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Pathos Vertex Semientire Graph of a Tree

Venkanagouda M. Goudar

Sri Gouthama Research centre (Affiliated to Kuvempu university),
Department of Mathematics, Sri Siddhartha Institute of Technology,
Tumkur, Karnataka- 572105.

Email: vmgouda@gmail.com

Abstract

In this communications, the concept of the pathos vertex semientire graph of a tree is introduced. We present characterization of graphs whose pathos vertex semientire graph of a tree is planar, outerplanar and minimally nonouterplanar. Also we establish a characterization of graphs whose pathos vertex semientire graph of a tree is noneularian , hamiltonian

Keywords: *Innervertex number, Line graph, Outerplanar, Pathos, Pathoslength, Vertex semientire graph.*

1 Introduction

By graph, we mean a finite, undirected graph without loops or multiple edges. We refer the terminology of [5].
The concept of pathos of a graph G was introduced by Harary[1] as a collection of minimum number of edge disjoint open paths whose union is G. The path number of a graph G is the number of paths in the pathos. The path number of a tree T is equal to k, where 2k is the number of odd degree vertices of T. In addition, the end vertices of each path of any pathos of a tree are of odd degree.
The edgedegree of an edge uv of a tree T is the sum of the degrees of u and v. The pathoslength is the number of edges that lie on a particular path P_i of pathos of T. A pendent pathos is a path P_i of pathos having unit length, which corresponds to a pendent edge in T. A pathosvertex is a vertex in $Pe_v(T)$ corresponding to a path of pathos of T. A regionvertex is a vertex in $Pe_v(T)$ corresponding to a region of T.

The inner vertex number i(G) of a planar graph G is the minimum number of vertices not belonging to the boundary of the exterior region in any embedding of G in the plane. A graph G is said to be minimally nonouterplanar if i(G) = 1.
A new concept of a graph valued functions called the vertex semientire graph $e_v(G)$ of a plane graph G was introduced by Kulli [4] and is defined as the graph whose vertex set can be put in one – to – one correspondence with the vertices and regions of G in such a way that two vertices of $e_v(G)$ are adjacent if and only if the corresponding elements of G are adjacent.
The pathos vertex semientire graph of a tree denoted by $Pe_v(T)$ is the graph whose vertex set is $V(T) \cup P_i \cup r$ and the two vertices are adjacent if and only if they are adjacent vertices, vertices lie on the path of pathos and vertices lie on the regions. Since the system of pathos for a tree is not unique, the corresponding pathos vertex semientire graph of a tree is also not unique.

2 Preliminaries

We need the following results to prove further results.

Theorem 2.1 [Ref 4]: *If G be a connected plane graph then $e_v(G)$ is planar if and only if G is a tree.*

Theorem 2.2 [Ref 3]: *Every maximal outerplanar graph G with p vertices has 2p – 3 edges.*

3 Pathos Vertex Semientire Graph of a Tree

We start with a preliminary result.

Remark 3.1: *For ant tree T,* $T \subseteq e_v(T) \subseteq Pe_v(T)$.

In the following theorem we obtain the number of vertices and edges in a pathos vertex semientire graph of a tree.

Theorem 3.1: *For any (p, q) graph T with k path of pathos, r regions, then pathos vertex semientire graph of a tree $Pe_v(T)$has (p + k + 1) vertices and*

$$p + q + \sum_{i=1}^{k} P_i \text{ edges.}$$

Proof.3.1: By the definition of pathos vertex semientire graph of a tree $Pe_v(T)$, the number of vertices is the union of the vertices, path of pathos and the region

vertices of T as the tree contain one region. Hence the number of vertices of pathos vertex semientire graph of a tree $Pe_v(T)$ is $(p + k + 1)$.
Further, by remark 1, tree T is a sub graph of $Pe_v(T)$, hence degree of regionvertex is the number of vertices of T. Also the degree of pathosvertex P_i is the number of vertices which lies on the path of pathos , which is $\sum_{i=1}^{k} P_i$. Hence the number of vertices of $Pe_v(T) = q + p + \sum_{i=1}^{k} P_i$.

Theorem 3.2: *For any tree T, pathos vertex semientire graph of a tree $Pe_v(T)$ is always planar.*

Proof. 3.2: By the Theorem 1, vertex semientire graph of a tree $e_v(T)$ is always planar. By the definition of pathos vertex semientire graph of a tree $Pe_v(T)$, the regionvertex *r* is adjacent to all vertices of T and the degree of pathosvertex P_i is the number of vertices which lies on the path of pathos of T. Clearly this pathosvertex does not loose their planarity. Hence pathos vertex semientire graph of a tree $Pe_v(T)$ is always planar.

