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Research Paper

7-cordial labeling of standard graphs

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Abstract

We contribute here some standard results for 7-cordial labeling. It has been proved that fans, friendship graphs, ladders, double fans, double wheels admit 7-cordial labeling.

Keywords: *Abelian Group; 7-Cordial Labeling.*

1. Introduction

Throughout this work, by a graph we mean finite, connected, undirected, simple graph $G = (V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$.
Definition 1.1. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s) If the domain of the mapping is the set of vertices(edges) then the labeling is called a *vertex labeling(an edge labeling.)*
Definition 1.2. Let $< A, * >$ be any Abelian group. A graph $G = (V(G), E(G))$ is said to be *A-cordial* if there is a mapping $f : V(G) \to A$ which satisfies the following two conditions when the edge $e = uv$ is labeled as $f(u) * f(v)$
(i) $|v_f(a) - v_f(b)| \leq 1$; for all $a, b \in A$,
(ii) $|e_f(a) - e_f(b)| \leq 1$; for all $a, b \in A$.
Where
$v_f(a)$=the number of vertices with label a;
$v_f(b)$=the number of vertices with label b;
$e_f(a)$=the number of edges with label a;
$e_f(b)$=the number of edges with label b.

We note that if $A =< Z_k, +_k >$, that is additive group of modulo k then the labeling is known as k-cordial labeling.

Here we consider $A =< Z_7, +_7 >$, that is additive group of modulo 7.

The concept of A-cordial labeling was introduced by Hovey[3] and proved the following results.

- All the connected graphs are 3-cordial.
- All the trees are 3-cordial.

- All the trees are 4-cordial.
- Cycles are k-cordial for all odd k.

Youssef[4] proved the following results.

- The *complete graph* K_n is 4-cordial $\iff n \leq 6$.
- The *complete bipartite graph* $K_{m,n}$ is 4-cordial $\iff morn \cong 2(mod4)$.
- The graph C_n^2 is 4-cordial $\iff n \cong 2(mod4)$.

Here we consider the following definitions of standard graphs.

- The *fan* f_n is $P_n + K_1$.
- The *friendship graph* F_n is one point union of n copies of cycle C_3.
- The *ladder graph* L_n is Planner grids $P_n \times P_m$ for $m = 2$.
- The *double fan* Df_n is obtained by $P_n + 2K_1$.
- The *double wheel* A double wheel graph DW_n of size n can be composed of $2C_n + K_1$. It consists of two cycles of size n where vertices of two cycles are all connected to a central vertex.

For any undefined term in graph theory we rely upon Gross and Yellen[2].

2. Main results

Theorem 2.1 *All the fans f_n are 7-cordial.*
Proof: Let $G = f_n$ be the fan. Let $v_1, v_2, ..., v_n$ be the path vertices of f_n and v_0 be the apex vertex. We note that $|V(G)| = n + 1$ and $|E(G)| = 2n - 1$.

To define 7-cordial labeling $f : V(G) \rightarrow < Z_7, +_7 >$ we consider the following cases.

Case 1: $n \equiv 0, 1, 2, 6(mod7)$
$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 0(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 6. \quad i \equiv 3(\text{mod } 7); \quad 1 \leq i \leq n$

Case 2: $n \equiv 3(mod7)$
$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 0(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 3(\text{mod } 7); \quad 1 \leq i \leq n - 2$
$f(v_{n-1}) = 6;$
$f(v_n) = 4.$

Case 3: $n \equiv 4(mod7)$
$f(v_0) = 0;$

$f(v_i) = 0; \quad i \equiv 0(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 3(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 5;$
$f(v_{n-1}) = 1;$
$f(v_n) = 3.$

Case 4: $n \equiv 5(mod 7)$
$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 0(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 3(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 1;$
$f(v_{n-1}) = 6;$
$f(v_n) = 3.$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for 7-cordial labeling as shown in *Table 2.1*. That is fan admits 7-cordial labeling.

Let $n = 7a + b$, $a, b \epsilon N \cup \{0\}$.

