Symmetry and asymmetry in certain atoms

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Abstract

Symmetry and asymmetry play a crucial role in the stability of the fundamental components of matter. Thus, the main objective of this work is to study the existence of symmetry, this universal geometric property of matter, in certain atoms, in order to provide a classification of the elements of the periodic table based on a symmetry criterion.

Keywords: Atom stability, atomic asymmetry, atomic symmetry, global atomic symmetry.

1. Introduction

Symmetry and asymmetry permeate the universe in multiple forms [3]. In the living world, this duality is expressed through a rich diversity of organic shapes [22]. The origin of this concept can be traced back to Greek mathematicians, who envisioned symmetric objects such as the cylinder, cone, torus, and sphere. These objects are invariant under continuous rotational symmetries. The five regular solids known as the Platonic solids (427-347 BCE) - tetrahedron, cube, octahedron, icosahedron, dodecahedron - idealize forms observable in nature, treated by [15], and define polyhedra whose multi-axial symmetries are the highest possible, see [18]. Gilles Cohen-Tannoudji emphasizes the crucial role of symmetry in the evolution of modern physics: " The consideration of symmetry properties is a guiding thread that runs through the entire history of modern physics, from Galileo (1564-1642) to Newton (1642-1727) on the unification of fundamental interactions using the Standard Model of particle physics... The Lagrangian formulation of classical rational mechanics highlights the fundamental role of symmetry properties. Noether's theorem states that for every symmetry, there corresponds a conservation law of a physical quantity... Properties of invariance under symmetry transformations are related to properties of relativity." [16]. By definition in [16], two figures are symmetrical with respect to a point if they can be superimposed by a half-turn around this point. This point is called the center of symmetry. And two figures are symmetrical with respect to a line when they can be folded onto each other around this line. Moreover, living beings possess common characteristics of symmetry and asymmetry [22]. These characteristics concern not only the external organization of living organisms but also involve all levels of their organization, down to the molecules that constitute them [19].

The term that opposes symmetry is asymmetry, representing contrasting categories with distinct ontological characteristics. Asymmetry refers to the absence of symmetry, as observed in the case of enantiomers.

Without asymmetry, or without symmetry breaking, without disruption of order, without an event that suddenly eliminates half of a landscape, there is no matter, no life, no bodies... Particles, molecules, cells, and even organs are, in their own way, not symmetrical, like two objects in a mirror. David Larousserie stated this in " Le Monde Science et Techno " on April 16, 2018. Beyond regular solids, symmetry has become a powerful tool to describe the fundamental laws of nature. Crystalline structures, floral patterns, and even mathematical equations are often imbued with symmetry. On the other hand, asymmetry has its own essential role. Where symmetry provides order and balance, asymmetry allows for diversity and adaptation [16]. In the organic world, asymmetry is the norm rather than the exception. From biological molecules to bodily forms, asymmetry is omnipresent and contributes to the complexity and variety of life. The principles of atomic symmetry are essential for understanding the structure and reactivity of chemical compounds, impacting various fields such as drug design and chemical catalysis [4], [6], [8], [16], [18]. Therefore, after establishing the necessary and sufficient condition for an atom to be symmetric or asymmetric, we propose a symmetric classification of atoms in the periodic table.

By exploring the world of symmetry, we unveil its crucial role in chemistry, particularly in atomic symmetry or asymmetry [4], [16]. Since atoms are fundamental components of molecules, we could extend our study to the molecular scale.

2. Preliminaries

Definition 2.1. *Democritus (Greek philosopher 460-371 BC) in his philosophical thought stated that matter is composed of extraordinarily small entities which he called atoms (derived from the Greek word " atomos " = indivisible) [5].*

2.1. Atomic structures

Definition 2.2. *An atom is an electrically neutral entity characterized by a number of electrons (negatively charged) orbiting around the nucleus and a number of nucleons that constitute its nucleus. An atom is composed of a nucleus and electrons; the nucleus is composed of protons and neutrons.*
Protons (positively charged) and neutrons (neutral) are composed of even more elementary particles called quarks.
Each atom is defined by its atomic number, which corresponds to the number of protons in its nucleus [1], [4], [5], [8], [11], [19], [24].

The interactions between these particles and their arrangements determine the chemical properties of each element.

Remark 2.1. *Chemistry focuses on changes in the electron cloud (not the nucleus), whereas nuclear physics focuses on the nucleus.*

Remark 2.2. *In nuclear physics, until* 1932 *only* 3 *elementary particles were known: the electron, the proton, and the neutron, but today, more than 100 elementary particles have been discovered (pions, positrons, antiparticles, neutrinos, quarks (6 flavors: up, down, charm, strange, top, bottom),* 3 *colors and* 3 *anticolors).*

2.1.1. On Atomic Orbitals

The atomic orbital is the representation of the volume within which there is a 95% chance of finding the electron.
They are characterized by sets of quantum numbers that determine the shape, energy, and orientation of each orbital:

- Principal quantum number (n): Indicates the size of the orbital and its energy level.
- Azimuthal quantum number (l): Determines the shape of the orbital (0 for s, 1 for p, 2 for d, 3 for f).
- Magnetic quantum number (m_l): Indicates the orientation of the orbital in space.
- Spin quantum number (m_s): Represents the electron's spin orientation.

s orbitals have a spherical shape, p orbitals have a dumbbell shape and exhibit rotational symmetry, while d and f orbitals have more complex shapes. Electrons fill these orbitals according to specific rules, such as the Pauli exclusion principle and Hund's rule.

2.1.2. Periodic Table

The periodic table [4] of elements classifies all chemical elements according to their atomic number, which corresponds to the number of protons in the nucleus. The horizontal rows of the table are called periods and indicate the number of electron shells, while the vertical columns are called groups and group elements with similar electron configurations, giving them comparable chemical properties.

2.1.3. Nucleus Stability

The ratio of the number of neutrons ($A - Z$) or (N) to protons (Z), expressed as $\dfrac{A - Z}{Z}$, can provide an indication of nucleus stability [14] as follows:

- If $\dfrac{A - Z}{Z} < 1$, the atom tends to be somewhat unstable in general.
- If $\dfrac{A - Z}{Z} = 1$, the atom is stable.
- If $1 < \dfrac{A - Z}{Z} < 1.5$, it can include both stable and unstable isotopes.
- If $\dfrac{A - Z}{Z} > 1.5$, the atom is radioactive.

