

Investigating the Propagation and Death of Information in Human Subpopulation Networks

Jomar F. Rabajante

Institute of Mathematical Sciences and Physics,
University of the Philippines Los Baños
Email: jfrabajante@up.edu.ph

Abstract

We introduce a conceptual-mathematical model that simulates the spread of information. We represent the states of the actors in the information flow using compartments. The compartment diagram is translated into a system of coupled ordinary differential equations. The diversity of social groups (subpopulations) is incorporated in the model, and the model is applicable for a heterogeneous community. We determine four possible spots that can be influenced to control the information propagation such as (1) controlling the inflow and outflow of people in each subpopulation, (2) regulating the media of communication, (3) influencing the belief system of the actors, and (4) introducing an antithesis to the circulating information.

Mathematics Subject Classification: 91D99

Keywords: *compartment model, correcting rumors, dynamical systems, information propagation, social contagion*

1 Introduction

Social interaction is part of the ecology of humans. Communication is one of the ways species interact. In humans, communication involves transfer of information. Studying information propagation may help create decisions such as in controlling organizational grapevines, minimizing damages caused by rumors, regulating spread of wrong information during disasters, and disseminating important news.

The speed of information propagation depends on the significance of the information to the receivers as well as on the level of awareness of the members of the community (such as in the case of advertised information). The truthfulness of the information based on evidences, the trustworthiness of the spreader of information, the need of any information (whether true or false) just to answer anxieties, and the degree of suggestibility and credulity of the people often influence information spread. The paper [7] discusses how rumors, gossips and urban legends are transmitted.

Various social scientists have already formulated theories about information propagation, and some of these theories are based on intuition and qualitative studies. However, experimental research is impractical and usually infeasible to validate such theories. The application of mathematics, especially dynamical systems, is one of the strategies in investigating social phenomenon — not to replace experimental validation but to observe more the implications of the existing theories and conjectures. Mathematical and computational simulations are commonly used to investigate in a conceptual and theoretical level the complex interactions during information flow.

Rumor spread is one of the common social phenomena that researchers study. Rosnow and Foster [11] explain the basic principles behind rumors and gossips. Various models were formulated to study rumor propagation, such as the Daley-Kendall and Maki-Thompson models which are inspired by the S-I-R (Susceptible-Infected-Removed) disease epidemics model [1, 2, 3, 6, 9]. Serge Galam, the father of sociophysics, as well as Kawachi analyze the dynamics of rumor spread using mathematics [4, 5, 8]. Umali and Rabajante [14], Salvania and Pabico [12], Suo and Chen [13], and Nekovee et al. [10] investigate the spread of information, such as gossips, in complex social networks. The study of information propagation is also useful in analyzing the behavior of financial markets [15].

We propose a new model that considers a community with different *subpopulations* where people have a choice to believe the shared information. This model also incorporates the possibility of introducing an information that opposes the circulating information.

In our model we consider a community of humans. We call the members of the population as *actors*, which are likewise members of subpopulations. A subpopulation can represent a clique in the social network or group of people in a specific geographic location. We assume that the subpopulations are mutually disjoint, that is, no member of one subpopulation is a member of the other.

We also consider the closeness, nature of fidelity and belief systems of each subpopulations. The interaction between two subpopulations may be more frequent than the interaction with the other subpopulations. Moreover, some subpopulations may consider the information from one subpopulation more

believable than the information from the other.

We refer to the actors who have not yet heard the circulating information as *ignorants*, actors who have believed the circulating information as *believers*, and believers that spread the information as *spreaders*. We represented and visualized the information propagation process using a compartment diagram, and the compartments in the diagram represent the *states* of the actors.

2 Assumptions and Compartment Diagram

Our model illustrates the behavior of transferring information from spreaders to other members of the community and the change in the states of the actors. The model is based on the classical I-S-S (Ignorant-Spreader-Stifler) rumor spreading model. The assumptions of the model are as follows:

- The model is deterministic and continuous in state space and time.
- There are only two circulated information that are unaltered during the process. We focus on one main information, and the other information is a total negation (opposite) of the main information. We refer to the opposite information as the *antithesis*.
- The subpopulations (groups, cliques, factions or clusters) are mutually disjoint; and no member can directly transfer to another group during the time frame of the simulation.
- Each subpopulation is homogeneous, that is, the members of a subpopulation share the same characteristics (e.g., ability to interact, fidelity, credulity, suggestibility).
- Rates of flow from one state to another follow the conservation law, that is, all actors in the model are accounted.
- All new members of the community are ignorants.
- Per unit of time, a person can only listen to a single spreader, but a spreader can transfer information to several people simultaneously (such as in the case of broadcast information). A spreader is always active in sharing the information.
- Believers of the main information do not spread the antithesis as well as non-believers do not spread the main information.
- A person decides to believe or not to believe the information immediately once the information is conveyed.

Let us define the following sets and parameters for $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, n$, where n is the number of mutually disjoint subpopulations in the community (note that P_i , P_iCS , P_iB and P_iNB compose the subpopulation i):

P_i : the set of ignorants in subpopulation i ;

P_iCS : the set of ignorants in subpopulation i having contact with spreaders of main information;

P_iB : the set of individuals in subpopulation i believing the main information;

P_iNB : the set of individuals in subpopulation i not believing the main information (non-believers);

a_i : the constant average number of new members of subpopulation i per unit of time t ;

b_i : the expected proportion of P_i exiting the community per unit of time;

c_i : the expected proportion of P_i having contact with spreaders of main information from all subpopulations per unit of time;

$prob_{ij}$: the probability that a spreader of main information from subpopulation j will have a contact with an actor from subpopulation i per unit of time;

$qrob_{ij}$: the probability that a spreader of antithesis from subpopulation j will have a contact with a believer in P_iB per unit of time;

S_{ji} : the average proportion of P_jB that are spreaders of main information who can interact with actors from subpopulation i at time t ;

Z_{ji} : the average proportion of P_jNB that are spreaders of antithesis who can interact with believers from P_iB at time t ;

L_{ij} : the average number of members of subpopulation i that a spreader of main information from subpopulation j can interact with per unit of time;

