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Research paper

Solving viscid Burgers equation by Adomian decomposition Method (ADM), Regular Pertubation Method (RPM) and Homotopy Perturbation Method (HPM)

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Abstract

In this paper the Adomian decomposition Method (ADM), Regular Pertubation Method (RPM) and the Homotopy Perturbation Method (HPM) are used to study Burgers equation. Then we compare the solutions obtained by these three methods.

Keywords: *Adomian decomposition Method (ADM), Burgers equation, Homotopy Perturbation Method (HPM), Regular Pertubation Method (RPM).*

1. Introduction

Burger's equation is a fundamental partial differential equation in fluid mechanics. It is also a very important model encountered in several areas of applied mathematics such as heat conduction, acoustic waves, gas dynamics and traffic flow [10, 11].
The general form of this equation is :

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=\varepsilon\frac{\partial^2 u}{\partial x^2}$$

Where ε is perturbation parameter which corresponds to the kinematic viscosity coefficient, $u(t,x)$ is some physical quantity, exactly the speed of the fluid, x the space variable and t stands for time.
When the diffusion term is absent that is $\varepsilon = 0$, Burgers' equation becomes the inviscid Burgers' equation :

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=0$$

In the case of the fluids (liquid or gas), the equation of Burgers modelizes their turbulence
Turbulence or turbulent flow is a flow regime in fluid dynamics characterized by chaotic changes in pressure and flow velocity. In general terms, in turbulent flow, unsteady vortices appear of many sizes which interact with each other, consequently drag due to friction effects increases. This would increase the energy needed to pump fluid through a pipe, for instance. However this effect can also be exploited by such as aerodynamic spoilers on aircraft [12].
In this paper, we examine the viscid Burgers equation with initial value :

$$\begin{cases} \dfrac{\partial u}{\partial t}+u\dfrac{\partial u}{\partial x}=\varepsilon\dfrac{\partial^2 u}{\partial x^2}, \quad 0\le x\le 1,\ 0\le t\le T \\ \\ u(0,x)=\varepsilon\cos x \end{cases} \tag{1}$$

Where $0<\varepsilon\ll 1$
The paper is organised as follows : in section 1, we start with the solving (1) by ADM [1, 2, 3, 4, 5, 10, 15]. In Section 2 and section 3, we construct the solution of (1) respectively by RPM [9] and HPM [6, 7, 8, 11, 16]. Section 4 contains numerical analysis of the solutions obtained by the different methods.

2. Application of ADM to Burgers equation

Defining the operators : $L_t = \frac{\partial}{\partial t}(.)$ $\quad L_x = \frac{\partial^2}{\partial x^2}(.)$ $\quad Nu = u\frac{\partial u}{\partial x}$ et $L_t^{-1} = \int_0^t (.)ds$
Equation (1) can be written as :

$$L_t u + Nu = \varepsilon L_x u \tag{2}$$

Applying L_t^{-1} to (2), we obtain :

$$u(t,x) = u(0,x) + \varepsilon L_t^{-1}(L_x u) - L_t^{-1}(Nu) \tag{3}$$

Assuming that the solution of (1) can be given by :

$$u(t,x) = \sum_{n=0}^{+\infty} u_n(t,x) \tag{4}$$

and

$$Nu(t,x) = \sum_{n=0}^{+\infty} A_n(t,x) \tag{5}$$

where A_n are the Adomian's polynomials with

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left(N\left(\sum_{i=0}^{+\infty}\lambda^i u_i\right)\right)\right]_{\lambda=0}$$

By substituting (4) and (5) into (3), we obtain the Adomian algorithm

$$\begin{cases} u_0(t,x) & = u(0,x) = \varepsilon\cos x \\ u_{n+1}(t,x) & = L_t^{-1}\left[\varepsilon L_x(u_n)\right] - L_t^{-1}(A_n) \end{cases} \tag{6}$$

with

$$\begin{aligned}
A_0 &= u_0\frac{\partial u_0}{\partial x} \\
A_1 &= u_0\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_0}{\partial x} \\
A_2 &= u_0\frac{\partial u_2}{\partial x} + u_1\frac{\partial u_1}{\partial x} + u_2\frac{\partial u_0}{\partial x} \\
A_3 &= u_0\frac{\partial u_3}{\partial x} + u_1\frac{\partial u_2}{\partial x} + u_2\frac{\partial u_1}{\partial x} + u_3\frac{\partial u_0}{\partial x} \\
A_4 &= u_0\frac{\partial u_4}{\partial x} + u_1\frac{\partial u_3}{\partial x} + u_2\frac{\partial u_2}{\partial x} + u_3\frac{\partial u_1}{\partial x} + u_4\frac{\partial u_0}{\partial x} \\
A_5 &= u_0\frac{\partial u_5}{\partial x} + u_1\frac{\partial u_4}{\partial x} + u_2\frac{\partial u_3}{\partial x} + u_3\frac{\partial u_2}{\partial x} + u_4\frac{\partial u_1}{\partial x} + u_5\frac{\partial u_0}{\partial x} \\
&\vdots
\end{aligned}$$

The Adomian's polynomials for Equation (1) are obtained from the recurrent relation :

$$A_n = \sum_{k=1}^{n} u_k \frac{\partial u_{n-k}}{\partial x}$$

Using the algorithm, we have :

$$\begin{aligned}
u_0(t,x) &= \varepsilon\cos x \\
u_1(t,x) &= \cos x(\sin x - 1)\varepsilon^2 t \\
u_2(t,x) &= \frac{3}{2}\cos x\sin x(\sin x - 2)\varepsilon^3 t^2 \\
u_3(t,x) &= \left(\frac{4}{3}\cos x\sin^3 x - \frac{37}{6}\cos x\sin^2 x - \frac{4}{3}\cos^3 x\sin x + \frac{13}{3}\cos x\sin x + \frac{7}{3}\cos^3 x\right)\varepsilon^4 t^3 \\
u_4(t,x) &= \left(\frac{17}{12}\cos x\sin^4 x - \frac{32}{3}\cos x\sin^3 x - \frac{83}{24}\cos^3 x\sin^2 x + \frac{431}{24}\cos x\sin^2 x + 13\cos^3 x\sin x\right. \\
&\quad \left. - \frac{13}{3}\cos x\sin x + \frac{1}{3}\cos^5 x - \frac{20}{3}\cos^3 x\right)\varepsilon^5 t^4
\end{aligned}$$

$$u_5(t,x) = \left(\frac{31}{20}\cos x\sin^5 x - \frac{169}{10}\cos x\sin^4 x - \frac{104}{15}\cos^3 x\sin^3 x + \frac{3007}{60}\cos x\sin^3 x + \frac{1737}{40}\cos^3 x\sin^2 x - \frac{4393}{120}\cos x\sin^2 x + \frac{139}{60}\cos^5 x\sin x - \frac{3551}{60}\cos^3 x\sin x + \frac{52}{15}\cos x\sin x - \frac{101}{20}\cos^5 x + \frac{155}{12}\cos^3 x\right)\varepsilon^6 t^5$$

$$u_6(t,x) = \left(\frac{149}{90}\cos x\sin^6 x - \frac{2263}{90}\cos x\sin^5 x - \frac{62}{5}\cos^3 x\sin^4 x + \frac{5158}{45}\cos x\sin^4 x + \frac{1137}{10}\cos^3 x\sin^3 x - \frac{15001}{90}\cos x\sin^3 x + \frac{2123}{240}\cos^5 x\sin^2 x - \frac{11311}{40}\cos^3 x\sin^2 x + \frac{41923}{720}\cos x\sin^2 x - \frac{1843}{45}\cos^5 x\sin x + \frac{1672}{9}\cos^3 x\sin x - \frac{104}{45}\cos x\sin x - \frac{53}{120}\cos^7 x + \frac{3959}{120}\cos^5 x - \frac{3593}{180}\cos^3 x\right)\varepsilon^7 t^6$$

$$u_7(t,x) = \left(\frac{2239}{1260}\cos x\sin^7 x - \frac{22499}{630}\cos x\sin^6 x - \frac{6413}{315}\cos^3 x\sin^5 x + \frac{581489}{2520}\cos x\sin^5 x + \frac{107543}{420}\cos^3 x\sin^4 x - \frac{696193}{1260}\cos x\sin^4 x + \frac{9283}{360}\cos^5 x\sin^3 x - \frac{249497}{252}\cos^3 x\sin^3 x + \frac{120913}{280}\cos x\sin^3 x - \frac{106927}{560}\cos^5 x\sin^2 x + \frac{649147}{504}\cos^3 x\sin^2 x - \frac{388013}{5040}\cos x\sin^2 x - \frac{10309}{2520}\cos^7 x\sin x + \frac{448411}{1260}\cos^5 x\sin x - \frac{232613}{504}\cos^3 x\sin x + \frac{416}{315}\cos x\sin x + \frac{4393}{420}\cos^7 x - \frac{366659}{2520}\cos^5 x + \frac{65561}{2520}\cos^3 x\right)\varepsilon^8 t^7$$