Table 2.1

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) + 1$ $= e_f(3) = e_f(4)$ $= e_f(5) = e_f(6)$
1	$v_f(0) = v_f(1) + 1 = v_f(2)$ $= v_f(3) + 1 = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2)$ $= e_f(3) + 1 = e_f(4) + 1$ $= e_f(5) + 1 = e_f(6) + 1$
2	$v_f(0) = v_f(1) + 1 = v_f(2)$ $= v_f(3) + 1 = v_f(4)$ $= v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2)$ $= e_f(3) + 1 = e_f(4)$ $= e_f(5) + 1 = e_f(6) + 1$
3	$v_f(0) = v_f(1) + 1 = v_f(2)$ $= v_f(3) + 1 = v_f(4)$ $= v_f(5) + 1 = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$ $= e_f(5) + 1 = e_f(6)$
4	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4) + 1$ $= v_f(5) = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$ $= e_f(5) = e_f(6)$
5	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$ $= v_f(5) + 1 = v_f(6)$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2)$ $= e_f(3) + 1 = e_f(4) + 1$ $= e_f(5) + 1 = e_f(6)$
6	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$ $= v_f(5) = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1$ $= e_f(3) = e_f(4)$ $= e_f(5) + 1 = e_f(6)$

Illustration 2.2 The fan f_9 and its 7-cordial labeling is shown in *Figure 2.1*.

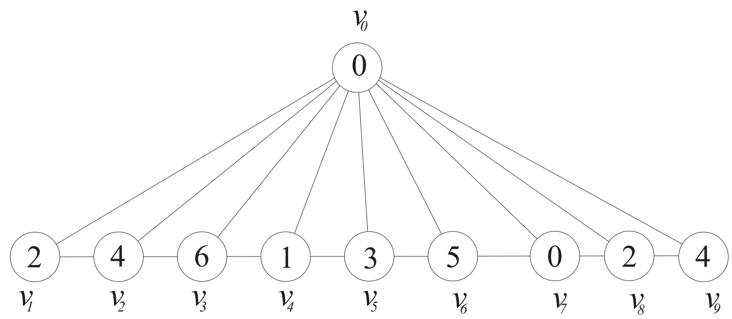

Fig. 2.1: 7-cordial labeling of fan f_9.

Theorem 2.3 *All the friendship graphs F_n are 7-cordial.*
Proof: Let $G = F_n$ be the Friendship graph.Let v_0 be the central vertex. Let $v_1, v_2, ..., v_{2n}$ be partition vertices of n triangles consecutively of F_n and v_0 be the central vertex. We note that $|V(G)| = 2n+1$ and $|E(G)| = 3n$.

To define 7-cordial labeling $f : V(G) \rightarrow < Z_7, +_7 >$ we consider the following cases.

Case 1: $n \equiv 0, 1, 4 (mod 7)$
$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6. \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq 2n$

Case 2: $n \equiv 2 (mod 7)$
$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq 2n-1$
$f(v_{2n}) = 2.$

Case 3: $n \equiv 3 (mod 7)$
$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq 2n-3$
$f(v_{2n-2}) = 2;$
$f(v_{2n-1}) = 4;$

$f(v_{2n}) = 6.$

<u>Case 4</u>: $n \equiv 5(mod 7)$
$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \le i \le 2n-1$
$f(v_{2n}) = 6.$

<u>Case 5</u>: $n \equiv 6(mod 7)$
$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \le i \le 2n-2$
$f(v_{2n-1}) = 2;$
$f(v_{2n}) = 4.$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for 7-cordial labeling as shown in *Table 2.2*. That is friendship graph admits 7-cordial labeling.

Let $n = 7a + b,\ a, b \epsilon N \cup \{0\}$.

Table 2.2

b	**Vertex conditions**	**Edge conditions**
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$ $= e_f(5) = e_f(6)$
1	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1$ $= e_f(3) = e_f(4)$ $= e_f(5) + 1 = e_f(6) + 1$
2	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4) + 1$ $= v_f(5) = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$ $= e_f(5) = e_f(6) + 1$
3	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$ $= v_f(5) = v_f(6)$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) + 1$ $= e_f(3) = e_f(4)$ $= e_f(5) + 1 = e_f(6) + 1$
4	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4)$ $= e_f(5) = e_f(6)$
5	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6)$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4) + 1$ $= e_f(5) + 1 = e_f(6)$
6	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$ $= v_f(5) = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4)$ $= e_f(5) = e_f(6)$

Illustration 2.4 The Friendship graph F_7 and its 7-cordial labeling is shown in *Figure 2.2*.