V_{ij} : the average number of members of subpopulation i that a spreader of antithesis from subpopulation j can interact with per unit of time;

d_i : the probability that a person from P_iCS will believe the conveyed main information (or the expected proportion of P_iCS that will believe the main information per unit of time);

- e_i : the probability that a person from P_iCS will not believe the conveyed main information (or the expected proportion of P_iCS that will not believe the main information per unit of time). Note that $d_i + e_i = 1$;
- pb_i : the probability that a believer in P_iB who had an interaction with a spreader of antithesis will believe the antithesis (or the expected proportion of P_iB who had an interaction with a spreader of antithesis that will believe the antithesis per unit of time);
- f_i : the expected proportion of P_iB exiting the community per unit of time;
- g_i : the expected proportion of P_iNB exiting the community per unit of time;
- h_i : the expected proportion of P_iB who listen and believe the spreaders of antithesis from all subpopulations per unit of time; and
- k_i : the expected proportion of P_iNB having contact with spreaders of main information from all subpopulations per unit of time.

Let $[\cdot]$ denote the cardinality of the set. We represent the states of the actors in the information flow using compartments. The compartment model of our information propagation model where the focus is the main information (not the antithesis) is shown in Figure 1.

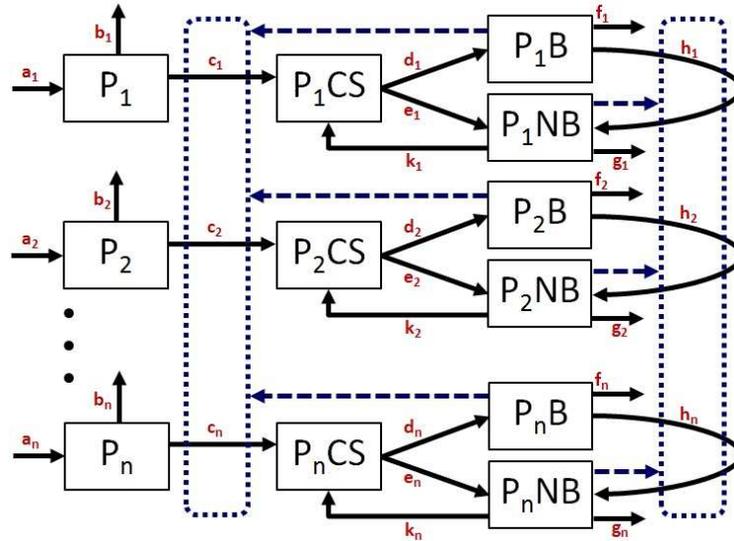


Figure 1: An Information Propagation Compartment Model. Solid lines denote flow of actors from one state to another while dashed lines denote influence.

There are actors who enter and exit P_i , and some members of P_i move to P_iCS . Actors that transfer from P_i to P_iCS are those who hear the main

information shared by the spreaders. The set of spreaders of main information is a subset of the set of believers.

Actors from P_iCS move to either P_iB or P_iNB . Actors who believe the main information transfer to P_iB , or otherwise to P_iNB .

There is a probability that a non-believer in P_iNB may believe the main information after interacting with a spreader. However, there is also a chance that a believer in P_iB may change his belief and transfer to P_iNB after interacting with a spreader of the antithesis. The set of spreaders of antithesis is a subset of the set of non-believers.

Spreaders of the main information or of the antithesis can influence actors in another subpopulation, e.g., a spreader from P_1B can interact and share the main information to an ignorant in P_2 . If $[P_iB] = 0$ for all i then it means the main information is no longer in circulation and we call this as “dead” information.

The formula for c_i is as follows:

$$\begin{aligned} c_i &= \frac{\sum_{j=1}^n (prob_{ij})(S_{ji})([P_jB])(L_{ij})}{[P_i]} \text{ if } [P_i] \neq 0 \\ c_i &= 0 \text{ if } [P_i] = 0 \\ i &= 1, 2, \dots, n. \end{aligned} \tag{1}$$

The contact rate c_i and k_i are proportional to the number of believers present since we define S_j as the average proportion of P_jB that are spreaders of main information at time t . L_{ij} can be interpreted as the broadcasting capacity of a spreader of main information, e.g., if $L_{12} = 4$ then, on the average, a spreader of main information from subpopulation 2 can talk to four ignorants in subpopulation 1.

The product $(prob_{ij})(S_{ji})([P_jB])(L_{ij})$ represents the number of ignorants in subpopulation i having interaction with spreaders of main information from subpopulation j . Since we assume that an actor listens to at most one spreader at time t then $W_{ij_1} \cap W_{ij_2} = \emptyset$ for all $i, j_1 \neq j_2$ (where W_{ij} denotes the set of ignorants in subpopulation i having interaction with spreaders of main information from subpopulation j at time t). It follows that the total number of ignorants in subpopulation i having interaction with spreaders of main information from all subpopulations is $\sum_{j=1}^n (prob_{ij})(S_{ji})([P_jB])(L_{ij})$.

Likewise, the formula for k_i and h_i are

$$k_i = \frac{\sum_{j=1}^n (prob_{ij})(S_{ji})([P_jB])(L_{ij})}{[P_iNB]} \text{ if } [P_iNB] \neq 0$$

$$\begin{aligned} k_i &= 0 \text{ if } [P_iNB] = 0 \\ i &= 1, 2, \dots, n, \text{ and} \end{aligned} \tag{2}$$

$$\begin{aligned} h_i &= \frac{\sum_{j=1}^n (prob_{ij})(Z_{ji})([P_jNB])(V_{ij})(pb_i)}{[P_iB]} \text{ if } [P_iB] \neq 0 \\ h_i &= 0 \text{ if } [P_iB] = 0 \\ i &= 1, 2, \dots, n. \end{aligned} \tag{3}$$

The rate at which an ignorant becomes a believer is $d_i c_i$ and the rate at which a non-believer becomes a believer is $d_i k_i$. Moreover, the rate at which an individual exits P_iB is $f_i + h_i$. Hence, the basic reproductive rate of the spread of the main information is $\frac{d_i c_i + d_i k_i}{f_i + h_i}$. It should be noted that the parameters c_i , k_i and h_i should not be greater than 1.