2.1.4. Stable Isotopes

Definition 2.3. *[22] Isotopes are classified as stable if they have a well-balanced number of protons and neutrons in their nucleus over very long, potentially indefinite periods of time.*

Example 2.1. *Here are some elements from the periodic table that possess stable isotopes [1], [4], [5] which do not undergo significant radioactive decay under normal conditions:*
Hydrogen (H), Oxygen (O), Carbon (C), Nitrogen (N), primarily composed of the stable isotope nitrogen$-$14 (14N), Calcium (Ca), predominantly found in nature as the stable isotope calcium$-$40 (40Ca), Iron (Fe), which exists largely as two stable isotopes, iron$-$56 (56Fe) and iron$-$54 (54Fe), Sodium (Na), primarily composed of the stable isotope sodium$-$23 (23Na), Copper (Cu), having two stable isotopes, copper$-$63 (63Cu) and copper$-$65 (65Cu), Argon (Ar), primarily composed of the stable isotope argon$-$40 (40Ar), Magnesium (Mg), which is predominantly composed of the stable isotope magnesium$-$24 (24Mg),...

2.1.5. Radioactive Isotopes

Definition 2.4. *Isotopes [14], [22] are classified as radioactive when their nuclei undergo spontaneous radioactive decay. Consequently, these isotopes are unstable by 2.1.3 and decay by emitting particles or energy in the form of radiation.*
The half-lives of radioactive isotopes range from fractions of a second to billions of years.

2.1.6. Modes of radioactive decay

We list some of the most common modes of radioactive decay for isotopes [1], [9], [17], [20], [19], [22]:

- **Alpha decay (α):** Radioactive isotopes emit alpha particles, consisting of two protons and two neutrons.
  For example: Uranium-238 undergoes alpha decay to become
  Thorium-234.
- **Beta decay (β):** Some isotopes undergo beta decay, where a neutron transforms into a proton or vice versa within the nucleus, accompanied by the emission of an electron ($\beta-$) or a positron ($\beta+$).
  For example: Carbon-14 decays to Nitrogen-14 by emitting an electron ($\beta-$).
- **Gamma decay (γ):** Following alpha or beta decay, the nucleus may be left in an excited state and return to its ground state by emitting gamma radiation. This radiation consists of high-energy photons.
  For example: Iodine-131 decays to Xenon-131 and emits gamma radiation.
- **Electron capture (EC):** Some nuclei capture an electron from the electron cloud surrounding the nucleus, transforming a proton into a neutron in the process.
  For example: Bismuth-214 decays via electron capture to become Polonium-214.
- **Spontaneous fission:** There are nuclei unstable enough to spontaneously split into two lighter nuclei, releasing energy and additional neutrons.
  For example: Uranium-235 and Plutonium-239.

Remark 2.3. *Apart from radioactive decay, there are processes such as excessive neutron capture, induced nuclear fission or fusion processes, which do not occur naturally on Earth but can potentially lead to instability of atoms or their isotopes.*
In fact, isotopes with irregular or unbalanced verifying 2.1.3 distributions of protons and neutrons are often associated with specific nuclear phenomena such as radioactivity, nuclear fission, or exotic properties, indicating their instability [13].

Theorem 2.1. *Chemical elements of the periodic table and their isotopes that do not undergo radioactive decay are considered stable at their nucleus level under normal conditions.*
Proof. *Consider non-radioactive isotopes. That is, these isotopes remain in their current state without spontaneously decaying by emitting particles or energy, regardless of the various decay modes. Therefore, by virtue of definitions 2.4 and 2.3 and remark 2.3, the result holds.*

2.2. Symmetry and Asymmetry of Atoms

Definition 2.5. *Symmetry in chemistry and physics refers to the invariance of an atomic structure under certain geometric transformations* namely [6], [8], [13], [23]:

1. *Axial Symmetry:* The atom is symmetrical around a central axis. For example, the p_z orbitals are symmetrical around the $z-$axis.
2. *Central Symmetry:* The atom exhibits symmetry around a central point, where each point on the atom has a corresponding point on the opposite side.
3. *Mirror Plane Symmetry:* The atom is symmetrical with respect to a plane, dividing the atom into two mirror halves.
4. *Translation symmetry [25]:* If the object is translated or moved from one position to another, the same orientation in the forward and backward motion is called translational symmetry. In other words, it is defined as the sliding of an object about an axis.
5. *Rotational symmetry [25]:* When an object is rotated in a particular direction, around a point, then it is known as rotational symmetry or radial symmetry. Rotational symmetry existed when a shape turned, and the shape is identical to the origin. The angle of rotational symmetry is the smallest angle at which the figure can be rotated to coincide with itself. The order of symmetry is how the object coincides with itself when it is in rotation.
6. *Reflexive symmetry [25]:* Reflection symmetry is a type of symmetry in which one half of the object reflects the other half of the object. It is also called mirror symmetry or line of symmetry.
7. *Glide symmetry [25]:* The combination of both translation and reflection transformations is defined as the glide reflection. A glide reflection is commutative in nature. If we change the combination's order, it will not alter the output of the glide reflection.
8. *Parity symmetry [3]:* It is a discrete symmetry involving a reflection of space around the origin.
9. *Local symmetry:* The element that uniquely determines the forces governing electromagnetic and nuclear phenomena themselves.
10. Asymmetry refers to the absence of symmetry or an irregular distribution of electron density or properties within the atom.

Remark 2.4. *Among the various symmetries, spherical, crystalline, nuclear, and cylindrical symmetries are crucial in the study of atoms. These symmetries strongly influence the electronic properties, chemical interactions, and physical behaviors of elements.*

3. Major Symmetries in an Atom

As atomic symmetry can take various forms, we will mention some types of atomic symmetry [1], [6], [7], [12]:

- *Spherical Symmetry:* Atoms are arranged symmetrically around a central point, as seen in noble gases.
- *Cubic Symmetry:* Atoms are arranged symmetrically along the x, y, and z coordinate axes. Cubic crystal structures include body-centered cubic (bcc), face-centered cubic (fcc), and face-centered cubic with additional atoms in the center (ccc).
- *Orthorhombic Symmetry:* Atoms are arranged symmetrically along three orthogonal axes. Many minerals exhibit this symmetry.
- *Monoclinic Symmetry:* Atoms are arranged along three axes, with two being perpendicular and the third inclined. This is typical in gypsum crystals.

- *Triclinic Symmetry:* No constraints on the orientation of the axes. This symmetry is often observed in minerals with rapid crystallization.
- *Nuclear Symmetry for Individual Atoms:* Refers to the distribution of protons and neutrons within its nucleus.
- *Crystalline Symmetry of Individual Atoms:* Refers to the periodic repetition of atomic motifs within a crystal.
- *Cylindrical Symmetry:* It can be observed in the distribution of electronic orbitals around the principal axis of an orbital, such as *p* orbitals which exhibit cylindrical symmetry along their axis.