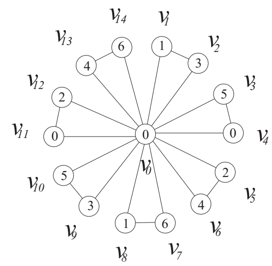

Fig. 2.2: 7-Cordial labeling of friendship graph F_7.

Theorem 2.5 *All the ladders L_n are 7-cordial.*

Proof: Let $G = L_n$ be the ladder.Let $v_1, v_2, ..., v_n$ be vertices of one path p_n and $v_1^{'}, v_2^{'}, ...v_n^{'}$ be vertices of another path $p_n^{'}$. We note that $|V(G)| = 2n$ and $|E(G)| = 3n - 2$.

To define 7-cordial labeling $f : V(G) \rightarrow < Z_7, +_7 >$ we consider the following cases.

Case 1: $n \equiv 0, 1, 2, 6(mod 7)$

$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6. \quad i \equiv 0(\text{mod } 7) \quad 1 \leq i \leq n$
$f(v_i^{'}) = 0; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 0(\text{mod } 7);$
$f(v_i^{'}) = 6. \quad i \equiv 4(\text{mod } 7); \quad 1 \leq i \leq n$

Case 2: $n \equiv 3(mod 7)$

$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n - 2$
$f(v_{n-1}) = 6;$
$f(v_n) = 3.$
$f(v_i^{'}) = 0; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 3(\text{mod } 7);$

$f(v_i^{'}) = 5; \quad i \equiv 0(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 4(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}^{'}) = 4;$
$f(v_n^{'}) = 2.$

<u>Case 3</u>: $n \equiv 4(mod 7)$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 2;$
$f(v_{n-1}) = 5;$
$f(v_n) = 0.$
$f(v_i^{'}) = 0; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 0(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 4(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}^{'}) = 4;$
$f(v_{n-1}^{'}) = 3;$
$f(v_n^{'}) = 6.$

<u>Case 4</u>: $n \equiv 5(mod 7)$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n) = 3.$
$f(v_i^{'}) = 0; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 0(\text{mod } 7);$
$f(v_i^{'}) = 6. \quad i \equiv 4(\text{mod } 7); \quad 1 \leq i \leq n$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for 7-cordial labeling as shown in *Table 2.3*. That is ladder admits 7-cordial labeling.

Let $n = 7a + b,\ a, b \epsilon N \cup \{0\}$.

Table 2.3

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$ $= v_f(5) = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$ $= e_f(5) + 1 = e_f(6)$
1	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4) + 1$ $= e_f(5) + 1 = e_f(6) + 1$
2	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2)$ $= e_f(3) + 1 = e_f(4)$ $= e_f(5) = e_f(6) + 1$
3	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$ $= v_f(5) + 1 = v_f(6)$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$ $= e_f(5) = e_f(6)$
4	$v_f(0) = v_f(1) + 1 = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4) + 1$ $= e_f(5) + 1 = e_f(6)$
5	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$ $= e_f(5) = e_f(6)$
6	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$ $= v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2)$ $= e_f(3) + 1 = e_f(4) + 1$ $= e_f(5) + 1 = e_f(6)$

Illustration 2.6 The ladder L_9 and its 7-cordial labeling is shown in *Figure 2.3.*

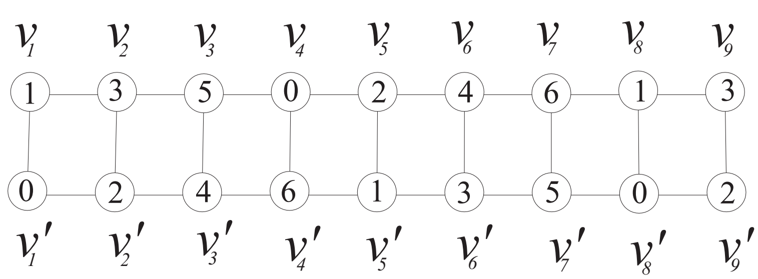

Fig. 2.3: 7-Cordial labeling of ladder L_9.

Theorem 2.7 All the Double fans Df_n are 7-cordial.