3 ODE Model and Control Spots

We translate the compartment diagram into an autonomous system of coupled ordinary differential equations (ODEs) with $[P_i]$, $[P_iB]$ and $[P_iNB]$, $i = 1, 2, \dots, n$ as the state variables. The actors in P_iCS , $i = 1, 2, \dots, n$ are in transient states, and we presume that a person decides to believe or not to believe the information immediately once the information is conveyed (i.e., $[P_iCS]$ quickly vanishes at each time t). Thus, for simplicity, we disregard P_iCS in the diagram.

The system of ordinary differential equations that corresponds to our simplified compartment model is

$$\frac{d[P_i]}{dt} = a_i - (b_i + c_i)[P_i] \tag{4}$$

$$\frac{d[P_iB]}{dt} = d_i(c_i[P_i] + k_i[P_iNB]) - (f_i + h_i)[P_iB] \tag{5}$$

$$\frac{d[P_iNB]}{dt} = e_i(c_i[P_i] + k_i[P_iNB]) - (g_i + k_i)[P_iNB] + h_i[P_iB] \tag{6}$$

$$i = 1, 2, \dots, n.$$

Substituting the formula of c_i , k_i and h_i , we now have the following piecewise (conditional) linear system of ODEs:

$$\frac{d[P_i]}{dt} = a_i - b_i[P_i] + \sum_{j=1}^n (prob_{ij})(S_{ji})([P_jB])(L_{ij}), [P_i] \neq 0 \tag{7}$$

$$\frac{d[P_i]}{dt} = a_i, [P_i] = 0 \quad (8)$$

$$\begin{aligned} \frac{d[P_i B]}{dt} &= 2d_i \sum_{j=1}^n (\text{prob}_{ij})(S_{ji})([P_j B])(L_{ij}) - f_i[P_i B] \\ &\quad - \sum_{j=1}^n (\text{qrob}_{ij})(Z_{ji})([P_j NB])(V_{ij})(pb_i), \\ &[P_i] \neq 0, [P_i B] \neq 0, [P_i NB] \neq 0 \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{d[P_i B]}{dt} &= d_i \sum_{j=1}^n (\text{prob}_{ij})(S_{ji})([P_j B])(L_{ij}) - f_i[P_i B] \\ &\quad - \sum_{j=1}^n (\text{qrob}_{ij})(Z_{ji})([P_j NB])(V_{ij})(pb_i), \\ &[P_i] = 0, [P_i B] \neq 0, [P_i NB] \neq 0 \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{d[P_i B]}{dt} &= d_i \sum_{j=1}^n (\text{prob}_{ij})(S_{ji})([P_j B])(L_{ij}) \\ &\quad - \sum_{j=1}^n (\text{qrob}_{ij})(Z_{ji})([P_j NB])(V_{ij})(pb_i), \\ &[P_i] \neq 0, [P_i B] = 0, [P_i NB] \neq 0 \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{d[P_i B]}{dt} &= 2d_i \sum_{j=1}^n (\text{prob}_{ij})(S_{ji})([P_j B])(L_{ij}) - f_i[P_i B], \\ &[P_i] \neq 0, [P_i B] \neq 0, [P_i NB] = 0 \end{aligned} \quad (12)$$

$$\begin{aligned} \frac{d[P_i B]}{dt} &= - \sum_{j=1}^n (\text{qrob}_{ij})(Z_{ji})([P_j NB])(V_{ij})(pb_i), \\ &[P_i] = 0, [P_i B] = 0, [P_i NB] \neq 0 \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{d[P_i B]}{dt} &= d_i \sum_{j=1}^n (\text{prob}_{ij})(S_{ji})([P_j B])(L_{ij}) - f_i[P_i B], \\ &[P_i] = 0, [P_i B] \neq 0, [P_i NB] = 0 \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{d[P_i B]}{dt} &= d_i \sum_{j=1}^n (\text{prob}_{ij})(S_{ji})([P_j B])(L_{ij}), \\ &[P_i] \neq 0, [P_i B] = 0, [P_i NB] = 0 \end{aligned} \quad (15)$$

$$\frac{d[P_i B]}{dt} = 0, [P_i] = 0, [P_i B] = 0, [P_i NB] = 0 \quad (16)$$

$$\begin{aligned} \frac{d[P_i NB]}{dt} &= 2e_i \sum_{j=1}^n (\text{prob}_{ij})(S_{ji})([P_j B])(L_{ij}) - g_i[P_i NB] \\ &\quad - \sum_{j=1}^n (\text{prob}_{ij})(S_{ji})([P_j B])(L_{ij}) \end{aligned} \quad (17)$$

$$\begin{aligned} & + \sum_{j=1}^n (qrob_{ij})(Z_{ji})([P_jNB])(V_{ij})(pb_i), \\ & [P_i] \neq 0, [P_iB] \neq 0, [P_iNB] \neq 0 \\ \frac{d[P_iNB]}{dt} = & e_i \sum_{j=1}^n (prob_{ij})(S_{ji})([P_jB])(L_{ij}) - g_i[P_iNB] \end{aligned} \quad (18)$$

$$\begin{aligned} & - \sum_{j=1}^n (prob_{ij})(S_{ji})([P_jB])(L_{ij}) \\ & + \sum_{j=1}^n (qrob_{ij})(Z_{ji})([P_jNB])(V_{ij})(pb_i), \\ & [P_i] = 0, [P_iB] \neq 0, [P_iNB] \neq 0 \\ \frac{d[P_iNB]}{dt} = & 2e_i \sum_{j=1}^n (prob_{ij})(S_{ji})([P_jB])(L_{ij}) - g_i[P_iNB] \end{aligned} \quad (19)$$

$$\begin{aligned} & - \sum_{j=1}^n (prob_{ij})(S_{ji})([P_jB])(L_{ij}), \\ & [P_i] \neq 0, [P_iB] = 0, [P_iNB] \neq 0 \\ \frac{d[P_iNB]}{dt} = & e_i \sum_{j=1}^n (prob_{ij})(S_{ji})([P_jB])(L_{ij}) \end{aligned} \quad (20)$$