$$u_8(t,x) = \left(\frac{2141}{1120}\cos x\sin^8 x - \frac{164719}{3360}\cos x\sin^7 x - \frac{635443}{20160}\cos^3 x\sin^6 x + \frac{1070803}{2520}\cos x\sin^6 x + \frac{1046791}{2016}\cos^3 x\sin^5 x - \frac{1083247}{720}\cos x\sin^5 x + \frac{1278317}{20160}\cos^5 x\sin^4 x - \frac{1437851}{504}\cos^3 x\sin^4 x + \frac{8398399}{4032}\cos x\sin^4 x - \frac{1348789}{2016}\cos^5 x\sin^3 x + \frac{2045231}{336}\cos^3 x\sin^3 x - \frac{1335767}{1440}\cos x\sin^3 x - \frac{853289}{40320}\cos^7 x\sin^2 x + \frac{28319159}{13440}\cos^5 x\sin^2 x - \frac{186707971}{40320}\cos^3 x\sin^2 x + \frac{505633}{5760}\cos x\sin^2 x + \frac{227707}{2016}\cos^7 x\sin x - \frac{10707467}{5040}\cos^5 x\sin x + \frac{9731791}{10080}\cos^3 x\sin x - \frac{208}{315}\cos x\sin x + \frac{6271}{10080}\cos^9 x - \frac{84667}{720}\cos^7 x + \frac{5101111}{10080}\cos^5 x - \frac{74231}{2520}\cos^3 x\right)\varepsilon^9 t^8$$

$$u_9(t,x) = \left(\frac{2329}{1134}\cos x\sin^9 x - \frac{743033}{11340}\cos x\sin^8 x - \frac{302899}{6480}\cos^3 x\sin^7 x + \frac{33221861}{45360}\cos x\sin^7 x + \frac{176812897}{181440}\cos^3 x\sin^6 x - \frac{20293387}{5670}\cos x\sin^6 x + \frac{310521}{2240}\cos^5 x\sin^5 x - \frac{162776741}{22680}\cos^3 x\sin^5 x + \frac{1395344693}{181440}\cos x\sin^5 x - \frac{59045789}{30240}\cos^5 x\sin^4 x + \frac{2046962759}{90720}\cos^3 x\sin^4 x - \frac{8439397}{1296}\cos x\sin^4 x - \frac{90379}{1120}\cos^7 x\sin^3 x + \frac{1648070731}{181440}\cos^5 x\sin^3 x - \frac{27949478}{945}\cos^3 x\sin^3 x + \frac{311749463}{181440}\cos x\sin^3 x + \frac{82026443}{120960}\cos^7 x\sin^2 x - \frac{93215201}{5760}\cos^5 x\sin^2 x + \frac{725058449}{51840}\cos^3 x\sin^2 x - \frac{32061937}{362880}\cos x\sin^2 x + \frac{64409}{8640}\cos^9 x\sin x - \frac{281394229}{181440}\cos^7 x\sin x + \frac{258845209}{25920}\cos^5 x\sin x - \frac{319274183}{181440}\cos^3 x\sin x + \frac{832}{2835}\cos x\sin x - \frac{259691}{12096}\cos^9 x + \frac{161833933}{181440}\cos^7 x - \frac{270092791}{181440}\cos^5 x + \frac{5360911\cos^3 x}{181440}\right)\varepsilon^{10} t^9$$

$$u_{10}(t,x) = \left(\frac{200203}{90720}\cos x\sin^{10} x - \frac{9719573}{113400}\cos x\sin^9 x - \frac{60801233}{907200}\cos^3 x\sin^8 x + \frac{362656681}{302400}\cos x\sin^8 x\right.$$

$$+\frac{86825969}{50400}\cos^3 x\sin^7 x-\frac{1165812791}{151200}\cos x\sin^7 x+\frac{14411417}{51840}\cos^5 x\sin^6 x$$
$$-\frac{29616964789}{1814400}\cos^3 x\sin^6 x+\frac{5321063}{225}\cos x\sin^6 x-\frac{10814557}{2160}\cos^5 x\sin^5 x$$
$$+\frac{8004463421}{113400}\cos^3 x\sin^5 x-\frac{986918897}{30240}\cos x\sin^5 x-\frac{339991}{1344}\cos^7 x\sin^4 x$$
$$+\frac{28914427123}{907200}\cos^5 x\sin^4 x-\frac{12818515657}{90720}\cos^3 x\sin^4 x+\frac{1768850857}{100800}\cos x\sin^4 x$$
$$+\frac{50084897}{16800}\cos^7 x\sin^3 x-\frac{982040333}{11340}\cos^5 x\sin^3 x+\frac{18242022469}{151200}\cos^3 x\sin^3 x$$
$$-\frac{53206451}{18900}\cos x\sin^3 x+\frac{11768993}{241920}\cos^9 x\sin^2 x-\frac{20161762577}{1814400}\cos^7 x\sin^2 x$$
$$+\frac{59294401319}{604800}\cos^5 x\sin^2 x-\frac{3180047321}{86400}\cos^3 x\sin^2 x+\frac{57891343}{725760}\cos x\sin^2 x$$
$$-\frac{21934733}{75600}\cos^9 x\sin x+\frac{2207919751}{151200}\cos^7 x\sin x-\frac{199434703}{5040}\cos^5 x\sin x$$
$$+\frac{185026441}{64800}\cos^3 x\sin x-\frac{1664}{14175}\cos x\sin x-\frac{570809}{604800}\cos^{11} x+\frac{654015613}{1814400}\cos^9 x$$
$$\left.-\frac{9589453187}{1814400}\cos^7 x+\frac{2323615621}{604800}\cos^5 x-\frac{24158863}{907200}\cos^3 x\right)\varepsilon^{11}t^{10}$$

Finally, the approximate solution of (1) is given by :

$$u(t,x) \simeq u_0(t,x)+u_1(t,x)+u_2(t,x)+\cdots+u_{10}(t,x)$$

3. The regular perturbation method

Let us suppose that the solution $u(t,x)$ of the initial value problem (1) has the following form [9] :

$$u(t,x)=\sum_{n=0}^{+\infty}\varepsilon^n u_n(t,x)+R_N(t,x,\varepsilon) \tag{7}$$

where $R_N(t,x,\varepsilon)$ is the remainder of the series.
Taking (7) into (1), and collecting equal powers of ε we obtain a system of recurrent initial value problems

$$\varepsilon^0 \quad : \quad \begin{cases} \dfrac{\partial u_0}{\partial t} = -u_0\dfrac{\partial u_0}{\partial x} \\ u_0(0,x)=0 \end{cases}$$

$$\varepsilon^1 \quad : \quad \begin{cases} \dfrac{\partial u_1}{\partial t} = -u_0\dfrac{\partial u_1}{\partial x}-u_1\dfrac{\partial u_0}{\partial x}+\dfrac{\partial^2 u_0}{\partial x^2} \\ u_1(0,x)=\cos x \end{cases}$$

$$\varepsilon^2 \quad : \quad \begin{cases} \dfrac{\partial u_2}{\partial t} = -u_0\dfrac{\partial u_2}{\partial x}-u_1\dfrac{\partial u_1}{\partial x}-u_2\dfrac{\partial u_0}{\partial x}+\dfrac{\partial^2 u_1}{\partial x^2} \\ u_2(0,x)=0 \end{cases}$$

$$\vdots$$

$$\varepsilon^j \quad : \quad \begin{cases} \dfrac{\partial u_j}{\partial t} = -\displaystyle\sum_{k=0}^{j} u_k\dfrac{\partial u_{j-k}}{\partial x}+\dfrac{\partial^2 u_{j-1}}{\partial x^2} \\ u_j(0,x)=0 \end{cases}$$