These are some types of atomic symmetry.

3.1. Spherical symmetry

Definition 3.1. *Spherical symmetry in atoms refers to an arrangement of atoms or electrons around a central nucleus that is symmetrical in all directions of space, like a sphere. More precisely, this means that the distribution of electric charge (protons in the nucleus and electrons around the nucleus) is uniform in all directions, creating spherical symmetry.*

In other words, an atom exhibits spherical symmetry when:

  i) the electron density (probability of finding an electron at a certain distance from the nucleus) is uniformly distributed around the nucleus in all directions, forming a sphere of electron density.
 ii) the probability of finding an electron at a certain distance from the nucleus is the same in all directions in space.
iii) the distribution of atomic orbitals (regions of space where electrons are most likely to be found) is symmetrical with respect to the nucleus in all directions, meaning the orbitals have spherical symmetry.

Spherical symmetry is often associated with *s* and *p* orbitals in atoms, especially in atoms where the outermost electron shell is filled. For example, the 1*s*, 2*s*, and 2*p* orbitals of helium, neon, and argon atoms exhibit spherical symmetry.

3.1.1. Characteristics of Spherical Symmetry in Atoms

Characteristics of spherical symmetry in atoms include:

   i) Uniform distribution of electric charge,
  ii) Symmetry of atomic orbitals,
 iii) Radial symmetry of charge densities,
  iv) Absence of directional preference,
   v) Symmetry in observable properties,
  vi) Isotropic movement of electrons,
 vii) Symmetry in atomic interactions,
viii) Absence of permanent dipole moment.

The latter three elements emphasize the homogeneity and uniformity of properties and interactions in an atom with spherical symmetry, highlighting the significance of this symmetry in understanding atomic structure.

Properties 3.1. *Spherical symmetry is an important property observed in many natural objects and structures that exhibit radial symmetry around a central point. Here are the properties of spherical symmetry:*

   i) *Uniformity in all directions,*
  ii) *Absence of preferred direction,*
 iii) *Ease of mathematical description,*
  iv) *Applications in physics and chemistry,*
   v) *Stability in physical systems,*
  vi) *Interaction with force fields,*
 vii) *Effect on scattering,*
viii) *Symmetry in biological systems,*
  ix) *Applications in engineering,*
   x) *Relations with quantum physics,*
  xi) *Stability in natural structures.*

These latter six properties underscore the importance and diversity of applications and implications of spherical symmetry across various fields of science, technology, and nature.
The following result provides a condition that is both necessary and sufficient for an atom to exhibit spherical symmetry:

Theorem 3.1. *An atom exhibits spherical symmetry if and only if the distribution of electric charge (protons in the nucleus and electrons around the nucleus) is uniform in all directions of space.*
Proof. *We proceed by demonstrating the two implications:*
Direct implication: *Suppose the distribution of electric charge in an atom is uniform in all directions of space. This implies that the electron density is evenly distributed around the nucleus in all directions, thereby creating spherical symmetry. In other words, the probability of finding an electron at a certain distance from the nucleus is the same in all directions. Therefore, the atom exhibits spherical symmetry.*
Inverse implication: *Conversely, assume now that the atom exhibits spherical symmetry. This means the distribution of electric charge (protons in the nucleus and electrons around the nucleus) is uniform in all directions of space. If the distribution of electric charge were not uniform, it would lead to asymmetry in the distribution of electrons around the nucleus, violating spherical symmetry. Therefore, the spherical symmetry of the atom implies that the distribution of electric charge is uniform in all directions of space. Hence the result.*

3.1.2. Possible Combination of Spherical Symmetry with Other Symmetries in Atoms

While spherical symmetry is often the dominant characteristic in atoms, it can coexist with other symmetry elements such as axial symmetry, reflection symmetry, rotational symmetry, and translational symmetry, depending on the specific nature of the atom and its electronic structure.

Remark 3.1. *There are certain atoms in the periodic table that are incompatible with spherical symmetry but compatible with other symmetries.*

Example 3.1. *Here are some examples:*

- *Boron (B): The boron atom has an electronic structure that does not lead to spherical symmetry due to its electronic configuration and orbital arrangement.*
- *Sodium (Na): Sodium has an electronic configuration that does not result in spherical symmetry, but rather cubic symmetry in certain crystalline cases.*
- *Carbon (C): The carbon atom can form covalent bonds in various arrangements, such as in diamond (cubic structure) or graphite (hexagonal structure), which are not spherical.*

Proposition 3.1. *The coexistence of spherical symmetry with other symmetries is possible in an atom if and only if the different symmetry elements do not contradict each other in their application to the atom's structure. In other words, each symmetry present in the atom must be compatible with the other symmetries present.*
Proof. We proceed by demonstrating the implications in both directions:
Direct implication:Suppose the different symmetry elements present in an atom do not contradict each other in their application to the atom's structure, meaning each symmetry is compatible with the others. In this case, the coexistence of spherical symmetry with other symmetries is possible. This implies that symmetry operations such as rotations, reflections, and translations are compatible and do not lead to contradictions in the overall symmetry of the atom. Therefore, spherical symmetry can coexist with other symmetries within the atom.
Inverse implication: Conversely, assume now that the coexistence of spherical symmetry with other symmetries is possible in an atom. This implies that the different symmetry elements present in the atom do not contradict each other in their application to the atom's structure. In other words, each symmetry present in the atom is compatible with the others. If this were not the case, the coexistence of spherical symmetry with other symmetries would not be possible because symmetry operations would conflict with each other, leading to an inconsistent global symmetry of the atomic structure.
Hence, the proof is complete.

3.2. Nuclear Symmetry

Definition 3.2. *Nuclear symmetry refers to the spatial distribution of protons and neutrons within the atomic nucleus. In an atomic nucleus, protons and neutrons are bound together by nuclear forces, and their arrangement can be regular and symmetrical in certain cases [6], [16].*

Nuclear symmetry is a fundamental property of atomic nuclei that influences their stability, structure, electromagnetic properties, and interactions with other particles. It encompasses various aspects of the structure and behavior of atomic nuclei resulting from their internal symmetry.