$$\begin{aligned} & + \sum_{j=1}^n (qrob_{ij})(Z_{ji})([P_jNB])(V_{ij})(pb_i), \\ & [P_i] \neq 0, [P_iB] \neq 0, [P_iNB] = 0 \\ \frac{d[P_iNB]}{dt} = & e_i \sum_{j=1}^n (prob_{ij})(S_{ji})([P_jB])(L_{ij}) - g_i[P_iNB] \end{aligned} \quad (21)$$

$$\begin{aligned} & - \sum_{j=1}^n (prob_{ij})(S_{ji})([P_jB])(L_{ij}), \\ & [P_i] = 0, [P_iB] = 0, [P_iNB] \neq 0 \\ \frac{d[P_iNB]}{dt} = & \sum_{j=1}^n (qrob_{ij})(Z_{ji})([P_jNB])(V_{ij})(pb_i), \\ & [P_i] = 0, [P_iB] \neq 0, [P_iNB] = 0 \\ \frac{d[P_iNB]}{dt} = & e_i \sum_{j=1}^n (prob_{ij})(S_{ji})([P_jB])(L_{ij}), \end{aligned} \quad (22)$$

$$\begin{aligned} & [P_i] \neq 0, [P_iB] = 0, [P_iNB] = 0 \\ \frac{d[P_iNB]}{dt} = & 0, [P_i] = 0, [P_iB] = 0, [P_iNB] = 0 \end{aligned} \quad (23)$$

$$i = 1, 2, \dots, n.$$

If $a_i = 0$ for all i , then we will have a non-homogeneous system of ODEs.

However, we can redefine a_i to be proportional to $[P_i] + [P_iB] + [P_iNB]$, that is, the number of new members entering the subpopulation is proportional to the number of actors in the subpopulation. On the other hand, we can redefine $b_i[P_i]$, $f_i[P_iB]$ and $g_i[P_iNB]$ to be constants.

We classify the parameters into four modules — the inflow and outflow module, the communication (for the main information) module, the belief module, and the counterattack module. See Figure 2 for the illustration. The following is the list of the parameters belonging to each module:

- Inflow and outflow module: a_i , b_i , f_i and g_i
- Communication module: c_i and k_i
- Belief module: d_i and e_i
- Counterattack module: h_i

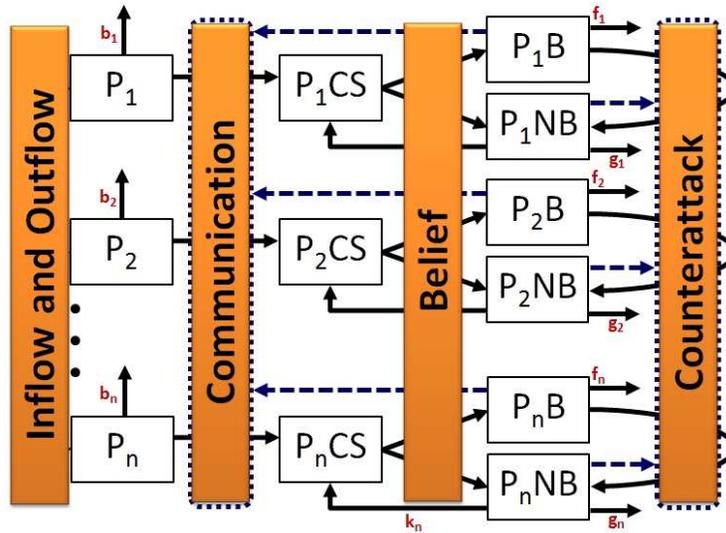


Figure 2: Possible Information Spread Control Spots.

These four modules are control spots that can be influenced to regulate the propagation of the main information. The inflow and outflow module describes the migration of actors in the community. As more new members continually enter the community through P_i , the longer the main information may circulate because P_i is the source of possible new believers. However, it is usually infeasible or impractical to control the migration of people inside a community.

If we increase c_i and k_i in the communication module, more actors will hear the main information and thus, the larger the chance $[P_iB]$ will increase.

Effective advertising and communication media can increase the value of c_i and k_i . On the other hand, in controlling rumor spread, inhibiting communication among actors (e.g., decreasing the value of c_i and k_i) may result to a bigger problem. Absence of communication drives the actors to create other stories to answer their curiosities and anxieties.

If we influence the belief module, such as decreasing d_i and increasing e_i , then we can regulate the flow of people to P_iB and P_iNB . However, it is very difficult to change the belief of people unless effective (usually costly) advertising campaigns and debates are done.

One strategy to decrease the number of believers is to introduce an antithesis to the main information. This strategy entails increased effort in broadcasting the antithesis and convincing the believers. In our model, increasing the number of non-believers may result to an effective antithesis campaign. The Conflict Research Consortium, University of Colorado [16] suggested strategies to control rumors, and one of these strategies is the introduction of an antithesis:

“The key to effective rumor control efforts is an ability to perform three functions. First, some mechanism is needed for determining what rumors are actually circulating. Second, an effective strategy is needed for determining which rumors are true, and which are false. Finally, mechanisms are needed for correcting inaccurate rumors and replacing them with reliable information.”

3.1 Simulation

Consider two subpopulations ($n = 2$). We numerically solved the system (Equations 7 to 23) for a given set of parameters. Figures 6 to 15 show some simulations in time series where the value of a specific parameter is varied.

Some of our numerical simulations (see Figure 3 for an illustration) show that the time series of $[P_iB]$ fluctuates at an early period and levels (usually approaching an equilibria) at a later period.