For j from 0 to 10, we have

$$\begin{aligned}
u_0(t,x) &= 0\\
u_1(t,x) &= \cos x\\
u_2(t,x) &= (\cos x\sin x-\cos x)\\
u_3(t,x) &= \frac{3}{2}\cos x\sin x(\sin x-2)t^2\\
u_4(t,x) &= \left(\frac{4}{3}\cos x\sin^3 x-\frac{37}{6}\cos x\sin^2 x-\frac{4}{3}\cos^3 x\sin x+\frac{13}{3}\cos x\sin x+\frac{7}{3}\cos^3 x\right)t^3\\
u_5(t,x) &= \left(\frac{17}{12}\cos x\sin^4 x-\frac{32}{3}\cos x\sin^3 x-\frac{83}{24}\cos^3 x\sin^2 x+\frac{431}{24}\cos x\sin^2 x+13\cos^3 x\sin x\right.
\end{aligned}$$

$$
\begin{aligned}
& -\frac{13}{3}\cos x\sin x+\frac{1}{3}\cos^5 x-\frac{20}{3}\cos^3 x\Big)t^4 \\
u_6(t,x) &= \Big(\frac{31}{20}\cos x\sin^5 x-\frac{169}{10}\cos x\sin^4 x-\frac{104}{15}\cos^3 x\sin^3 x+\frac{3007}{60}\cos x\sin^3 x \\
&+\frac{1737}{40}\cos^3 x\sin^2 x-\frac{4393}{120}\cos x\sin^2 x+\frac{139}{60}\cos^5 x\sin x-\frac{3551}{60}\cos^3 x\sin x \\
&+\frac{52}{15}\cos x\sin x-\frac{101}{20}\cos^5 x+\frac{155}{12}\cos^3 x\Big)t^5 \\
u_7(t,x) &= \Big(\frac{149}{90}\cos x\sin^6 x-\frac{2263}{90}\cos x\sin^5 x-\frac{62}{5}\cos^3 x\sin^4 x+\frac{5158}{45}\cos x\sin^4 x \\
&+\frac{1137}{10}\cos^3 x\sin^3 x-\frac{15001}{90}\cos x\sin^3 x+\frac{2123}{240}\cos^5 x\sin^2 x-\frac{11311}{40}\cos^3 x\sin^2 x \\
&+\frac{41923}{720}\cos x\sin^2 x-\frac{1843}{45}\cos^5 x\sin x+\frac{1672}{9}\cos^3 x\sin x-\frac{104}{45}\cos x\sin x \\
&-\frac{53}{120}\cos^7 x+\frac{3959}{120}\cos^5 x-\frac{3593}{180}\cos^3 x\Big)t^6 \\
u_8(t,x) &= \Big(\frac{2239}{1260}\cos x\sin^7 x-\frac{22499}{630}\cos x\sin^6 x-\frac{6413}{315}\cos^3 x\sin^5 x+\frac{581489}{2520}\cos x\sin^5 x \\
&+\frac{107543}{420}\cos^3 x\sin^4 x-\frac{696193}{1260}\cos x\sin^4 x+\frac{9283}{360}\cos^5 x\sin^3 x-\frac{249497}{252}\cos^3 x\sin^3 x \\
&+\frac{120913}{280}\cos x\sin^3 x-\frac{106927}{560}\cos^5 x\sin^2 x+\frac{649147}{504}\cos^3 x\sin^2 x-\frac{388013}{5040}\cos x\sin^2 x \\
&-\frac{10309}{2520}\cos^7 x\sin x+\frac{448411}{1260}\cos^5 x\sin x-\frac{232613}{504}\cos^3 x\sin x+\frac{416}{315}\cos x\sin x \\
&+\frac{4393}{420}\cos^7 x-\frac{366659}{2520}\cos^5 x+\frac{65561}{2520}\cos^3 x\Big)t^7 \\
u_9(t,x) &= \Big(\frac{2141}{1120}\cos x\sin^8 x-\frac{164719}{3360}\cos x\sin^7 x-\frac{635443}{20160}\cos^3 x\sin^6 x+\frac{1070803}{2520}\cos x\sin^6 x \\
&+\frac{1046791}{2016}\cos^3 x\sin^5 x-\frac{1083247}{720}\cos x\sin^5 x+\frac{1278317}{20160}\cos^5 x\sin^4 x-\frac{1437851}{504}\cos^3 x\sin^4 x \\
&+\frac{8398399}{4032}\cos x\sin^4 x-\frac{1348789}{2016}\cos^5 x\sin^3 x+\frac{2045231}{336}\cos^3 x\sin^3 x-\frac{1335767}{1440}\cos x\sin^3 x \\
&-\frac{853289}{40320}\cos^7 x\sin^2 x+\frac{28319159}{13440}\cos^5 x\sin^2 x-\frac{186707971}{40320}\cos^3 x\sin^2 x+\frac{505633}{5760}\cos x\sin^2 x \\
&+\frac{227707}{2016}\cos^7 x\sin x-\frac{10707467}{5040}\cos^5 x\sin x+\frac{9731791}{10080}\cos^3 x\sin x-\frac{208}{315}\cos x\sin x \\
&+\frac{6271}{10080}\cos^9 x-\frac{84667}{720}\cos^7 x+\frac{5101111}{10080}\cos^5 x-\frac{74231}{2520}\cos^3 x\Big)t^8 \\
u_{10}(t,x) &= \Big(\frac{2329}{1134}\cos x\sin^9 x-\frac{743033}{11340}\cos x\sin^8 x-\frac{302899}{6480}\cos^3 x\sin^7 x+\frac{33221861}{45360}\cos x\sin^7 x \\
&+\frac{176812897}{181440}\cos^3 x\sin^6 x-\frac{20293387}{5670}\cos x\sin^6 x+\frac{310521}{2240}\cos^5 x\sin^5 x \\
&-\frac{162776741}{22680}\cos^3 x\sin^5 x+\frac{1395344693}{181440}\cos x\sin^5 x-\frac{59045789}{30240}\cos^5 x\sin^4 x \\
&+\frac{2046962759}{90720}\cos^3 x\sin^4 x-\frac{8439397}{1296}\cos x\sin^4 x-\frac{90379}{1120}\cos^7 x\sin^3 x \\
&+\frac{1648070731}{181440}\cos^5 x\sin^3 x-\frac{27949478}{945}\cos^3 x\sin^3 x+\frac{311749463}{181440}\cos x\sin^3 x \\
&+\frac{82026443}{120960}\cos^7 x\sin^2 x-\frac{93215201}{5760}\cos^5 x\sin^2 x+\frac{725058449}{51840}\cos^3 x\sin^2 x \\
&-\frac{32061937}{362880}\cos x\sin^2 x+\frac{64409}{8640}\cos^9 x\sin x-\frac{281394229}{181440}\cos^7 x\sin x \\
&+\frac{258845209}{25920}\cos^5 x\sin x-\frac{319274183}{181440}\cos^3 x\sin x+\frac{832}{2835}\cos x\sin x-\frac{259691}{12096}\cos^9 x \\
&+\frac{161833933}{181440}\cos^7 x-\frac{270092791}{181440}\cos^5 x+\frac{5360911}{181440}\cos^3 x\Big)t^9
\end{aligned}
$$

Hence, the approximate solution of (1) is given by :

$$u(t,x) \simeq u_0(t,x)+\varepsilon u_1(t,x)+\varepsilon^2 u_2(t,x)+\cdots+\varepsilon^{10}u_{10}(t,x)$$

4. Application of HPM to Burgers equation

In order to apply the HPM [6, 7, 8], we construct a homotopie $H(v,p)$ which satisfies :

$$H(v,p) = (1-p)\left[\frac{\partial v}{\partial t} - \frac{\partial u_0}{\partial t}\right] + p\left[\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} - \varepsilon\frac{\partial^2 v}{\partial x^2}\right]$$

As $H(v,p) = 0$, then we have :

$$\frac{\partial v}{\partial t} - \frac{\partial u_0}{\partial t} + p\frac{\partial u_0}{\partial t} + pv\frac{\partial v}{\partial x} - p\varepsilon\frac{\partial^2 v}{\partial x^2} = 0 \tag{8}$$

Suppose the solution of (1) has the form

$$v = v_0 + pv_1 + p^2v_2 + p^3v_3 + p^4v_4 + p^5v_5 + \cdots \tag{9}$$

Substituting (9) into (8), and equating the terms with the identical powers of p,

$$p^0 \quad : \quad \begin{cases} \dfrac{\partial v_0}{\partial t} = \dfrac{\partial u_0}{\partial t} \\ v_0(0,x) = \varepsilon\cos x \end{cases}$$

$$p^1 \quad : \quad \begin{cases} \dfrac{\partial v_1}{\partial t} = -\dfrac{\partial u_0}{\partial t} - v_0\dfrac{\partial v_0}{\partial x} + \varepsilon\dfrac{\partial^2 v_0}{\partial x^2} \\ v_1(0,x) = 0 \end{cases}$$

$$p^2 \quad : \quad \begin{cases} \dfrac{\partial v_2}{\partial t} = -v_0\dfrac{\partial v_1}{\partial x} - v_1\dfrac{\partial v_0}{\partial x} + \varepsilon\dfrac{\partial^2 v_1}{\partial x^2} \\ v_2(0,x) = 0 \end{cases}$$

$$\vdots$$

$$p^j \quad : \quad \begin{cases} \dfrac{\partial v_j}{\partial t} = -\displaystyle\sum_{k=0}^{j-1} v_k\dfrac{\partial v_{j-k-1}}{\partial x} + \varepsilon\dfrac{\partial^2 v_{j-1}}{\partial x^2} \\ v_j(0,x) = 0 \end{cases}$$