Properties 3.2. *Here are the properties of nuclear symmetry:*

- *i) Nuclear stability,*
- *ii) Nuclear isospin,*
- *iii) Multipole moments,*
- *iv) Nuclear spectra,*
- *v) Nuclear decay and reactions*
- *vi) Structure of exotic nuclei,*
- *vii) Nuclear magnetic moment,*
- *viii) Nuclear deformation,*
- *ix) Modes of decay,*
- *x) Nuclear phase transition,*
- *xi) Effect on nuclear chemistry.*

The last five properties highlight the importance of nuclear symmetry in understanding and modeling atomic nuclei and their interactions. It is noteworthy that nuclear symmetry is a complex characteristic that can vary depending on the number of protons and neutrons, as well as other factors such as nuclear binding energy and nuclear interactions.

3.2.1. Characteristic Elements of Nuclear Symmetry

Here are its characteristic elements:

- *i*) Spherical symmetry,
- *ii*) Axial symmetry,
- *iii*) Paired nucleons,
- *iv*) Deformation effects,
- *v*) Nuclear force effects,
- *vi*) Octupole symmetry,
- *vii*) Tetrahedral symmetry,

viii) Parity symmetry,
ix) Symmetry breaking,
x) Effects of nuclear dynamics.

These elements demonstrate the diversity of manifestations of nuclear symmetry and how it can be studied through various phenomena and properties of atomic nuclei.

Remark 3.2. *Nuclear symmetry is important because it influences the properties and behavior of atomic nuclei, including their stability, reactivity, and modes of decay. Theoretical models of nuclear structure, such as the shell model or liquid drop model, attempt to describe and explain the observed nuclear symmetry in different atomic nuclei.*

Proposition 3.2. *For an atom to exhibit crystal symmetry, it is necessary and sufficient for it to be incorporated into a crystal where its position repeats periodically in a regular spatial arrangement:*

1. *Protons and neutrons must be uniformly and regularly distributed within the nucleus, without any excessive concentration in a specific region.*
2. *The spatial distribution of particles must exhibit symmetry about a center, meaning that the structural characteristics of the nucleus are identical in all directions.*

Proof. Necessity: *Suppose the atomic nucleus exhibits nuclear symmetry. This implies that the structural characteristics of the nucleus are identical in all directions. Therefore, protons and neutrons are uniformly and regularly distributed within the nucleus, without any excessive concentration in a specific region. Moreover, nuclear symmetry implies that the spatial distribution of particles exhibits symmetry about a center, confirming that the structural characteristics of the nucleus are identical in all directions.*
Sufficiency: *Suppose protons and neutrons are uniformly and regularly distributed within the nucleus. This ensures that the structural characteristics of the nucleus are identical in all directions, implying nuclear symmetry. Furthermore, if the spatial distribution of particles exhibits symmetry about a center, this confirms that the structural characteristics of the nucleus are identical in all directions, which is a necessary condition for nuclear symmetry. Thus, the proof.*

Example 3.2. *Here are some examples illustrating atoms with nuclear symmetry based on the balanced distribution of protons and neutrons within their nucleus, leading to a stable and often spherical configuration:*

- *Helium-4 (4He): Its nucleus consists of two protons and two neutrons arranged symmetrically, forming a stable configuration with spherical symmetry.*
- *Neon-20 (20Ne): Its nucleus consists of ten protons and ten neutrons, and its distribution of nucleons in the nucleus can exhibit spherical or axial symmetry depending on the energy configuration.*
- *Magnesium-24 (24Mg): Its nucleus consists of twelve protons and twelve neutrons, and its nuclear symmetry can be influenced by slight deformation effects due to its even number of nucleons.*
- *Argon-36 (36Ar): Its nucleus consists of eighteen protons and eighteen neutrons, and its nuclear symmetry can be predominantly spherical due to its stable configuration.*

3.3. Crystal Symmetry

Definition 3.3. *Crystal symmetry refers to the regularity and geometric order observed in the arrangement of atoms within a crystal [10], [13], [18], [21], [23].*
It is characterized by a set of symmetric transformations that preserve the crystal structure. These transformations may include reflections, rotations, and translations that leave the crystalline structure unchanged.

3.3.1. Characteristic elements or properties of crystal symmetry in atoms

Crystal symmetry manifests through several characteristic elements:

i) Periodic repetition of atoms,
ii) Bravais lattices;
iii) Point groups of symmetry;
iv) Symmetry planes;
v) Axis of symmetry;
vi) Center of symmetry.

These characteristic elements describe crystal symmetry from various aspects, from the periodic spatial arrangement of atoms to specific symmetry operations that preserve this structure within a crystal.

Remark 3.3. *Crystal symmetry is important because it influences many physical and chemical properties of crystalline materials, such as electrical conductivity, magnetism, transparency, and mechanical strength. It is also used to determine the atomic structure of crystals using experimental techniques such as X$-$ray diffraction and point group spectroscopy.*
The properties of crystal symmetry in atoms and its characteristic elements are closely related because they both describe how atoms are arranged in a crystal and how this arrangement generates specific symmetry.

Proposition 3.3. *For an atom to exhibit crystal symmetry, it is necessary and sufficient that it is incorporated into a crystal where its position repeats periodically in a regular spatial arrangement.*
Proof. *We proceed with our demonstration in two steps:*
Necessity : First, suppose an atom exhibits crystal symmetry. This means it is located in a crystal whose structure is characterized by

periodic and regular repetition of its atoms or ions in space. Therefore, for an atom to be included in this structure, its position must also repeat periodically within the crystal. In other words, the presence of crystal symmetry implies that atoms occupy positions that repeat regularly in the crystal.

Sufficiency : Conversely, if an atom is incorporated into a crystal where its position repeats periodically in a regular spatial arrangement, then this ensures that the crystal exhibits crystal symmetry. Indeed, the regular repetition of atomic positions in the crystal results in a crystalline structure that exhibits specific symmetry properties, such as planes, axes, and centers of symmetry. Thus, the condition that the atom is incorporated into a crystal where its position repeats periodically in space is both necessary and sufficient for it to exhibit crystal symmetry.

This completes our demonstration.

We next provide a necessary and sufficient condition for crystalline symmetry to not be influenced by molecular symmetry.

Theorem 3.2. *Crystal symmetry is primarily dictated by the geometry of constituent atoms or ions, rather than by any molecular symmetry of discrete entities within the crystal.*

Proof. *Due to the periodic repetition of the crystal's unit cell and the interactions between the atoms or ions comprising it, the overall symmetry described by the symmetry operations of the crystal lattice (such as translations, rotations, and reflections) is mainly determined by the spatial geometry of these fundamental constituents. However, the molecular symmetry of discrete entities (such as individual molecules) may not be preserved within the crystal if their arrangement and orientation do not align with the symmetry of the crystal lattice. Therefore, crystal symmetry is primarily dictated by the geometry of the constituent atoms or ions of the crystal, rather than by the molecular symmetry of discrete entities present within it.*

This completes our proof.