In some cases, the parameters a_i and b_i in the inflow and outflow module do not significantly affect $[P_iB]$ and $[P_iNB]$ unlike the other parameters. Based on our system (Equations 7 to 23), increasing a_i and decreasing b_i do not necessarily increase the number of believers and non-believers because the parameters a_i and b_i do not appear in $\frac{d[P_iB]}{dt}$ and $\frac{d[P_iNB]}{dt}$. However, a_i and b_i affect $[P_i]$ and as we increase a_i and decrease b_i , the number of ignorants increases. As the number of ignorants increases, the life span of the main information may increase since P_i is the source of possible new believers and spreaders. The effect of the inflow of new members is significantly noticeable when there is minimal number of actors exiting the community (outflow). As

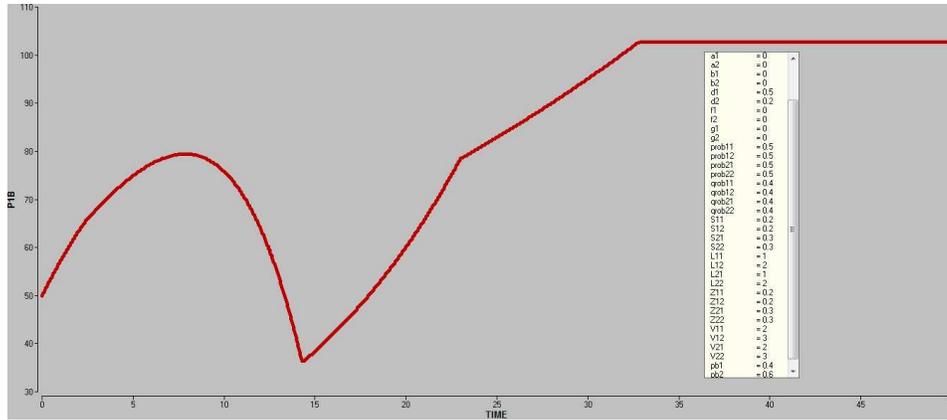


Figure 3: An example where the time series of $[P_i B]$ fluctuates at an early period and levels at a later period.

shown in Figure 4, the change in the value of a_i may induce variation in the behavior of $[P_i B]$ especially when there is inhibited outflow.

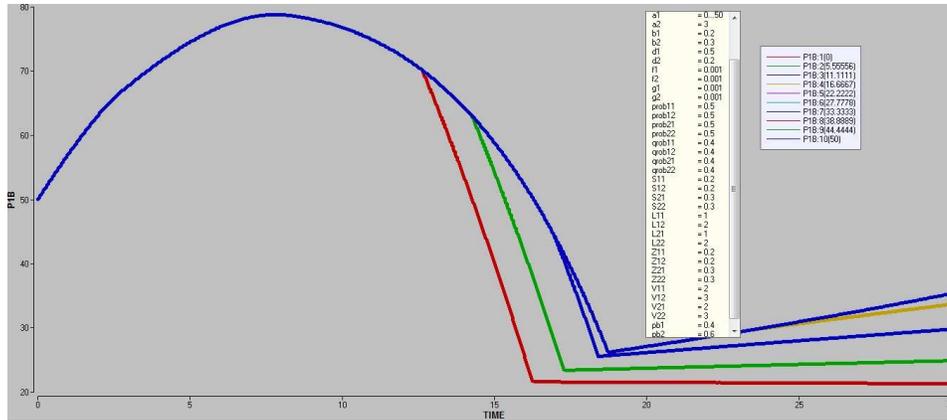


Figure 4: An instance when a_i affects $[P_i B]$.

In our system (Equations 7 to 23) where the number of ignorants is large, the propagation of the main information does not significantly depend on the number of ignorants but on the number of spreaders and their ability to disseminate the information. It also depends on the chance that actors would listen and believe the spreaders. However, if we use another formula for c_i (e.g., c_i is equal to a positive constant) then we can make the number of ignorants matter. In reality, sometimes the number of ignorants may or may not matter in information propagation.

We investigate another case where we assumed c_i , k_i and h_i are constants. This assumption is only valid when $[P_i B]$ is always nonzero if $c_i > 0$ or $k_i > 0$ because if $[P_i B] = 0$ then there will be no spreader of main information.

Solving the equilibrium points $([P_i]^*, [P_iB]^*, [P_iNB]^*)$ of our system (Equations 4 to 6) leads to $[P_i]^* = \frac{a_i}{b_i+c_i}$ where either b_i or c_i is nonzero. It follows that

$$\frac{d_i c_i a_i}{b_i + c_i} + d_i k_i [P_iNB]^* - (f_i + h_i) [P_iB]^* = 0 \tag{24}$$

$$\frac{e_i c_i a_i}{b_i + c_i} + (e_i k_i - g_i - k_i) [P_iNB]^* + h_i [P_iB]^* = 0. \tag{25}$$

which imply that

$$[P_iB]^* = \frac{\frac{d_i c_i a_i}{b_i+c_i} + d_i k_i [P_iNB]^*}{f_i + h_i} \tag{26}$$

$$[P_iNB]^* = \frac{\frac{e_i c_i a_i}{b_i+c_i} (f_i + h_i) + h_i \frac{d_i c_i a_i}{b_i+c_i}}{-((e_i k_i - g_i - k_i)(f_i + h_i) + h_i d_i k_i)}. \tag{27}$$

Notice that it is necessary for either f_i or h_i to be nonzero for the above $[P_iB]^*$ to be defined. It can easily be seen that if either $b_i = c_i = 0$ or $f_i = h_i = 0$ then no equilibrium exists (assuming $a_i > 0, c_i > 0$ and $d_i > 0$). Moreover, $(e_i k_i - g_i - k_i)(f_i + h_i) + h_i d_i k_i < 0$ for $[P_iNB]^*$ to be nonnegative and defined.

From our analytic investigation we can observe that all parameters affect the behavior of our system. Notice that $[P_iB]^*$ and $[P_iNB]^*$ are in terms of all the parameters. However, note that $([P_i]^*, [P_iB]^*, [P_iNB]^*)$ may be stable or unstable given some parameter values.