For j from 0 to 10, we have

$$\begin{aligned}
v_0(t,x) &= \varepsilon\cos x \\
v_1(t,x) &= (\cos x\sin x - \cos x)\,\varepsilon^2 t \\
v_2(t,x) &= \frac{3}{2}\cos x\sin x(\sin x - 2)\varepsilon^3 t^2 \\
v_3(t,x) &= \left(\frac{4}{3}\cos x\sin^3 x - \frac{37}{6}\cos x\sin^2 x - \frac{4}{3}\cos^3 x\sin x + \frac{13}{3}\cos x\sin x + \frac{7}{3}\cos^3 x\right)\varepsilon^4 t^3 \\
v_4(t,x) &= \left(\frac{17}{12}\cos x\sin^4 x - \frac{32}{3}\cos x\sin^3 x - \frac{83}{24}\cos^3 x\sin^2 x + \frac{431}{24}\cos x\sin^2 x + 13\cos^3 x\sin x\right. \\
&\quad \left. - \frac{13}{3}\cos x\sin x + \frac{1}{3}\cos^5 x - \frac{20}{3}\cos^3 x\right)\varepsilon^5 t^4 \\
v_5(t,x) &= \left(\frac{31}{20}\cos x\sin^5 x - \frac{169}{10}\cos x\sin^4 x - \frac{104}{15}\cos^3 x\sin^3 x + \frac{3007}{60}\cos x\sin^3 x + \frac{1737}{40}\cos^3 x\sin^2 x\right. \\
&\quad - \frac{4393}{120}\cos x\sin^2 x + \frac{139}{60}\cos^5 x\sin x - \frac{3551}{60}\cos^3 x\sin x + \frac{52}{15}\cos x\sin x \\
&\quad \left. - \frac{101}{20}\cos^5 x + \frac{155}{12}\cos^3 x\right)\varepsilon^6 t^5 \\
v_6(t,x) &= \left(\frac{149}{90}\cos x\sin^6 x - \frac{2263}{90}\cos x\sin^5 x - \frac{62}{5}\cos^3 x\sin^4 x + \frac{5158}{45}\cos x\sin^4 x\right. \\
&\quad + \frac{1137}{10}\cos^3 x\sin^3 x - \frac{15001}{90}\cos x\sin^3 x + \frac{2123}{240}\cos^5 x\sin^2 x - \frac{11311}{40}\cos^3 x\sin^2 x \\
&\quad + \frac{41923}{720}\cos x\sin^2 x - \frac{1843}{45}\cos^5 x\sin x + \frac{1672}{9}\cos^3 x\sin x - \frac{104}{45}\cos x\sin x \\
&\quad \left. - \frac{53}{120}\cos^7 x + \frac{3959}{120}\cos^5 x - \frac{3593}{180}\cos^3 x\right)\varepsilon^7 t^6 \\
v_7(t,x) &= \left(\frac{2239}{1260}\cos x\sin^7 x - \frac{22499}{630}\cos x\sin^6 x - \frac{6413}{315}\cos^3 x\sin^5 x + \frac{581489}{2520}\cos x\sin^5 x\right.
\end{aligned}$$

$$
\begin{aligned}
&+\frac{107543}{420}\cos^3 x\sin^4 x-\frac{696193}{1260}\cos x\sin^4 x+\frac{9283}{360}\cos^5 x\sin^3 x-\frac{249497}{252}\cos^3 x\sin^3 x\\
&+\frac{120913}{280}\cos x\sin^3 x-\frac{106927}{560}\cos^5 x\sin^2 x+\frac{649147}{504}\cos^3 x\sin^2 x-\frac{388013}{5040}\cos x\sin^2 x\\
&-\frac{10309}{2520}\cos^7 x\sin x+\frac{448411}{1260}\cos^5 x\sin x-\frac{232613}{504}\cos^3 x\sin x+\frac{416}{315}\cos x\sin x\\
&+\frac{4393}{420}\cos^7 x-\frac{366659}{2520}\cos^5 x+\frac{65561}{2520}\cos^3 x\Big)\varepsilon^8 t^7
\end{aligned}
$$

$$
\begin{aligned}
v_8(t,x) \;=\; &\Big(\frac{2141}{1120}\cos x\sin^8 x-\frac{164719}{3360}\cos x\sin^7 x-\frac{635443}{20160}\cos^3 x\sin^6 x+\frac{1070803}{2520}\cos x\sin^6 x\\
&+\frac{1046791}{2016}\cos^3 x\sin^5 x-\frac{1083247}{720}\cos x\sin^5 x+\frac{1278317}{20160}\cos^5 x\sin^4 x-\frac{1437851}{504}\cos^3 x\sin^4 x\\
&+\frac{8398399}{4032}\cos x\sin^4 x-\frac{1348789}{2016}\cos^5 x\sin^3 x+\frac{2045231}{336}\cos^3 x\sin^3 x-\frac{1335767}{1440}\cos x\sin^3 x\\
&-\frac{853289}{40320}\cos^7 x\sin^2 x+\frac{28319159}{13440}\cos^5 x\sin^2 x-\frac{186707971}{40320}\cos^3 x\sin^2 x+\frac{505633}{5760}\cos x\sin^2 x\\
&+\frac{227707}{2016}\cos^7 x\sin x-\frac{10707467}{5040}\cos^5 x\sin x+\frac{9731791}{10080}\cos^3 x\sin x-\frac{208}{315}\cos x\sin x\\
&+\frac{6271}{10080}\cos^9 x-\frac{84667}{720}\cos^7 x+\frac{5101111}{10080}\cos^5 x-\frac{74231}{2520}\cos^3 x\Big)\varepsilon^9 t^8
\end{aligned}
$$

$$
\begin{aligned}
v_9(t,x) \;=\; &\Big(\frac{2329}{1134}\cos x\sin^9 x-\frac{743033}{11340}\cos x\sin^8 x-\frac{302899}{6480}\cos^3 x\sin^7 x+\frac{33221861}{45360}\cos x\sin^7 x\\
&+\frac{176812897}{181440}\cos^3 x\sin^6 x-\frac{20293387}{5670}\cos x\sin^6 x+\frac{310521}{2240}\cos^5 x\sin^5 x\\
&-\frac{162776741}{22680}\cos^3 x\sin^5 x+\frac{1395344693}{181440}\cos x\sin^5 x-\frac{59045789}{30240}\cos^5 x\sin^4 x\\
&+\frac{2046962759}{90720}\cos^3 x\sin^4 x-\frac{8439397}{1296}\cos x\sin^4 x-\frac{90379}{1120}\cos^7 x\sin^3 x\\
&+\frac{1648070731}{181440}\cos^5 x\sin^3 x-\frac{27949478}{945}\cos^3 x\sin^3 x+\frac{311749463}{181440}\cos x\sin^3 x\\
&+\frac{82026443}{120960}\cos^7 x\sin^2 x-\frac{93215201}{5760}\cos^5 x\sin^2 x+\frac{725058449}{51840}\cos^3 x\sin^2 x\\
&-\frac{32061937}{362880}\cos x\sin^2 x+\frac{64409}{8640}\cos^9 x\sin x-\frac{281394229}{181440}\cos^7 x\sin x\\
&+\frac{258845209}{25920}\cos^5 x\sin x-\frac{319274183}{181440}\cos^3 x\sin x+\frac{832}{2835}\cos x\sin x\\
&-\frac{259691}{12096}\cos^9 x+\frac{161833933}{181440}\cos^7 x-\frac{270092791}{181440}\cos^5 x+\frac{5360911}{181440}\cos^3 x\Big)\varepsilon^{10} t^9
\end{aligned}
$$

$$
\begin{aligned}
v_{10}(t,x) \;=\; &\Big(\frac{200203}{90720}\cos x\sin^{10} x-\frac{9719573}{113400}\cos x\sin^9 x-\frac{60801233}{907200}\cos^3 x\sin^8 x\\
&+\frac{362656681}{302400}\cos x\sin^8 x+\frac{86825969}{50400}\cos^3 x\sin^7 x-\frac{1165812791}{151200}\cos x\sin^7 x\\
&+\frac{14411417}{51840}\cos^5 x\sin^6 x-\frac{29616964789}{1814400}\cos^3 x\sin^6 x+\frac{5321063}{225}\cos x\sin^6 x\\
&-\frac{10814557}{2160}\cos^5 x\sin^5 x+\frac{8004463421}{113400}\cos^3 x\sin^5 x-\frac{986918897}{30240}\cos x\sin^5 x\\
&-\frac{339991}{1344}\cos^7 x\sin^4 x+\frac{28914427123}{907200}\cos^5 x\sin^4 x-\frac{12818515657}{90720}\cos^3 x\sin^4 x\\
&+\frac{1768850857}{100800}\cos x\sin^4 x+\frac{50084897}{16800}\cos^7 x\sin^3 x-\frac{982040333}{11340}\cos^5 x\sin^3 x\\
&+\frac{18242022469}{151200}\cos^3 x\sin^3 x-\frac{53206451}{18900}\cos x\sin^3 x+\frac{11768993}{241920}\cos^9 x\sin^2 x\\
&-\frac{20161762577}{1814400}\cos^7 x\sin^2 x+\frac{59294401319}{604800}\cos^5 x\sin^2 x-\frac{3180047321}{86400}\cos^3 x\sin^2 x\\
&+\frac{57891343}{725760}\cos x\sin^2 x-\frac{21934733}{75600}\cos^9 x\sin x+\frac{2207919751}{151200}\cos^7 x\sin x\\
&-\frac{199434703}{5040}\cos^5 x\sin x+\frac{185026441}{64800}\cos^3 x\sin x-\frac{1664}{14175}\cos x\sin x-\frac{570809}{604800}\cos^{11} x\\
&+\frac{654015613}{1814400}\cos^9 x-\frac{9589453187}{1814400}\cos^7 x+\frac{2323615621}{604800}\cos^5 x-\frac{24158863}{907200}\cos^3 x\Big)\varepsilon^{11} t^{10}
\end{aligned}
$$

