathrm{Sin}\left(\frac{\pi}{2}x\right) - \left(\frac{\pi^2}{16}\right)^3 \frac{t^3}{3!}\,\mathrm{Sin}\left(\frac{\pi}{2}x\right) + \cdots \quad (58)$$

The exact solution is

$$u(x,t) = \mathrm{Sin}\,\frac{\pi}{2}x\left(1 - \frac{\pi^2}{16}t + \left(\frac{\pi^2}{16}\right)^2 \frac{t^2}{2!} - \left(\frac{\pi^2}{16}\right)^3 \frac{t^3}{3!} + \cdots\right) \quad (59)$$

$$u(x,t) = e^{-\frac{\pi^2}{16}t}\,\mathrm{Sin}\,\frac{\pi}{2}x.$$

### 4.4.2. Elzaki transform

By using the ELzaki transform for Eq (49)

$$vT''(x,v) - 4T(x,v) = -4v^2\sin\frac{\pi}{2}x \quad (60)$$

Solve for $T(x,v)$ we find that the particular solution is

$$T(x,v) = \frac{v^2\sin\frac{\pi}{2}x}{1+\frac{\pi^2 v}{16}} \quad (61)$$

And similarly if we take the inverse ELzaki transform for Eq (61), we obtain the Solution of (60) in the form

$$u(x,t) = e^{-\frac{\pi^2}{16}t}\,\mathrm{Sin}\,\frac{\pi}{2}x \quad (62)$$

## 5. Conclusion and discussions

The main goal of this paper is to conduct a comparative study between Adomain decomposition and Elzaki Transform. The two methods are powerful and efficient.

An important conclusion can made here. Adomain decomposition methods for solving linear partial differential equations, the same problems are solved by Elzaki Transform. Adomain decomposition method provides the components of exact solution, where these components should follow the summation given in (1). However, Application of the new transform ‘’Elzaki Transform’’ to Solutions of linear PDEs has been demonstrated.

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