The representative Jacobian of the system (Equations 4, 5 and 6) where c_i, k_i and h_i are constants is as follows:

$$\begin{bmatrix} -(b_i + c_i) & 0 & 0 \\ d_i c_i & -(f_i + h_i) & d_i k_i \\ e_i c_i & h_i & e_i k_i - g_i - k_i \end{bmatrix}. \tag{28}$$

There are three eigenvalues of the above Jacobian matrix which will give us an idea if $([P_i]^*, [P_iB]^*, [P_iNB]^*)$ is stable. If the real part of the eigenvalues are all negative then the equilibrium point is stable. The eigenvalues are

$$\lambda_1 = -b_i - c_i \tag{29}$$

$$\lambda_2 = -\frac{1}{2}g_i - \frac{1}{2}h_i - \frac{1}{2}k_i - \frac{1}{2}f_i + \frac{1}{2}k_i e_i + \frac{1}{2}A \tag{30}$$

$$\lambda_3 = -\frac{1}{2}g_i - \frac{1}{2}h_i - \frac{1}{2}k_i - \frac{1}{2}f_i + \frac{1}{2}k_i e_i - \frac{1}{2}A \tag{31}$$

where

$$A = \left(k_i^2 e_i^2 - 2k_i^2 e_i - 2f_i g_i + 2f_i h_i - 2g_i h_i - 2f_i k_i + 2g_i k_i - 2h_i k_i + f_i^2 + g_i^2 + h_i^2 + k_i^2 + 4d_i h_i k_i + 2f_i k_i e_i - 2g_i k_i e_i + 2h_i k_i e_i \right)^{\frac{1}{2}}.$$

We can explore other possible formulas for computing c_i , such as hyperbolic and sigmoidal (Hill) equations. However, if we use a hyperbolic or a sigmoidal equation then our system (Equations 4, 5 and 6) will be nonlinear. A possible formula for c_i is

$$c_i = \frac{[P_i] + \sum_{j=1}^n S_{ji}[P_j B]}{1 + [P_i] + \sum_{j=1}^n S_{ji}[P_j B]} \tag{32}$$

Notice that the above formula incorporates the effect of the number of ignorants and spreaders in communicating the main information. Similar formula can also be used for k_i and h_i . A sample simulation is shown in Figure 5 where c_i , k_i and h_i are of hyperbolic-type.

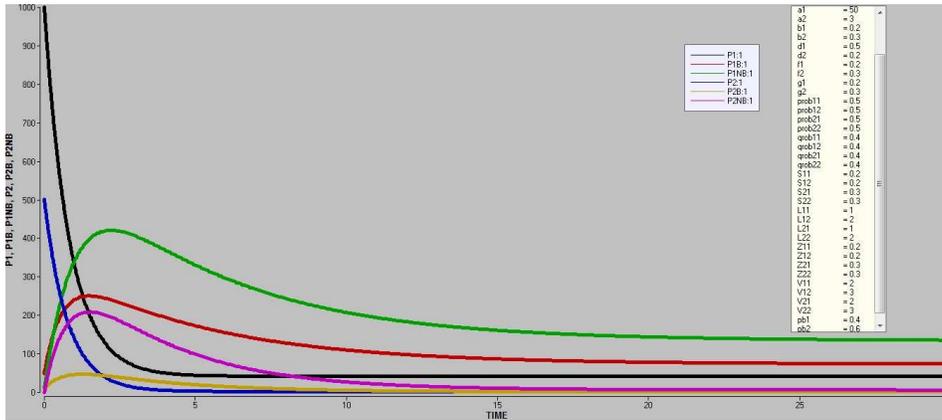


Figure 5: An example simulation where c_i , k_i and h_i are of hyperbolic-type.

4 Concluding Remarks

Even without performing an actual experiment to study information propagation, theoretical and conceptual modeling help us understand the general behavior of the information propagation process. In this research, we determined four possible control spots to regulate information spread — the migration of

actors, the interaction with spreaders, the degree of skepticism of receivers, and the introduction of an opposite information. However, one of the difficult tasks in using theoretical models is the determination of the actual values of the parameters and input values. Furthermore, we did not include the effect of time delays and presence of stochastic noise. We recommend to extend our mathematical model to include more social factors, such as presence of multiple information, to further understand the propagation and death of information.

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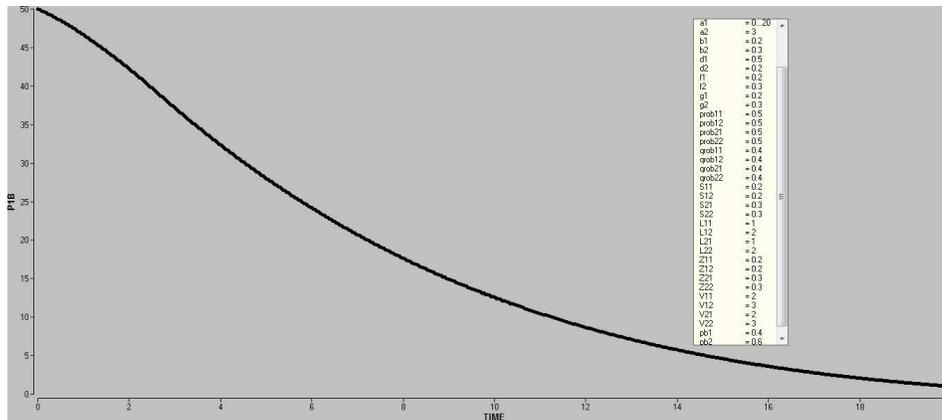


Figure 6: An example where varying the value of a_i does not affect $[P_i B]$.

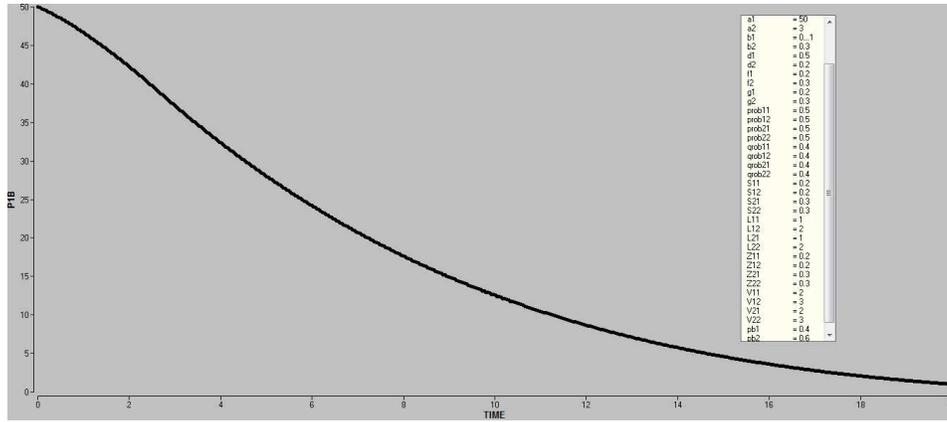


Figure 7: An example where varying the value of b_i does not affect $[P_i B]$.