Hence, the approximate solution of (1) is given by :

$$
u(t,x) \;\simeq\; \lim_{p\to 1}\left[v_0(t,x)+pv_1(t,x)+p^2 v_2(t,x)+\cdots+p^{10}v_{10}(t,x)\right]
$$

$$\simeq \quad v_0(t,x)+v_1(t,x)+v_2(t,x)+\cdots+v_{10}(t,x)$$

5. Solutions analysis

In this section we analyze the approximate solutions of (1) obtained by the three numerical methods (ADM, RPM and ADM).
We note that the approximate solutions obtained by HPM and DAM are the same ones. On the other hand the solution obtained by RPM differ from two methods (HPM and ADM) of an order.

Computations of Absolute Errors

Consider the equation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=\varepsilon\frac{\partial^2 u}{\partial x^2}$$

Assuming that the approximate solution of (1) in order n at the grid point (t_i,x_i) is $u_n(t_i,x_i)$. Thus the absolute errors, were computed by use of the formula :

$$err=\left|\frac{\partial u_n(t_i,x_i)}{\partial t}+u_n(t_i,x_i)\frac{\partial u_n(t_i,x_i)}{\partial x}-\varepsilon\frac{\partial^2 u_n(t_i,x_i)}{\partial x^2}\right|$$

Tables (**??**), (2), (3) give some values of the approximate solutions of order 10 for different values of ε.
It is noted that the numerical values given by the three methods are almost the same ones. However, we note that a difference on the absolute errors.

Table 1: Approximate solutions by ADM, RPM and HPM, errors for $t=0.2$ and $\varepsilon=0.001$

x	u_{adm}	u_{rpm}	u_{hpm}	err_{adm}	err_{rpm}	err_{hpm}
0	0.9998×10^{-3}	0.9998×10^{-3}	0.9998×10^{-3}	0.1512×10^{-37}	0.0568×10^{-34}	0.1512×10^{-37}
0.1	0.9948×10^{-3}	0.9948×10^{-3}	0.9948×10^{-3}	0.1510×10^{-37}	0.1392×10^{-34}	0.1510×10^{-37}
0.2	0.9799×10^{-3}	0.9799×10^{-3}	0.9799×10^{-3}	0.0999×10^{-37}	0.1557×10^{-34}	0.0999×10^{-37}
0.3	0.9552×10^{-3}	0.9552×10^{-3}	0.9552×10^{-3}	0.0330×10^{-37}	0.1217×10^{-34}	0.0330×10^{-37}
0.4	0.9209×10^{-3}	0.9209×10^{-3}	0.9209×10^{-3}	0.0209×10^{-37}	0.0644×10^{-34}	0.0209×10^{-37}
0.5	0.8775×10^{-3}	0.8775×10^{-3}	0.8775×10^{-3}	0.0483×10^{-37}	0.0092×10^{-34}	0.0483×10^{-37}
0.6	0.8253×10^{-3}	0.8253×10^{-3}	0.8253×10^{-3}	0.0501×10^{-37}	0.0282×10^{-34}	0.0501×10^{-37}
0.7	0.7648×10^{-3}	0.7648×10^{-3}	0.7648×10^{-3}	0.0361×10^{-37}	0.0437×10^{-34}	0.0361×10^{-37}
0.8	0.6967×10^{-3}	0.6967×10^{-3}	0.6967×10^{-3}	0.0172×10^{-37}	0.0415×10^{-34}	0.0172×10^{-37}
0.9	0.6216×10^{-3}	0.6216×10^{-3}	0.6216×10^{-3}	0.0015×10^{-37}	0.0294×10^{-34}	0.0015×10^{-37}
1	0.5403×10^{-3}	0.5403×10^{-3}	0.5403×10^{-3}	0.0078×10^{-37}	0.0150×10^{-34}	0.0078×10^{-37}

Table 2: Approximate solutions by ADM, RPM and HPM, errors for $t=0.5$ and $\varepsilon=0.1$

x	u_{adm}	u_{rpm}	u_{hpm}	err_{adm}	err_{rpm}	err_{hpm}
0	0.0950	0.0950	0.0950	0.1431×10^{-9}	0.2071×10^{-9}	0.1431×10^{-9}
0.1	0.0950	0.0950	0.0950	0.1440×10^{-9}	0.5257×10^{-9}	0.1440×10^{-9}
0.2	0.0940	0.0940	0.0940	0.0960×10^{-9}	0.5931×10^{-9}	0.0960×10^{-9}
0.3	0.0920	0.0920	0.0920	0.0323×10^{-9}	0.4664×10^{-9}	0.0323×10^{-9}
0.4	0.0891	0.0891	0.0891	0.0194×10^{-9}	0.2486×10^{-9}	0.0194×10^{-9}
0.5	0.0853	0.0853	0.0853	0.0459×10^{-9}	0.0375×10^{-9}	0.0459×10^{-9}
0.6	0.0805	0.0805	0.0805	0.0479×10^{-9}	0.1065×10^{-9}	0.0479×10^{-9}
0.7	0.0749	0.0749	0.0749	0.0346×10^{-9}	0.1667×10^{-9}	0.0346×10^{-9}
0.8	0.0685	0.0685	0.0685	0.0165×10^{-9}	0.1588×10^{-9}	0.0165×10^{-9}
0.9	0.0613	0.0613	0.0613	0.0014×10^{-9}	0.1128×10^{-9}	0.0014×10^{-9}
1	0.0534	0.0534	0.0534	0.0074×10^{-9}	0.0574×10^{-9}	0.0074×10^{-9}

Approximate solution of order k

Let k be a natural number. We indicate by φ_k the approximate solution of (1) of order k. One computes it, in the following way :

$$\varphi_k(t,x)=\sum_{i=0}^{k}U_i(t,x)$$

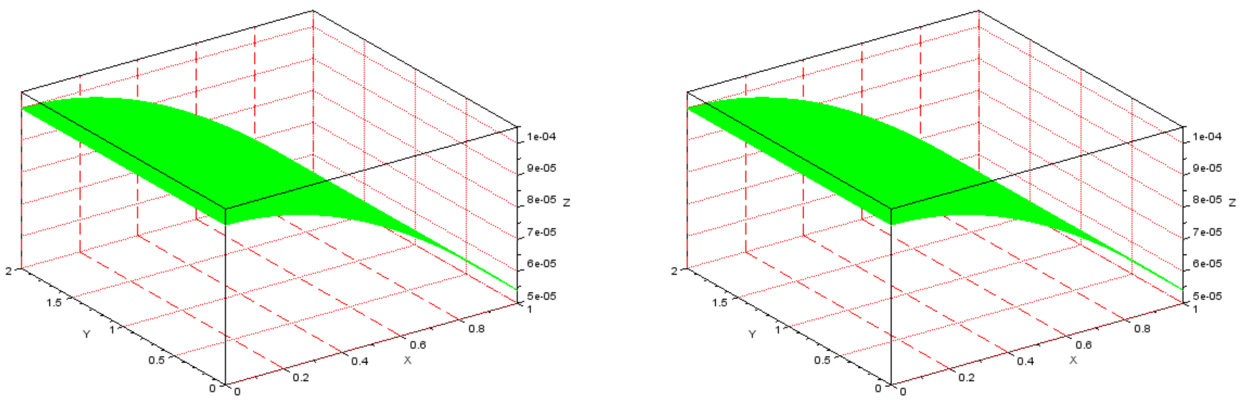
where $U_i(t,x)$ is the i th term of the approximate solution.
For $k=0,\,1,\cdots,\,10$, we give in tables (4), (5), (6) approximate solutions of order k obtained by the various methods with the corresponding absolute errors.
Figures (1) to (6) give the curves of the solutions approached by ADM, RPM and HPM for various values of ε