Example 3.3. *Here are some examples that highlight substances where crystalline symmetry results from the ordered and periodic arrangement of atoms or ions in a three-dimensional crystal lattice:*

- *Sodium (Na): It is an alkali metal that forms body-centered cubic crystals at room temperature. In a sodium crystal, sodium atoms are arranged regularly in a cubic structure with crystalline symmetry.*
- *Diamond (C): Diamond is a crystalline form of carbon where each carbon atom is bonded to four other carbon atoms, forming a tetrahedral crystalline structure. The crystalline symmetry of diamond is based on the ordered and regular arrangement of these covalent bonds.*
- *Iron (Fe): Iron crystallizes in a face-centered cubic structure at room temperature. In an iron crystal, iron atoms are arranged regularly in a cubic structure with crystalline symmetry.*

Remark 3.4. *From [18], a crystal is composed of atoms, yet it can also be formed from molecules. The distinction between these two cases depends on the nature of the crystalline substances. Molecular symmetry can impact crystalline symmetry if molecules exhibit symmetric organization within the crystal. There are crystals composed of atoms, molecules, as well as a combination of both atoms and molecules.*

Example 3.4. • ***Crystals composed of atoms:*** *Some crystals are formed by the three-dimensional repetition of a periodic arrangement of atoms. For example, diamond is a crystal composed exclusively of carbon atoms, bonded together covalently in an ordered crystalline structure.*
- ***Crystals composed of molecules:*** *Other crystals can be formed by the three-dimensional repetition of a periodic arrangement of molecules. For instance, halite (sodium chloride) is a crystal composed of NaCl molecules, where sodium and chloride ions are arranged in an orderly manner within the crystalline structure.*
- ***Mixed crystals:*** *Crystals can be composed of both atoms and molecules. For example, copper sulfate pentahydrate ($CuSO_4 5H_2O$) is a crystal where copper ions (Cu^{2+}) and sulfate ions (SO_4^{2-}) form a crystalline structure, while water molecules (H_2O) are incorporated into this structure.*

3.4. Cylindrical symmetry in atoms

Definition 3.4. *Cylindrical symmetry in an atom refers to a specific arrangement of atomic components around a central axis, where the arrangement of atoms or orbitals remains unchanged when rotated around this axis. More precisely, an atom exhibits cylindrical symmetry when its atomic structure retains its appearance or configuration upon rotation around a central axis, typically parallel to a specific direction within the atom [10], [21], [23].*

In an atomic context, cylindrical symmetry can be observed in several aspects:

1. **Arrangement of electrons:** Electrons in atomic orbitals can be arranged to exhibit cylindrical symmetry around the axis of symmetry. For example, certain atomic orbitals such as $dz^2 dz^2$ or $p_z p_z$ can exhibit cylindrical symmetry along the $z-$axis.
2. **Arrangement of nucleons:** Nucleons (protons and neutrons) in the atomic nucleus can also be arranged to exhibit cylindrical symmetry. This can occur in specific isotopes or in situations where the atomic nucleus has a cylindrical geometric shape.
3. **Interaction of electrons and nuclei:**: Cylindrical symmetry can influence the interactions between electrons and nuclei within the atom. For instance, in atoms with cylindrical symmetry, certain electronic or nuclear interactions may be facilitated or hindered due to this symmetry.

Indeed, cylindrical symmetry in an atom can influence various aspects of the atom's structure and behavior in different physical and chemical contexts.

3.4.1. Characteristics elements of cylindrical symmetry in an atom

In an atom, the characteristic elements of cylindrical symmetry include:

1. Distribution of atomic orbitals.
2. Valence electrons.
3. Distribution of electron density.
4. Shape of the atom.
5. Electric potential.
6. Magnetic properties.
7. Interactions with light.

These characteristic elements contribute to understanding atomic structure and predicting chemical properties of atoms, particularly where cylindrical symmetry predominates. They also demonstrate that cylindrical symmetry can have significant implications across various fields of chemistry and physics, affecting different aspects of atomic behavior.

Properties 3.3. *Cylindrical symmetry in an individual atom exhibits several important properties:*

1. *Distribution of atomic orbitals.*
2. *Electron density.*
3. *Dipole moments.*
4. *Electric potential.*
5. *Orbital angular momentum.*
6. *Shape of the atom.*
7. *Magnetic interactions.*
8. *Atomic spectroscopy.*
9. *Zeeman effect.*
10. *Nuclear magnetic resonance (NMR).*
11. *Diamagnetism and paramagnetism.*
12. *Polarization of emitted light.*

Remark 3.5. *These properties help define cylindrical symmetry in an individual atom and are essential for understanding its behavior and interactions with its environment. They underscore the importance of cylindrical symmetry in various aspects of atomic physics and chemistry, and how it can influence the behavior and interactions of individual atoms.*

Proposition 3.4. *For an atom to exhibit cylindrical symmetry, its structure must necessarily have an arrangement that does not change when rotated around a central axis.*
Proof. *Suppose an atom has cylindrical symmetry, but its structure changes upon rotation around a central axis. This implies that the distribution of electrons or atomic orbitals in the atom is not symmetric under rotation around this axis.*
If the atom's structure changes upon rotation around the central axis, it indicates there is a preferred direction where the atom's structure differs from other directions. This contradicts the principle of cylindrical symmetry, which requires symmetric distribution of electrons around the central axis.
In other words, if the arrangement of electrons or atomic orbitals in an atom changes upon rotation around a central axis, then the atom cannot have cylindrical symmetry. Therefore, for an atom to exhibit cylindrical symmetry, its structure must have an arrangement that remains unchanged when rotated around a central axis. This demonstrates the proposal.