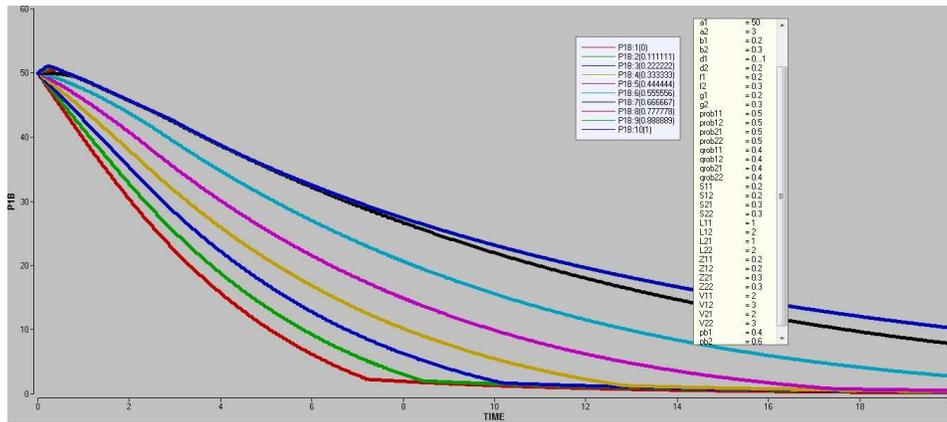


Figure 8: An example where varying the value of d_i affects $[P_i B]$.

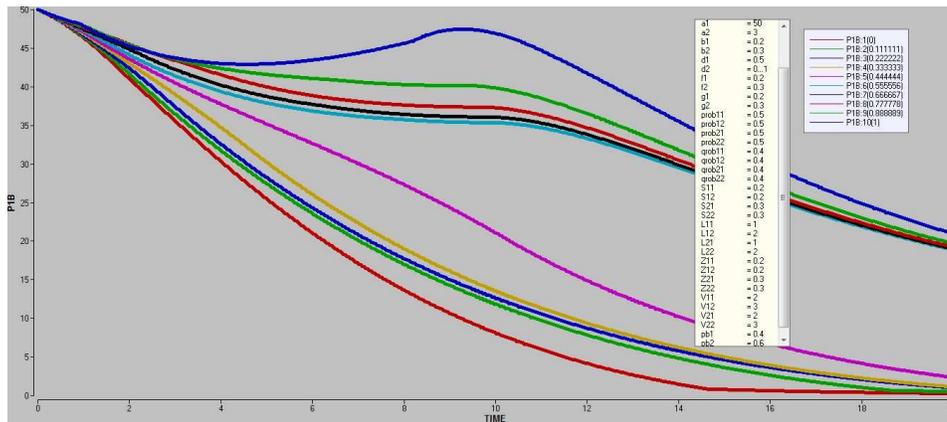


Figure 9: An example where varying the value of d_j $j \neq i$ affects $[P_i B]$.

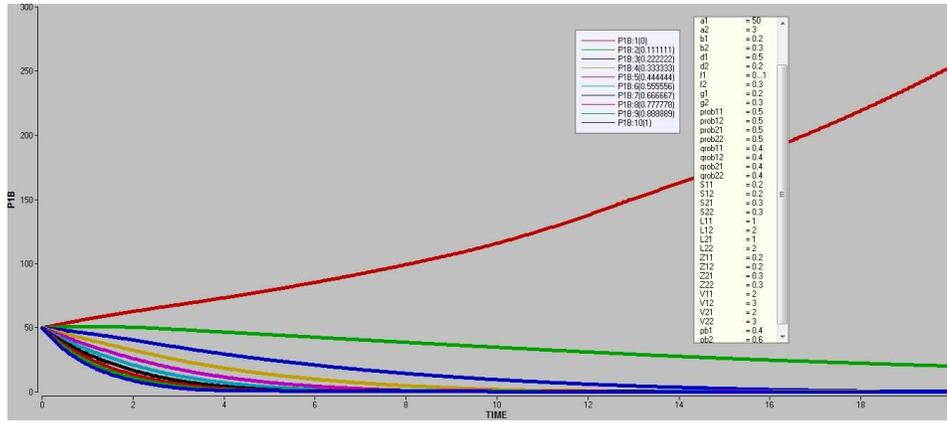


Figure 10: An example where varying the value of f_i affects $[P_iB]$.

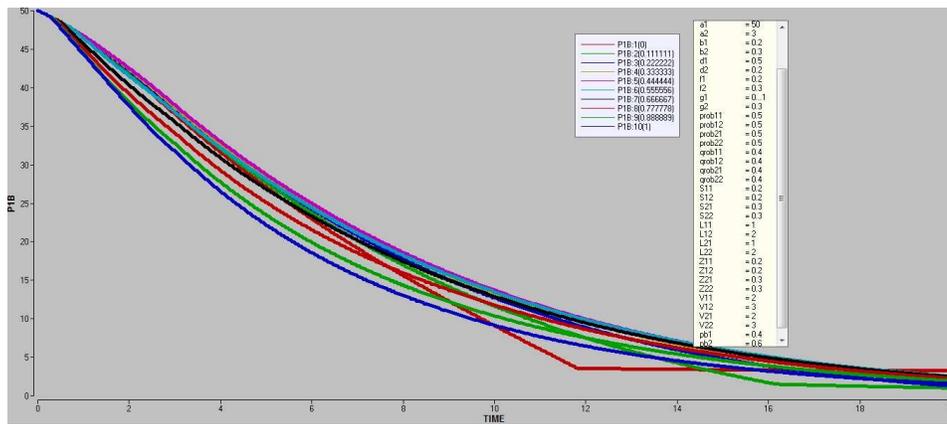


Figure 11: An example where varying the value of g_i affects $[P_iB]$.