Table 3: Approximate solutions by ADM, RPM and HPM, errors for $t = 0.8$ and $\varepsilon = 0.3$

x	u_{adm}	u_{rpm}	u_{hpm}	err_{adm}	err_{rpm}	err_{hpm}
0	0.2330	0.2332	0.2330	0.0081	0.0021	0.0081
0.1	0.2352	0.2357	0.2352	0.0084	0.0061	0.0084
0.2	0.2352	0.2358	0.2352	0.0057	0.0071	0.0057
0.3	0.2329	0.2334	0.2329	0.0020	0.0057	0.0020
0.4	0.2281	0.2283	0.2281	0.0010	0.0031	0.0010
0.5	0.2207	0.2207	0.2207	0.0026	0.0006	0.0026
0.6	0.2106	0.2104	0.2106	0.0028	0.0012	0.0028
0.7	0.1978	0.1976	0.1978	0.0020	0.0020	0.0020
0.8	0.1825	0.1823	0.1825	0.0010	0.0019	0.0010
0.9	0.1647	0.1646	0.1647	0.0001	0.0014	0.0001
1	0.1647	0.1446	0.1647	0.0004	0.0007	0.0004

Table 4: Solution truncated obtained by ADM, RPM and HPM for $x = 0.1$, $t = 0$ and $\varepsilon = 0.001$

k	u_{adm}	u_{rpm}	u_{hpm}	err_{adm}	err_{rpm}	err_{hpm}
0	0.9950×10^{-3}	0	0.9950×10^{-3}	0.8957×10^{-6}	0	0.8957×10^{-6}
1	0.9950×10^{-3}	0.9950×10^{-3}	0.9950×10^{-3}	0	0.8957×10^{-6}	0
2	0.9950×10^{-3}	0.9950×10^{-3}	0.9950×10^{-3}	0	0	0
3	0.9950×10^{-3}	0.9950×10^{-3}	0.9950×10^{-3}	0	0	0
4	0.9950×10^{-3}	0.9950×10^{-3}	0.9950×10^{-3}	0	0	0
5	0.9950×10^{-3}	0.9950×10^{-3}	0.9950×10^{-3}	0	0	0
6	0.9950×10^{-3}	0.9950×10^{-3}	0.9950×10^{-3}	0	0	0
7	0.9800×10^{-3}	0.9800×10^{-3}	0.9800×10^{-3}	0	0	0
8	0.9800×10^{-3}	0.9800×10^{-3}	0.9800×10^{-3}	0	0	0
9	0.9800×10^{-3}	0.9800×10^{-3}	0.9800×10^{-3}	0	0	0
10	0.9800×10^{-3}	0.9800×10^{-3}	0.9800×10^{-3}	0	0	0



(a) HPM solution

(b) RPM solution

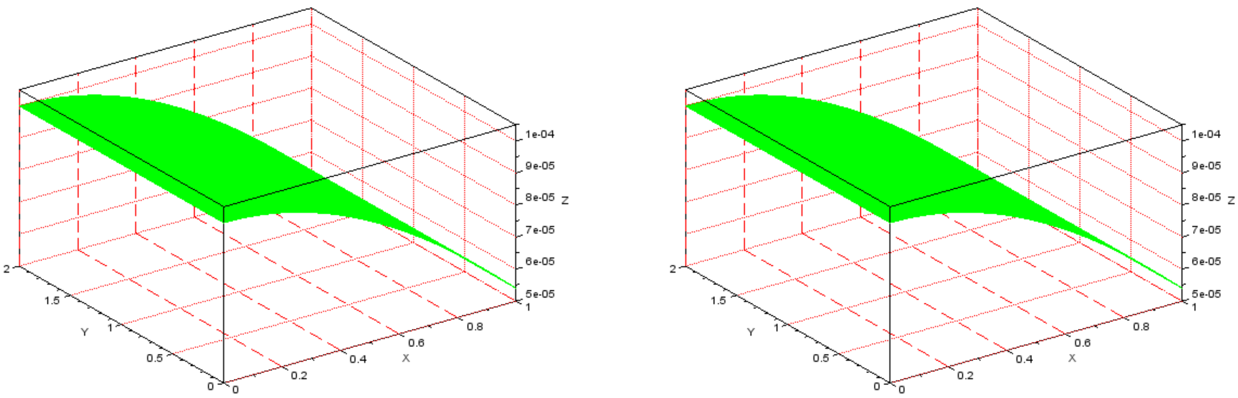
Figure 1: Comparison of the HPM solution with RPM solution for $\varepsilon = 0.0001$

Table 5: Solution truncated obtained by ADM, RPM and HPM for $x = 0.2$, $t = 0.05$ and $\varepsilon = 0.001$

k	u_{adm}	u_{rpm}	u_{hpm}	err_{adm}	err_{rpm}	err_{hpm}
0	0.9801×10^{-3}	0	0.9801×10^{-3}	0.7854×10^{-6}	0	0.7854×10^{-6}
1	0.9800×10^{-3}	0.9801×10^{-3}	0.9800×10^{-3}	0.0001×10^{-6}	0.7854×10^{-6}	0.0001×10^{-6}
2	0.9800×10^{-3}	0.9800×10^{-3}	0.9800×10^{-3}	0	0.0001×10^{-6}	0
3	0.9800×10^{-3}	0.9800×10^{-3}	0.9800×10^{-3}	0	0	0
4	0.9800×10^{-3}	0.9800×10^{-3}	0.9800×10^{-3}	0	0	0
5	0.9800×10^{-3}	0.9800×10^{-3}	0.9800×10^{-3}	0	0	0
6	0.9800×10^{-3}	0.9800×10^{-3}	0.9800×10^{-3}	0	0	0
7	0.9800×10^{-3}	0.9800×10^{-3}	0.9800×10^{-3}	0	0	0
8	0.9800×10^{-3}	0.9800×10^{-3}	0.9800×10^{-3}	0	0	0
9	0.9800×10^{-3}	0.9800×10^{-3}	0.9800×10^{-3}	0	0	0
10	0.9800×10^{-3}	0.9800×10^{-3}	0.9800×10^{-3}	0	0	0

Table 6: Solution truncated obtained by ADM, RPM and HPM for $x = 0.5$, $t = 0.1$ and $\varepsilon = 0.01$

k	u_{adm}	u_{rpm}	u_{hpm}	err_{adm}	err_{rpm}	err_{hpm}
0	0.0088	0	0.0088	0.4568×10^{-4}	0	0.4568×10^{-4}
1	0.0088	0.0088	0.0088	0.0019×10^{-4}	0.4568×10^{-4}	0.0019×10^{-4}
2	0.0088	0.0088	0.0088	0	0.0019×10^{-4}	0
3	0.0088	0.0088	0.0088	0	0	0
4	0.0088	0.0088	0.0088	0	0	0
5	0.0088	0.0088	0.0088	0	0	0
6	0.0088	0.0088	0.0088	0	0	0
7	0.0088	0.0088	0.0088	0	0	0
8	0.0088	0.0088	0.0088	0	0	0
9	0.0088	0.0088	0.0088	0	0	0
10	0.0088	0.0088	0.0088	0	0	0

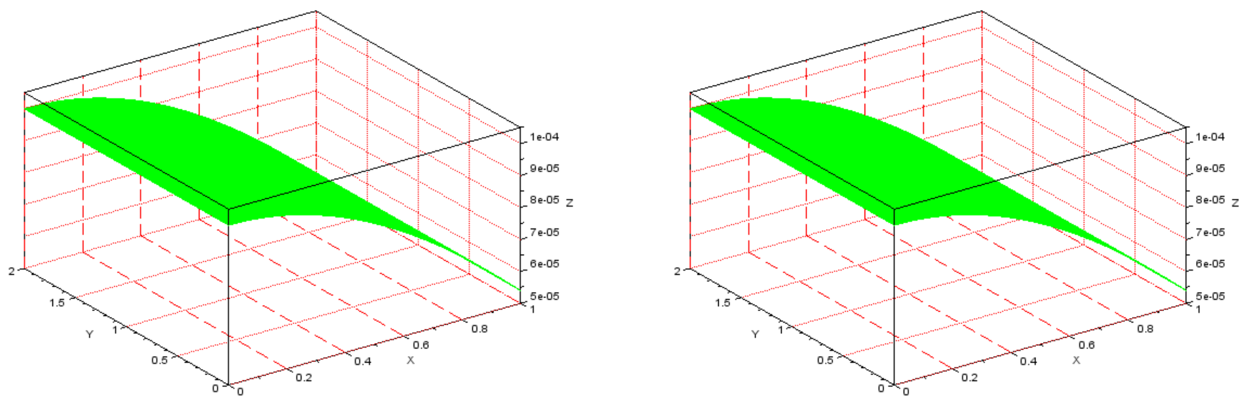


(a) HPM solution (b) ADM solution

Figure 2: Comparison of the HPM solution with ADM solution for $\varepsilon = 0.0001$