Example 3.5. *Here are some examples of individual atoms that exhibit cylindrical symmetry:*

1. *Hydrogen (H): The hydrogen atom has a $1s$ orbital that is spherical around the nucleus, but it can also have $2s$ and $2p$ orbitals that exhibit cylindrical symmetry, especially the $2s$ orbital.*
2. *Carbon (C): Carbon has an electron configuration of $1s^2\ 2s^2\ 2p^2$. The $2s$ orbital of carbon has a cylindrical shape, giving it cylindrical symmetry.*
3. *Nitrogen (N): Nitrogen has an electron configuration of $1s^2\ 2s^2\ 2p^3$. The $2s$ orbital and some of the $2p$ orbitals (e.g., p_z) exhibit cylindrical symmetry.*
4. *Oxygen (O): Oxygen has an electron configuration of $1s^2\ 2s^2\ 2p^4$. Similar to nitrogen, the $2s$ orbital and some of the $2p$ orbitals exhibit cylindrical symmetry.*

3.5. Global symmetry in an individual atom

Definition 3.5. *Global symmetry [16] of an atom refers to the set of symmetry operations that can be applied to the atom without altering its spatial configuration [8]. These operations include rotation, reflection, inversion, and any combination thereof.*

Understanding global symmetry in an atom is crucial for predicting its physical and chemical properties, as well as interpreting experimental results in spectroscopy and crystallography.
The properties of global symmetry in an atom are important for understanding its physical, chemical behavior, and interactions with other particles.

Properties 3.4. *Here are some of the properties associated with global symmetry:*

1. *Stability and robustness.*
2. *Predictability of physical and chemical properties.*
3. *Interpretation of experimental results.*
4. *Classification of structures.*
5. *Materials engineering.*
6. *Understanding vibration modes.*

7. *Determination of crystalline structures.*
8. *Analysis of electronic states.*
9. *Development of theoretical models.*
10. *Selection of electronic transitions.*
11. *Detection of crystalline defects.*

In summary, the properties of global symmetry in an atom are diverse and significantly influence its physical, chemical behavior, and interactions with the environment, making it an essential concept in various fields of science and engineering.

Overall, global symmetry in an atom plays a crucial role in many aspects of physics, chemistry, and materials science, and its properties influence how we understand and manipulate the microscopic world around us.

3.5.1. Characteristic elements of global symmetry in an atom

The characteristic elements of global symmetry in an individual atom include:

1. *Spatial arrangement of atomic orbitals.*
2. *Centers of symmetry.*
3. *Rotation axes.*
4. *Symmetry planes.*
5. *Point symmetry.*
6. *Point groups of symmetry.*
7. *Crystal lattice symmetry.*
8. *Mixed symmetry operations.*
9. *Chiral or non-chiral symmetry.*

Remark 3.6. *These latter four additional elements, in the above paragraph 3.5.1, contribute to a more comprehensive understanding of global symmetry in an individual atom, considering its characteristics both in an isolated atomic context and in a crystalline context. The characteristic elements of global symmetry in an individual atom include the spatial arrangement of atomic orbitals, centers of symmetry, rotation axes, symmetry planes, and point symmetries. These elements help define the global symmetry of the atom and predict its physical and chemical properties.*

A necessary and sufficient condition is given for an individual atom to possess global symmetry.

Theorem 3.3. *For an individual atom to have global symmetry, it is necessary and sufficient that the following conditions are satisfied:*

1. *It must have at least one symmetry element, such as a center of symmetry, a rotation axis, or a plane of symmetry.*
2. *This symmetry element must preserve the atom's symmetry, meaning the atom retains its shape and properties when subjected to a symmetry operation.*
3. *All parts of the atom must be symmetrically arranged relative to the symmetry element, implying a balanced distribution of electron density and a coherent arrangement of atomic orbitals.*

Proof. Necessity:
First, suppose an atom has global symmetry. This implies there are symmetry elements, such as a center of symmetry, a rotation axis, or a plane of symmetry, that preserve the atom's symmetry. Furthermore, for this symmetry to be maintained, all parts of the atom must be symmetrically organized relative to these symmetry elements. Otherwise, the atom's symmetry would be broken. Therefore, if the atom's symmetry is broken, it cannot have global symmetry. Thus, satisfaction of all three conditions is necessary for an atom to have global symmetry.

Sufficiency :
Suppose an atom satisfies the following three conditions. First, the presence of at least one symmetry element (condition 1) ensures that there are symmetry operations preserving the atom's symmetry. Second, when the atom undergoes these symmetry operations, it retains its shape and properties (condition 2). Finally, all parts of the atom are symmetrically organized relative to these symmetry elements (condition 3). Consequently, the atom possesses global symmetry. Therefore, the satisfaction of all three conditions is sufficient for an atom to exhibit global symmetry, thus proving the statement.

Remark 3.7. *All types of symmetry in an individual atom, without exception, contribute to the global symmetry of that atom.*

Indeed, generally, all types of symmetry present in an individual atom are part of its global symmetry. Global symmetry encompasses all symmetry operations that preserve the structure and properties of the atom as a whole, including all specific symmetry elements such as spherical symmetry, crystalline symmetry, nuclear symmetry, cylindrical symmetry, and others. Each type of symmetry uniquely contributes to the global symmetry, and together, they determine the overall symmetry of the atom. For example, an atom may exhibit spherical symmetry in its electron orbitals, crystalline symmetry in a crystal where it is included, cylindrical symmetry in certain molecules, and nuclear symmetry in the distribution of protons and neutrons within the nucleus.

Theorem 3.4. *Global symmetry represents the combination of all symmetry elements present in an individual atom, without exception, and is crucial for characterizing its overall symmetry.*

Proof. *According to definition 3.5, global symmetry can be characterized by various symmetry operations such as rotations, reflections, and translations that preserve the atom's structure and properties. In other words, global symmetry describes how different parts of the atom organize symmetrically relative to a certain point, axis, plane, or other symmetry elements. In an atom, various symmetry elements may be present, such as mirror planes, rotation axes, or inversion centers. According to the theorem 3.3, each of these symmetry elements represents a specific operation that leaves the atom unchanged. Moreover, the global symmetry of the atom is determined by combining all individual symmetry elements present within it. This means that each symmetry operation contributes to defining the atom's global symmetry. Finally, we know that global symmetry is essential for fully characterizing symmetry because it considers all possible symmetry operations within the*

atom. This allows for a comprehensive description of the atom's symmetry, taking into account all its spatial characteristics. Thus, the proposition is demonstrated.