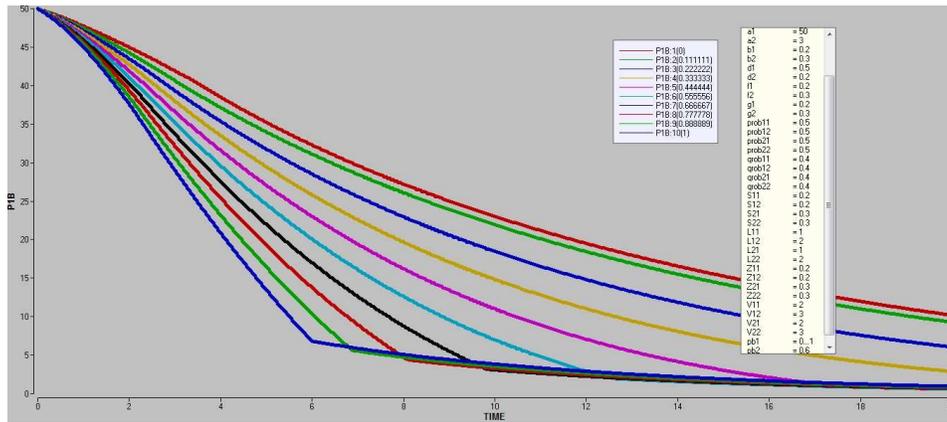


Figure 12: An example where varying the value of pb_i affects $[P_iB]$.

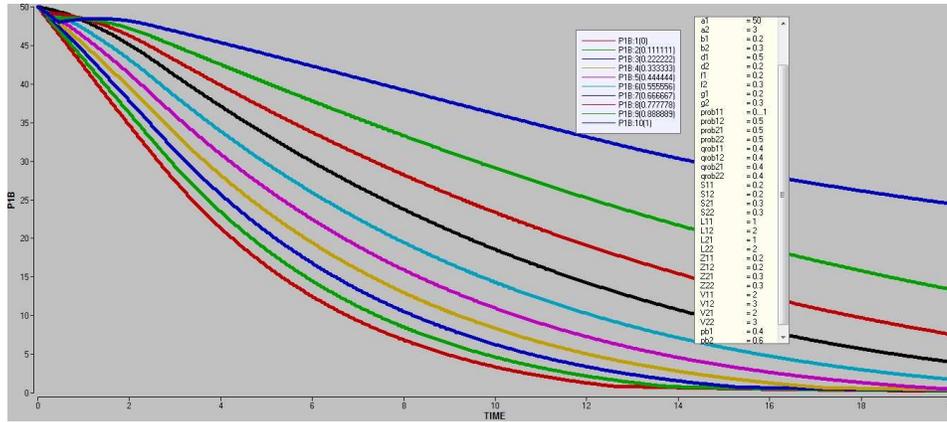


Figure 13: An example where varying the value of $prob_{ii}$ affects $[P_i B]$.

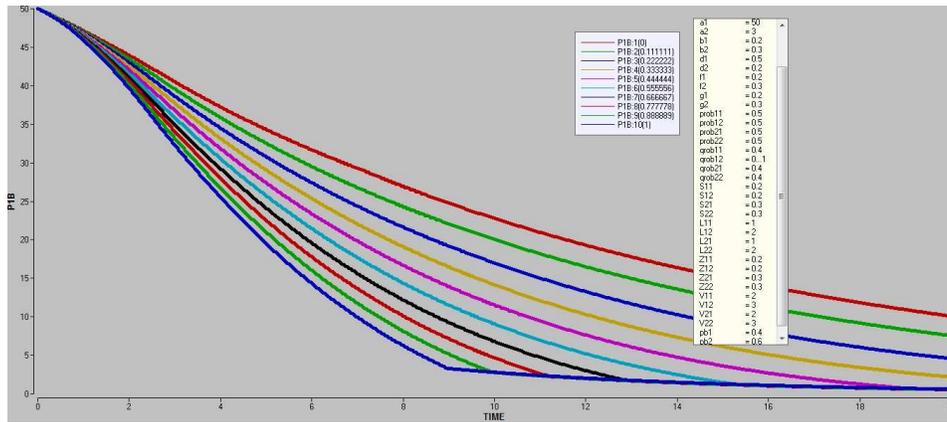


Figure 14: An example where varying the value of $qrob_{ij}$ $j \neq i$ affects $[P_i B]$.

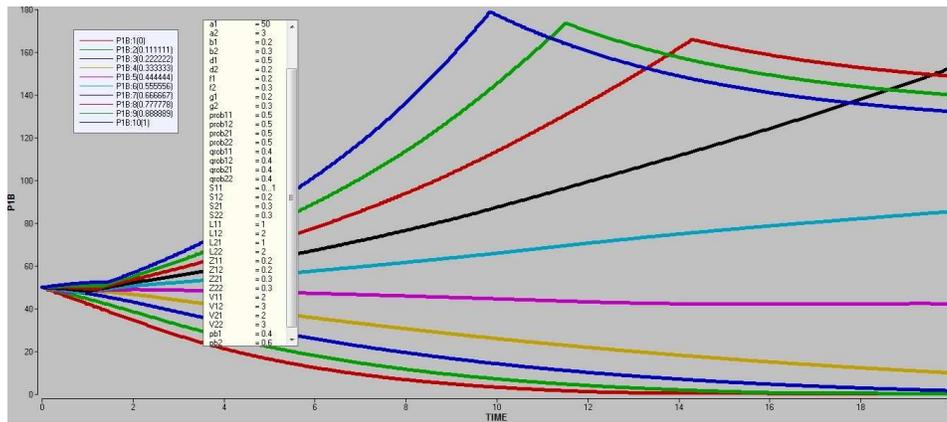


Figure 15: An example where varying the value of S_{ii} affects $[P_i B]$.