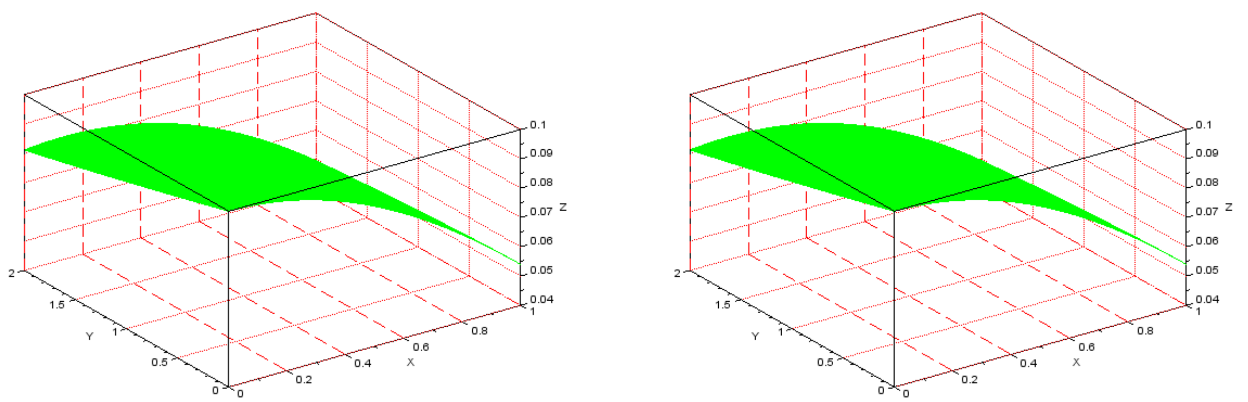
Table 7: Solution truncated obtained by ADM, RPM and HPM for $x = 0.75$, $t = 0.3$ and $\varepsilon = 0.1$

k	u_{adm}	u_{rpm}	u_{hpm}	err_{adm}	err_{rpm}	err_{hpm}
0	0.0732	0	0.0732	0.0023	0	0.0023
1	0.0725	0.0732	0.0725	0.0006	0.0023	0.0006
2	0.0724	0.0725	0.0724	0	0.0006	0
3	0.0724	0.0724	0.0724	0	0	0
4	0.0724	0.0724	0.0724	0	0	0
5	0.0724	0.0724	0.0724	0	0	0
6	0.0724	0.0724	0.0724	0	0	0
7	0.0724	0.0724	0.0724	0	0	0
8	0.0724	0.0724	0.0724	0	0	0
9	0.0724	0.0724	0.0724	0	0	0
10	0.0724	0.0724	0.0724	0	0	0



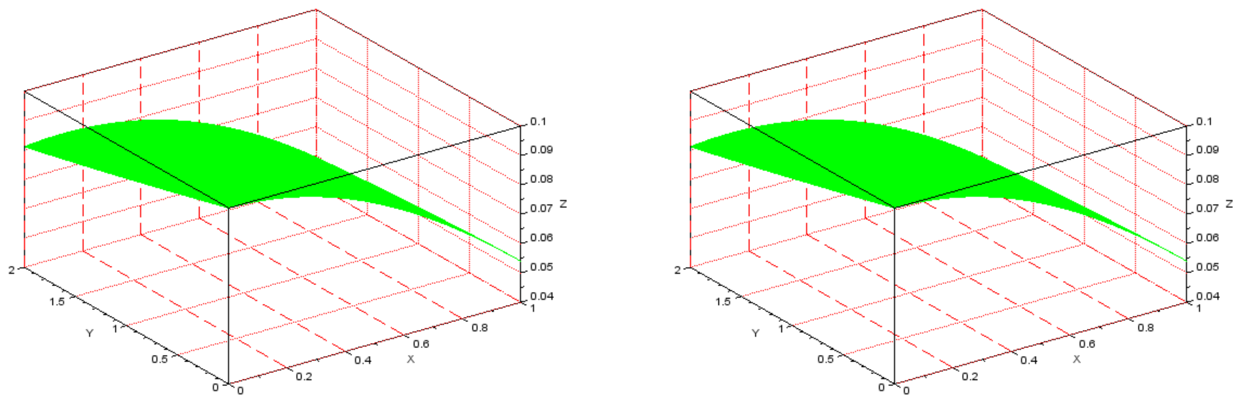
(a) ADM solution (b) RPM solution

Figure 3: Comparison of the ADM solution with RPM solution for $\varepsilon = 0.0001$



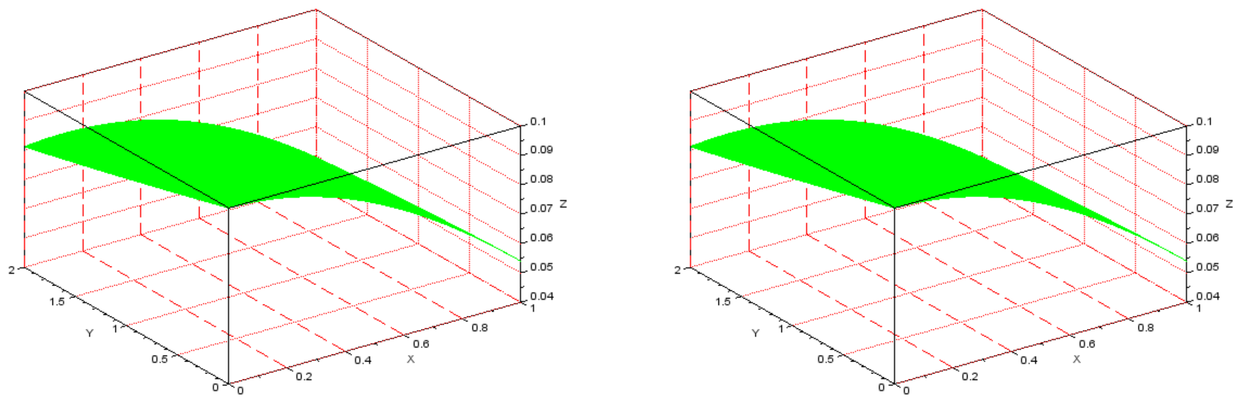
(a) HPM solution (b) RPM solution

Figure 4: Comparison of the HPM solution with RPM solution for $\varepsilon = 0.1$



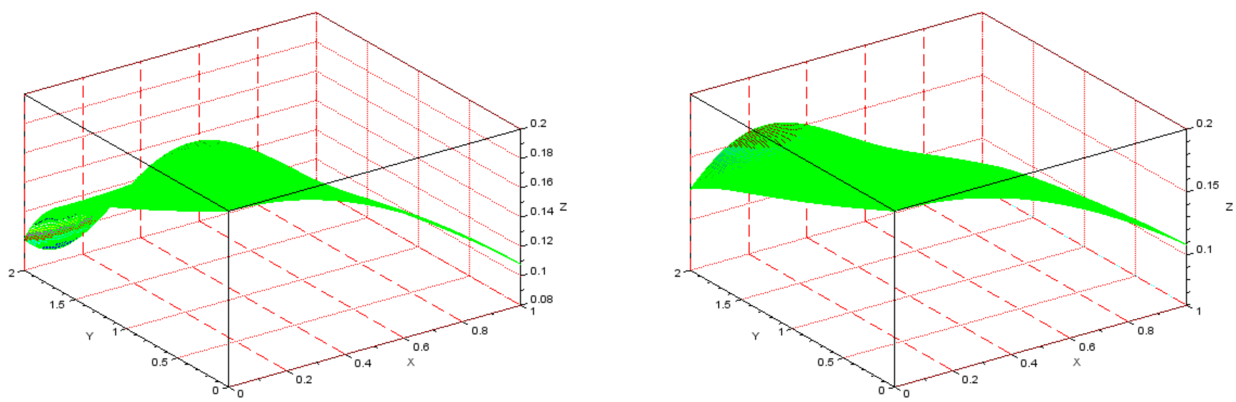
(a) HPM solution (b) ADM solution

Figure 5: Comparison of the HPM solution with ADM solution for $\varepsilon = 0.1$