Example 3.6. *Here are some examples of individual atoms that exhibit global symmetry, maintained by the uniform distribution of electrons around the nucleus, making them symmetric under all possible rotations and reflections:*

- *Helium atom (He): Helium exhibits spherical symmetry. Its global symmetry is preserved when it undergoes rotation around any axis passing through its center. Additionally, it has a plane of symmetry through which it can be reflected without altering its appearance.*
- *Argon atom (Ar): Argon, like helium (He) and neon (Ne), exhibits spherical global symmetry due to the uniform distribution of electrons in its orbitals.*
- *Xenon atom (Xe): As a noble gas, xenon also exhibits spherical global symmetry due to the uniform distribution of electrons in its orbitals, similar to the other noble gases mentioned above.*

Remark 3.8. *It is important to note that in some cases, specific defects or distortions may be considered in the analysis of a system's global symmetry. For example, in crystals with point defects such as vacancies or interstitials, global symmetry may be locally disrupted around these defects. However, the overall global symmetry of the crystal is still determined by symmetry operations that preserve the regular crystalline structure.*

3.5.2. Symmetry condition of an atom

To confirm the symmetry of an individual atom, here are the most important conditions:

1. Uniform distribution of electrons,
2. Completely filled orbitals,
3. Symmetry elements in the natural structure of the atom,
4. Uniform distribution of electrons in all orbitals,
5. Absence of significant distortions in the orbitals,
6. Presence of all expected symmetry elements,
7. Maintenance of symmetry during interaction with other atoms or molecules,
8. Maintenance of symmetry in different phases and environments.

Remark 3.9. *Although asymmetry may exist in an individual atom, it is generally not considered part of its overall symmetry, which is defined by the set of symmetry operations preserving the structure and properties of the atom as a whole.*

3.5.3. Variable (Conditional) Symmetry of Atoms

The symmetry properties of an atom can change depending on external conditions such as temperature, pressure, or the presence of electric or magnetic fields. Thus, an atom can be symmetric, asymmetric, or neither symmetric nor asymmetric.

These conditional variations in symmetry play a key role in chemical reactivity, atomic spectroscopy, and other physical and chemical phenomena.

3.6. Atom classification.

We will present another way to classify atoms from the periodic table in this subsection.

Theorem 3.5. *Alkali metals, alkaline earth metals, noble gases, and rare gases are naturally symmetric.*
Proof. *The natural symmetry of atoms in groups 1, 2, 8, and 18 of the periodic table primarily arises from their outer electronic configurations, namely:*
Group 1 elements, which include alkali metals, have an outer electronic configuration of the form ns^1, where n is the electron shell number and s^1 indicates a single electron in the s subshell. This electron is often present in a symmetric spherical orbital around the atomic nucleus.
Group 2 elements, including alkaline earth metals, have an outer electronic configuration of ns^2, where n is the electron shell number and s^2 indicates two electrons in the s subshell. These electrons occupy symmetric orbitals around the nucleus, which also contributes to their natural symmetry.
Group 8 atoms such as noble gases have complete outer electronic configurations, meaning their outer electron shells are fully occupied. This complete configuration makes these atoms highly stable and unreactive. Their symmetry arises from the balanced distribution of electrons in their outer electron shells.
Group 18, composed of rare gases, is a subset of Group 8 and shares the same stability and symmetry characteristics as noble gases.
In fact, this symmetry is a consequence of the spatial distribution of electrons around the atomic nucleus, which is dictated by principles of quantum mechanics. Thus concludes the demonstration.

Theorem 3.6. *Atoms in groups other than those containing alkali metals, alkaline earth metals, noble gases, and rare gases in the periodic table may exhibit varied symmetry depending on their electronic configuration, molecular geometry, and spatial arrangement within molecules.*
Proof. *Let's briefly examine the electronic configurations of atoms in groups other than groups 1, 2, 8 and 18 of the periodic table [4].*
Firstly, groups other than 1, 2, 8 and 18 include a variety of elements, such as transition metals, inner transition metals, lanthanides, actinides, and other p−block elements.
Atoms in these groups have diverse outer electronic configurations due to the complexity of their electron shells. For instance, transition metals have incomplete outer electronic configurations and can form metallic bonds, whereas p−block elements have valence electrons in p

orbitals.

Additionally, the symmetry of atoms in these groups can vary based on their electronic structure and molecular geometry. Some atoms may exhibit axial symmetry due to d orbitals (as seen in transition metals), while others may have planar or tetrahedral symmetry due to p orbitals (as seen in p−block elements).

Moreover, certain molecular compounds containing atoms from these groups can display asymmetry due to the spatial arrangement of atoms and bonds. For example, organic molecules containing carbon, hydrogen, oxygen, and nitrogen atoms can have asymmetric molecular geometries due to the presence of functional groups.

In summary, for atoms in groups other than groups 1, 2, 8 and 18 of the periodic table, the presence of varied symmetry depends either on their electronic configuration, molecular geometry, and/or spatial arrangement in molecules. In other words, some may exhibit symmetry, others asymmetry, and some may have symmetry that is neither clearly defined as asymmetric nor symmetric, depending on their chemical and physical context.

3.7. Implications of atomic symmetry

The diversity of atomic symmetry observed across different groups of the periodic table reflects the complexity of electronic structures and atomic interactions underlying the behavior of elements and their compounds in nature and in laboratory settings, namely:

3.7.1. Atomic Symmetry and Molecular Geometry

Elements of alkali metals, alkaline earth metals, noble gases, and rare gases often possess simple external electronic configurations and well-defined atomic orbitals [6], [8], [12]. This typically results in molecules with high symmetry, as electrons are evenly distributed around the nucleus. For instance, noble gases often form spherical or linear molecules that reflect their symmetry.

3.7.2. Variability of symmetry for the other groups of the periodic table

Groups such as transition metals, metalloids, and non-metals have more complex electronic configurations with partially filled d and f orbitals [6], [12]. This can lead to significant variability in the atomic symmetry and molecular geometry of the compounds they form. For example, transition metals can form complexes with a wide range of geometries, from octahedral to tetrahedral, depending on ligands and environmental conditions.

3.7.3. Impact on Chemical and Physical Properties

Atomic symmetry directly influences the chemical and physical properties of substances. Molecules with high symmetry may exhibit specific properties such as characteristic melting or boiling points, distinct chemical reactivities, and unique electrical transport capabilities. Conversely, molecules with variable symmetry can display a wider range of behaviors depending on their molecular structure [6], [12].

3.7.4. Applications in Chemistry and Physics

Understanding atomic symmetry is crucial in various fields of chemistry and physics, including catalysis, materials chemistry, spectroscopy, and drug design [6], [8], [12]. The ability to predict the symmetry of molecules and complexes can guide the design of new materials and compounds with desired specific properties.

4. Atomic Stability

Theorem 4.1. *Any naturally occurring object that is globally symmetrical or asymmetrical is inherently stable, and vice versa [16].*

4.1. Atomic stability through symmetric structures

Atomic stability [1], [9], [22] can be enhanced by symmetric electronic structures, although this is not always a sufficient condition. Here are some aspects of atomic stability related to symmetric structures:

4.1.1. Stability through spherical symmetry

By virtue of paragraph 3.1.1, theorem 3.1 and the symmetry conditions of atoms according to 3.5.2, atoms with spherical symmetry, such as noble gases, tend to be more stable. This symmetry allows for a uniform distribution of electron density around the nucleus, thereby minimizing electrostatic energy and contributing to atom stability.