Theorem 3.3.: *For any tree T the pathos vertex semientire graph of a tree $Pe_v(T)$ outerplanar if and only if T is a path P_2.*

Proof. 3.3: Suppose pathos vertex semientire graph of a tree $Pe_v(T)$ is outerplanar. Assume that a tree T is not P_2. If a tree T is P_3, then by the definition of $Pe_v(T)$, $Pe_v(P_3) = W_{1,4.}$, which is nonouterplanar, a contradiction. In general $Pe_v(T)$ contains $W_{1,n}$ as a sub graph, which is nonouterplanar. Hence T must be P_2.
Conversely, suppose T is P_2. By definition of $Pe_v(T)$, the regionvertex *r* is adjacent to two vertices v_1 ,v_2 to form K_3 and a pathosvertex P_i is adjacent to two vertices v_1,v_2 fo K_3 to form $K_4 – x$, which is outerplanar.

Theorem 3.4.: *For any tree T, pathos vertex semientire graph of a tree $Pe_v(T)$ is maximal outerplanar if and only if T is a path P_2.*

Proof. 3.4: Suppose, pathos vertex semientire graph of a tree $Pe_v(T)$ is maximal outerplanar, then $Pe_v(T)$ is connected. Let T be a path P_2, it contains p=2 vertices and q=1 edge. Given $Pe_v(T)$ is maximal outerplanar, by Theorem 2, if has 2p – 3 edges. By the definition of $Pe_v(T)$, $V[Pe_v(P_2)]= 4$ and $E[Pe_v(T)]=5$.

$\Rightarrow 2p-3=q$

$\Rightarrow 2X4-3=5$

$5=5$ *is satisfied*.

Clearly, $T=P_2$ is a nonempty path. Hence necessity is proved.

Theorem 3.5: *For any tree T, pathos vertex semientire graph of a tree $Pe_v(T)$ is minimally nonouterplanar if and only if T is a path P_3.*

Proof. 3.5: Suppose pathos vertex semientire graph of a tree $Pe_v(T)$ is minimally nonouterplanar. Assume that a tree T is not a path P_3. Let $T=P_4$, by the definition of pathos vertex semientire graph of a tree $Pe_v(T)$, the regionvertex r_1 is adjacent to four vertices v_1,v_2,v_3,v_4. Also the pathosvertex is adjacent to these four vertices, each set $\{v_1, v_2, r_1, P_1\}$ forms an induced subgraph as $K_4 - x$. Clearly v_2, v_3 are the inner vertices of $Pe_v(T)$, hence $i[Pe_v(P_4)]=2$, which is not minimally nonouterplanar, a contradiction.

Conversely, suppose a tree T is P_3. By the theorem 5, $Pe_v(P_3) = W_{1,4}$ and $i[W_{1,4}]=1$. Hence $Pe_v(P_3)$ is minimally nonouterplanar.

Theorem 3.6: *For any tree T, pathos vertex semientire graph of a tree $Pe_v(T)$ is non Eulerian.*

Proof. 3.6: Suppose a tree T is a path P_n:$u_1,u_2,\ldots u_n$, $n > 1$. Further $V[e_v(T)= \{u_1,u_2,\ldots u_n,r_1\}$ where r_1 is only one regionvertex. In $Pe_v(T)$ of a path P_n, each set $\{u_1,u_2,r_1,P_1\}$, $\{u_2,u_3,r_1,P_2\}$,… $\{u_{n-1},u_n,r_1,P_n\}$ forms an induced subgraph as $K_4 - x$. Clearly $\deg(u_1) = \deg(u_n)$ = odd in number. Hence $Pe_v(T)$ is nonEulerian.

Suppose a tree T is not a path. By the above case, the degree of pendent vertices is odd and hence $Pe_v(T)$ is nonEulerian. Hence $Pe_v(T)$ is always nonEulerian.

Theorem 3.7: *For any tree T, pathos vertex semientire graph of a tree $Pe_v(T)$ is always Hamiltonian.*

Proof. 3.7: Suppose a tree T is a path P_n:$u_1,u_2,\ldots u_n$, $n > 1$. Clearly T has exactly one path of pathos P_1 and one regionvertex r_1. Further $V[e_v(T)= \{u_1,u_2,\ldots u_n,r_1\}$. Hence in $Pe_v(T)$ of a path P_n, each set $\{u_1,u_2,r_1,P_1\}$, $\{u_2,u_3,r_1,P_1\}$,… $\{u_{n-1},u_n,r_1,P_1\}$ forms an induced subgraph as $K_4 - x$. Clearly $r_1,u_1,P_1,u_2,u_3,\ldots u_n$, r_1 form a cycle containing all the vertices of $Pe_v(T)$. Hence $Pe_v(T)$ is Hamiltonian.

Also if a tree T is not a path and contain n vertices. Let $V[e_v(T)= \{u_1,u_2,\ldots u_n,r_1\}$ where r_1 is only one regionvertex. In $Pe_v(T)$, each set $\{r_1, u_1,u_2,P_1\}$, $\{r_1, u_2,u_3,P_2\}$,…$\{r_1, u_{n-1},u_n,P_n\}$ forms an induced subgraph as $K_4 - x$. Clearly these vertices r_1, u_1, P_1, u_2, u_3, P_2,…r_1 form a cycle containing all the vertices of $Pe_v(T)$ and it is Hamiltonian. Hence $Pe_v(T)$ is always Hamiltonian.

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