Proof: Let $G = Df_n$ be the double fan. Let $v_1, v_2, ..., v_n$ be the path vertices of Df_n and v_0 and $v_0^{'}$ be two apex vertices. We note that $|V(G)| = n + 2$ and $|E(G)| = 3n - 1$.

To define 7-cordial labeling $f : V(G) \to < Z_7, +_7 >$ we consider the following cases.

Case 1: $n \equiv 0, 1, 2, 6(mod 7)$

$f(v_0) = 0;$

$f(v_0^{'}) = 6;$

$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$

$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$

$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$

$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$

$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$

$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$

$f(v_i) = 6. \quad i \equiv 0(\text{mod } 7); \quad 1 \le i \le n$

Case 2: $n \equiv 3(mod 7)$
$f(v_0) = 0;$
$f(v_0^{'}) = 6;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 5;$
$f(v_n) = 3.$

Case 3: $n \equiv 4(mod 7)$
$f(v_0) = 0;$
$f(v_0^{'}) = 6;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 5;$
$f(v_{n-1}) = 3;$
$f(v_n) = 4.$

Case 4: $n \equiv 5(mod 7)$
$f(v_0) = 0;$
$f(v_0^{'}) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-11$
$f(v_{n-10}) = 6;$
$f(v_{n-9}) = 2;$
$f(v_{n-8}) = 5;$
$f(v_{n-7}) = 4;$
$f(v_{n-6}) = 3;$
$f(v_{n-5}) = 1;$
$f(v_{n-4}) = 5;$
$f(v_{n-3}) = 2;$
$f(v_{n-2}) = 3;$
$f(v_{n-1}) = 4;$
$f(v_n) = 6.$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for 7-cordial labeling as shown in *Table 2.4*. That is double fan admits 7-cordial labeling.

Let $n = 7a + b,\ a, b \epsilon N \cup \{0\}$.

Table 2.4

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$ $= e_f(5) = e_f(6)$
1	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6)$	$e_f(0) = e_f(1) = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4) + 1$ $= e_f(5) + 1 = e_f(6) + 1$
2	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6)$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$ $= e_f(5) + 1 = e_f(6) + 1$
3	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4) + 1$ $= v_f(5) = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4) + 1$ $= e_f(5) + 1 = e_f(6) + 1$
4	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4)$ $= v_f(5) = v_f(6)$	$e_f(0) = e_f(1) = e_f(2) + 1$ $= e_f(3) = e_f(4)$ $= e_f(5) + 1 = e_f(6) + 1$
5	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$ $= v_f(5) = v_f(6)$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$ $= e_f(5) = e_f(6)$
6	$v_f(0) = v_f(1) + 1 = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2)$ $= e_f(3) + 1 = e_f(4)$ $= e_f(5) + 1 = e_f(6) + 1$

Illustration 2.8 The Double fan Df_{12} and its 7-cordial labeling is shown in *Figure 2.4*.

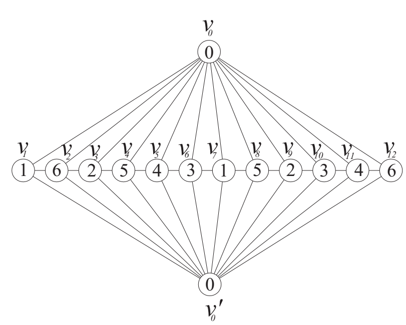

Fig. 2.4: 7-cordial labeling of Double fan Df_{12}.

Theorem 2.9 *All the Double wheels DW_n are 7-cordial.*

Proof: Let $G = DW_n$ be the double wheel.Let v_0 be the central vertex. Let $v_1, v_2, ..., v_n$ be inner rim vertices of DW_n. and $v_1', v_2', ...v_n'$ be outer rim vertices of Dw_n. We note that $|V(G)| = 2n + 1$ and $|E(G)| = 4n$.

To define 7-cordial labeling $f : V(G) \rightarrow < Z_7, +_7 >$ we consider the following cases.