4.1.2. Complete electronic configuration

Atoms with complete electron shells, like noble gases, are particularly stable. When all atomic orbitals are filled, the atom achieves a stable electronic configuration, which can be favored by symmetry in electron distribution.

4.1.3. Absence of unpaired electrons

Atoms with paired electrons in all orbitals, such as noble gases, tend to be more stable because there are no unpaired electrons to cause reactive interactions with other atoms.

4.1.4. Balanced electron interaction

In some cases, symmetry in electron distribution can promote balanced electron interaction, thereby reducing electronic repulsion forces and contributing to atom stability.

Indeed, atomic stability through symmetric structures is often observed in atoms with spherical symmetry and complete electronic configurations. However, other factors such as the presence of unpaired electrons and specific electronic interactions can also influence atomic stability.

4.2. Atomic stability through asymmetric structures

Atomic stability can also be favored by asymmetric electronic structures in certain cases, in according to theorem 3.6. Here are some aspects of atomic stability related to asymmetric structures:

4.2.1. Stability of partially filled orbitals

It is often evaluated considering orbital filling principles, symmetry effects, electronic interactions, and chemical reactivity by vertue of theorem 3.6. These factors contribute to undermining the relative stability of atoms with partially filled electronic configurations in different contexts.

4.2.2. Stability concerning chemical reactivities

Even though chemical reactivity depends on numerous complex and variable factors, there are general principles and theories to explain and predict the reactivity of reagents in different situations. Chemical reactivity is a complex and multidimensional field often studied qualitatively and quantitatively using a combination of these principles and theories.

4.2.3. Stability in specific environments.

Stability in specific environments depends on multiple factors, including intermolecular interactions, environmental conditions, phase, coordination in metal complexes, and solvent effects. There is no single formula to describe this stability, but rather a combination of principles and considerations that must be taken into account to understand and predict stability in particular situations.

4.2.4. Principles of stability in specific crystalline phases

Stability in specific crystalline phases depends on several factors, including crystalline symmetry, binding energy, environmental conditions, grain size, crystal growth strategies, and order/disorder in the crystalline structure.

5. Discussion

Recall that an atomic orbital represents the volume within which there is a high probability of finding the electron [10], [23]. In the periodic classification, s orbitals have a spherical shape, p orbitals have a dumbbell shape and exhibit rotational symmetry, while d and f orbitals have more complex shapes. Under certain geometric transformations, the invariance and stability of an atomic structure depend primarily on symmetry in chemistry and physics [23]. However, among the various symmetries, it is crucial to note that spherical, crystalline, nuclear, cylindrical, and overall symmetries play significant roles in the study of atoms and matter. These symmetries strongly influence electronic properties, chemical interactions, and physical behaviors of elements.

An atom can be symmetric, asymmetric, or neither symmetric nor asymmetric. In other words, the symmetry properties of an individual atom can change depending on external conditions such as temperature, pressure, or the presence of electric or magnetic fields, thus resulting in variable or conditional symmetries of individual atoms. These conditional variations in symmetry play a key role in chemical reactivity, atomic spectroscopy, and other physical and chemical phenomena.

Although spherical symmetry is among the dominant characteristics in atoms, there are certain atoms in the periodic table that are incompatible with spherical symmetry but compatible with other symmetries. The importance of nuclear symmetry is undeniable as it influences the properties and behavior of atomic nuclei, including their stability, reactivity, and modes of decay [7], [9], [19], [20], [22]. Theoretical models of nuclear structure, such as the shell model or liquid drop model, attempt to describe and explain the observed nuclear symmetry in various atomic nuclei [18].

Many physical and chemical properties of crystalline materials, such as electrical conductivity, magnetism, transparency, and mechanical strength, can be influenced by crystalline symmetry [10], [13], [18], [19]. This underscores its importance in atoms. This symmetry helps determine the atomic structure of crystals using experimental techniques like $X-$ray diffraction and point group spectroscopy. It should be noted that the properties of crystalline symmetry in atoms and its characteristic elements are closely related because they both describe how atoms are arranged in a crystal and how this arrangement generates specific symmetry.

Through the characteristic elements and properties of cylindrical symmetry of individual atoms, this symmetry can have significant implications not only in various areas of chemistry and physics, affecting various aspects of atomic behavior, but also in influencing the behavior and interactions of individual atoms.

Certainly, the overall symmetry of an atom, which refers to all symmetry operations - rotation, reflection, inversion, and any possible combination of these operations that can be applied to the atom without altering its spatial configuration - is essential for predicting its physical and chemical properties, as well as interpreting experimental results in spectroscopy and crystallography [8], [18].

Furthermore, while asymmetry may exist in an individual atom [3], [13], it is generally not considered part of its overall symmetry. Additionally, symmetric electronic structures are a necessary condition for atomic stability, such as stability through spherical symmetry, complete electronic configuration, absence of unpaired electrons, and balanced electronic interaction, even though this condition is not

sufficient. In some cases, symmetry in the distribution of electrons can be conditioned by asymmetric structures, such as stability of currently filled orbitals, stability concerning chemical reactivities, stability in specific environments, stability of partially filled orbitals, as well as principles of stability in specific crystalline phases.

6. Conclusion

Atomic symmetry and asymmetry are important concepts in chemistry, physics, and other fields because they greatly influence the properties and behavior of materials. In this study, we examined different forms of symmetry such as spherical symmetry, crystalline symmetry, nuclear symmetry, cylindrical symmetry, and more broadly, overall symmetry, as well as atomic asymmetry, highlighting their implications in structuring atoms and molecules. We observed that atomic symmetry can be observed in crystalline structures where atoms are arranged in a regular and repetitive manner.

We emphasized the necessary and sufficient condition for atoms to exhibit symmetry or asymmetry. This symmetry can be described using point groups and symmetry symbols that help determine the physical and chemical properties of materials. On the other hand, atomic asymmetry can result from various factors such as structural defects, impurities, or interactions with other atoms. This asymmetry leads to variations in material properties such as electrical conductivity, mechanical strength, or chemical reactivity.

Therefore, the study of atomic symmetry and asymmetry is essential not only for understanding the structure and properties of materials but also for their stability. By identifying symmetry patterns and considering sources of asymmetry, it becomes possible to predict and interpret atomic and molecular systems, opening new perspectives for the design of innovative and high-performance materials.

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