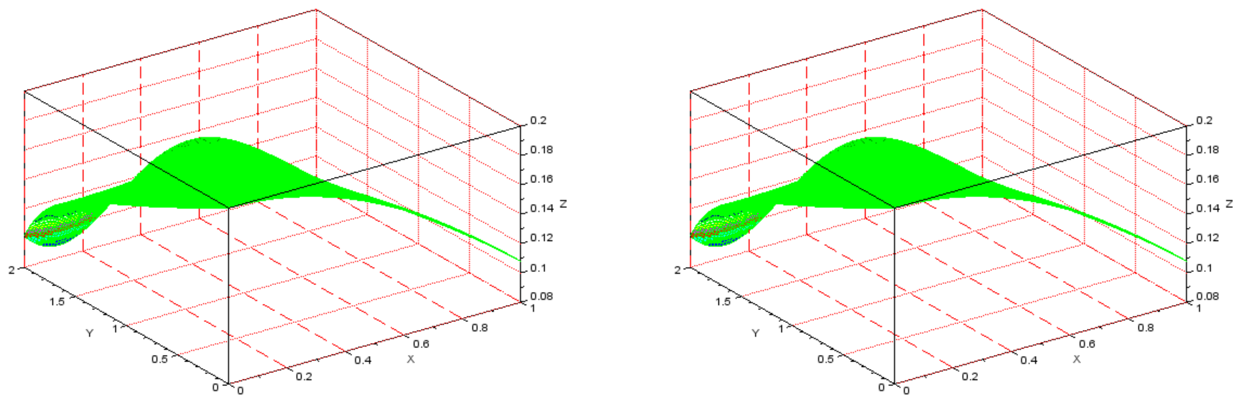
(a) ADM solution (b) RPM solution

Figure 6: Comparison of the ADM solution with RPM solution for $\varepsilon = 0.1$



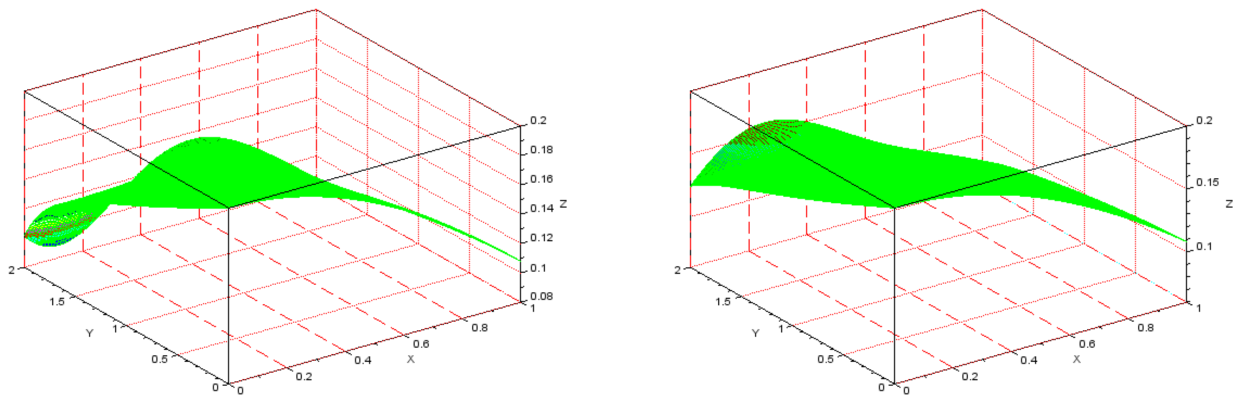
(a) HPM solution (b) RPM solution

Figure 7: Comparison of the HPM solution with RPM solution for $\varepsilon = 0.2$



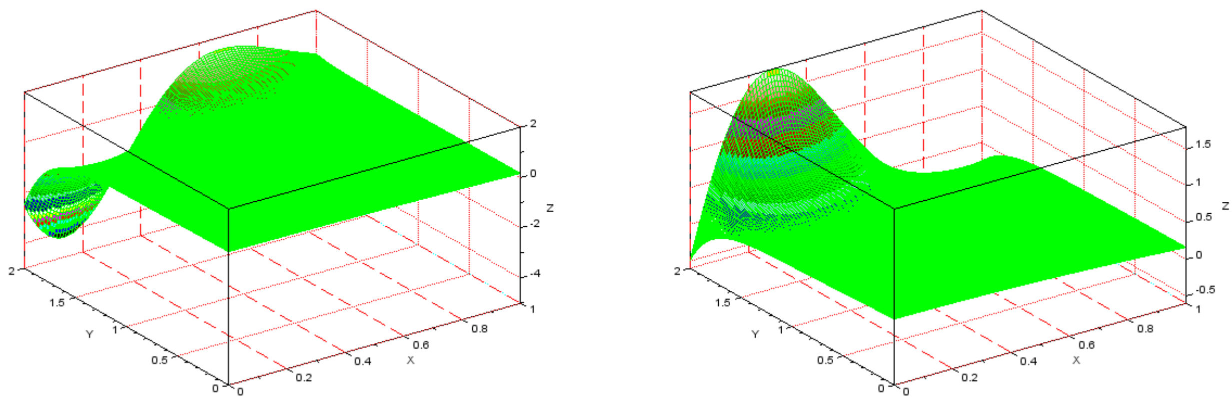
(a) HPM solution (b) ADM solution

Figure 8: Comparison of the HPM solution with ADM solution for $\varepsilon = 0.2$



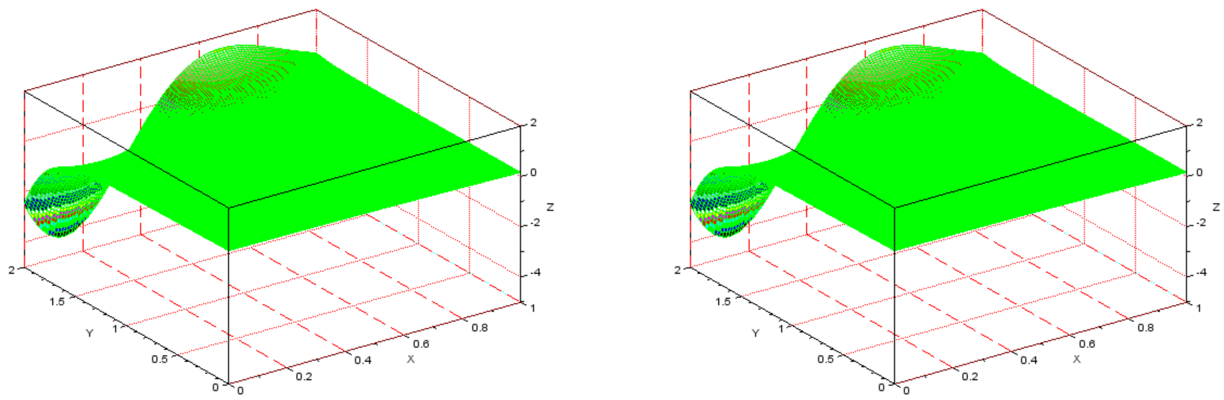
(a) ADM solution (b) RPM solution

Figure 9: Comparison of the ADM solution with RPM solution for $\varepsilon = 0.2$



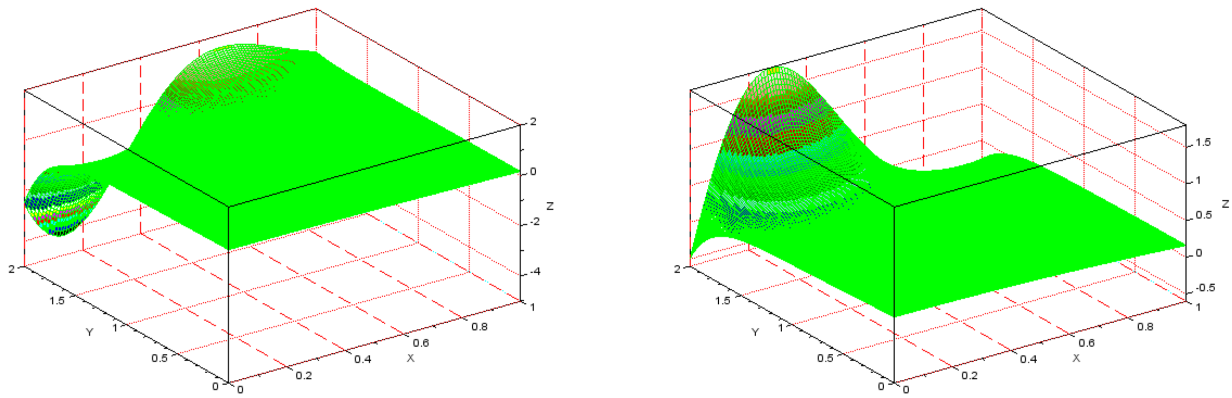
(a) HPM solution (b) RPM solution

Figure 10: Comparison of the HPM solution with RPM solution for $\varepsilon = 0.3$



(a) HPM solution (b) ADM solution

Figure 11: Comparison of the HPM solution with ADM solution for $\varepsilon = 0.3$



(a) HPM solution (b) ADM solution

Figure 12: Comparison of the RPM solution with ADM solution for $\varepsilon = 0.3$

6. Conclusion

In this paper we solved the viscid Burgers equation with ADM (Adomian Decomposition Method), RPM (Regular Perturbation Method) and HPM (Homotopy Perturbation Method). HPM and ADM give the same approximate solutions. The approximate solutions given by RPM are shifted of an order compared to those of two other methods (ADM and HPM).

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