<u>Case 1</u>: $n \equiv 0(mod7)$

$f(v_0) = 0;$

$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$

$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$

$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$

$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$

$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$

$f(v_i) = 5; \quad i \equiv 3 (\text{mod } 7);$
$f(v_i) = 6. \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n$
$f(v_i^{'}) = 0; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3 (\text{mod } 7);$
$f(v_i^{'}) = 6. \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n$

Case 2: $n \equiv 1(mod 7)$
$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3 (\text{mod } 7);$
$f(v_i) = 6. \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n$
$f(v_i^{'}) = 0; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3 (\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n^{'}) = 6.$

Case 3: $n \equiv 2(mod 7)$
$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3 (\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n) = 6.$
$f(v_i^{'}) = 0; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6 (\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3 (\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0 (\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}^{'}) = 3;$
$f(v_n^{'}) = 4.$

Case 4: $n \equiv 3(mod 7)$
$f(v_0) = 6;$
$f(v_i) = 0; \quad i \equiv 4 (\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1 (\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5 (\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2 (\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6 (\text{mod } 7);$

$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 0;$
$f(v_{n-1}) = 5;$
$f(v_n) = 2.$
$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-1$
$f(v_n^{'}) = 4.$

Case 5: $n \equiv 4(mod7)$
$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-2$
$f(v_{n-1}) = 2;$
$f(v_n) = 5.$
$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}^{'}) = 6;$
$f(v_{n-1}^{'}) = 3;$
$f(v_n^{'}) = 4.$

Case 6: $n \equiv 5(mod7)$
$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 2;$
$f(v_{n-1}) = 5;$
$f(v_n) = 6.$
$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$

$f(v_i^{'}) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-4$
$f(v_{n-3}^{'}) = 2;$
$f(v_{n-2}^{'}) = 4;$
$f(v_{n-1}^{'}) = 3;$
$f(v_n^{'}) = 5.$

Case 7: $n \equiv 6(mod7)$
$f(v_0) = 0;$
$f(v_i) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i) = 6; \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n-3$
$f(v_{n-2}) = 2;$
$f(v_{n-1}) = 4;$
$f(v_n) = 6.$
$f(v_i^{'}) = 0; \quad i \equiv 4(\text{mod } 7);$
$f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 7);$
$f(v_i^{'}) = 2; \quad i \equiv 5(\text{mod } 7);$
$f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 7);$
$f(v_i^{'}) = 4; \quad i \equiv 6(\text{mod } 7);$
$f(v_i^{'}) = 5; \quad i \equiv 3(\text{mod } 7);$
$f(v_i^{'}) = 6. \quad i \equiv 0(\text{mod } 7); \quad 1 \leq i \leq n$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for 7-cordial labeling as shown in *Table 2.5*. That is double wheel admits 7-cordial labeling.

Let $n = 7a + b,\ a, b\epsilon N \cup \{0\}$.

Table 2.5

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$ $= e_f(5) = e_f(6)$
1	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6)$	$e_f(0) + 1 = e_f(1) = e_f(2)$ $= e_f(3) + 1 = e_f(4) + 1$ $= e_f(5) = e_f(6)$
2	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4)$ $= v_f(5) + 1 = v_f(6)$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4) + 1$ $= e_f(5) + 1 = e_f(6) + 1$
3	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$ $= v_f(5) = v_f(6)$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1$ $= e_f(3) = e_f(4)$ $= e_f(5) = e_f(6)$
4	$v_f(0) + 1 = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4) + 1$ $= v_f(5) + 1 = v_f(6) + 1$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4) + 1$ $= e_f(5) = e_f(6) + 1$
5	$v_f(0) + 1 = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4) + 1$ $= v_f(5) = v_f(6) + 1$	$e_f(0) = e_f(1) = e_f(2) + 1$ $= e_f(3) = e_f(4)$ $= e_f(5) = e_f(6)$
6	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$ $= v_f(5) = v_f(6) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4)$ $= e_f(5) = e_f(6) + 1$

Illustration 2.10 The Double wheel DW_{10} and its 7-Cordial labeling is shown in *Figure 2.5.*

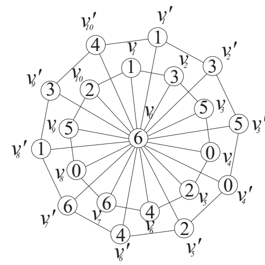

Fig. 2.5: 7-Cordial labeling of Double wheel DW_{10}.

3. Conclusion

Here we have contributed five general results to the theory of 7-cordial labeling. To derive similar results for other graph families is an open area of research.

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