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Research paper

Mathematical modelling and analysis of Kidnapping dynamics

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Abstract

The problem of kidnapping as a social menace to a society is increasing in some African countries such as Nigeria. We therefore proposed a new deterministic mathematical model for the dynamics of Kidnapping in a community. This menace is considered like a two strains communicable disease with kidnapping propagation mission by kidnappers as one strain and adoption mission for kidnapped victims as the other strain to assess the impact of super-infection. The model exhibits four equilibrium points each of which is unique and asymptotically stable both locally and globally under certain conditions. We obtain the kidnapping propagation number $\mathscr{C}_p = \max_{i\in\{1,2\}}\{\mathscr{C}_i\}$ where $\mathscr{C}_i$ is the propagation number associated with strain i. Another important threshold parameters associated with respective strains 1 and 2 are $\mathscr{C}_{p_{1,2}}$ and $\mathscr{C}_{p_{2,1}}$. Indeed, we show that at most one strain invades the population if one of these parameters is less than unity. while the two strains coexist at endemic state when both $\mathscr{C}_{p_{1,2}}$ and $\mathscr{C}_{p_{2,1}}$ are greater than unity. The global stability results of the model equilibria are established by numerical simulations. This simulations results indicate that the super-infection destabilize the coexistence equilibrium.

Keywords: *Kidnapping; Kidnap Propagation Number; Stability; Super-infection.*

1. Introduction

Social epidemic like ideas, behaviors, messages etc., spread almost the same ways as infectious diseases do such as Tuberculosis, HIV/AIDS and sexually transmitted diseases (STDs) from one person to another with similar characteristics associated with the epidemic. Gladwell pointed out in his book that social epidemic like those mentioned above, spread over population same way virus spread [18]. Kidnapping which can be defined as taking away or transportation of a person by force against his/her will and holding him/her in false imprisonment/confinement without legal authority. It is usually done for a motive or ransom [17, 26]. Moreover, kidnapping has various definitions in the literature with varying degree and success [6]. It is one of such ideas/behaviours that can correctly be described as a social epidemic which spread across vulnerable humans over time, for more examples (see [8, 12, 21, 29, 30, 31, 32]). Several types of kidnapping are commonly seen in the world. These include Kidnap for Ransom, Tiger Kidnapping/Proxy Bombings, Express Kidnapping, Political and Ideological Kidnapping, and Virtual Kidnapping [23]. The first type is common and rampant in Nigeria nowadays and so, we are going to consider it here. Kidnap for ransom is the one for which a criminal detain the hostage to receive a payment from their family, employer, or country in exchange for his/her release. It is a major source of income for criminal gangs that rely on ransom to finance their operations and is on the rise worldwide.

It is well known that on kidnap for ransom, criminals target the rich, elite and expatriates, but in Nigeria they no longer just targeting them as ordinary citizens are now becoming victims . It was reported that most victims are now ordinary citizens especially poor villagers, a large proportion of which are kidnapped indiscriminately and are much more likely to be killed by the kidnappers in the event of failure to pay the ransom. This is simply because Nigerian ruralities are less secure then its cities [22, 26]. Some problems which cause the social crime of kidnapping include: politicians, poverty, terrorism, lack of stiffer punishment by government, negligence on the part of the well to do in families and quick money [34]. Other causes are the problem of idleness, greed for money, the nature of kidnapping network contributes to the youth's involvement, wrong moral choices aid youth's involvements, peer-group pressure, lack of proper orientation in the home front by parents and guardians, unnecessary public display of wealth, wrong societal values, and lack of integrity/corrupt practices of government officers and others [26].

It was reported by SB Morgen (SBM) that in the last 9 years, the total money spent for ransom in Nigeria is more than four billion Naira (NGN 4 Billion, equivalent to 18.34 Million USD). Moreover, a huge chunk of the figure, about 11 Million USD was paid out between January 2016 and March 2020 which is worrisome. Nigeria has the highest rate of kidnaps for ransom in Africa and it records more than a thousand kidnapping incidents a year [7]. Over the past ten years, kidnapping is most rampant in the South-South geopolitical zone of the country. The most affected states in the country are Rivers, Kaduna, Delta, and Bayelsa [26]. It was presented in [22] that Rivers state recorded 120 kidnap cases between 2016 and 2020, followed by Kaduna, with 117, then Delta with 96. Bayelsa is fourth with 85 kidnap cases and Borno fifth, with 82 cases. As a consequence, kidnapping traumatises the victim and victim's family and is also accompanied with huge economic or financial implications. In Nigeria, kidnapping has led to the loss of thousands of lives and huge sums of money. Many of the victims of the crime have been killed in the course of their abduction, custody or release and many more have been injured. In fact, it was estimated that globally, ransom payments could worth US$500 million annually [6].

Applications of mathematical epidemic theories to social phenomena were proposed for a long time [11, 19]. However, Mathematical models have also been applied to investigate a variety of "contagious" social phenomena like crime, opinions, addiction and fanaticism (see for instance, [3, 4, 5, 13, 14, 15, 16, 24, 25, 35]. A mathematical model of crime epidemic is proposed in [13], to gain insights into the best policies regarding criminality. The analysis of the model shows that when the threshold parameter R_0 is less than unity, the criminality disappears, while the reverse results in persistence of the criminality in the society. Numerical simulations using a set of estimated parameter values for the USA scenario showed that criminality can decrease. Sensitivity analysis reveals that prevention is the most important policy to decrease criminality. Furthermore, elasticity analysis shows that having an honest subpopulation of judges play a significant role on the decrease of criminality in the society. Recently, mathematical model of illicit drugs and banditry dynamics in a population was proposed in [4]. Asymptotic stability analysis of the model equilibria was carried out which reveals that it is difficult to eliminate the menace due to the occurrence of backward bifurcation phenomenon. Moreover, sensitivity analysis shows that illicit drug and banditry-free population depends on the reduction of the influence rate on susceptible individuals and the parameter that measures the effectiveness of banditry compared to illicit drug use. Also, in 2023 a deterministic mathematical model for armed banditry to get insights on controlling the spread of the menace using job creation and efforts to make the crime unprofitable [16]. Numerical simulations of the model reveals that the applying any of the two control strategies is effective in reducing the population profile of the informers and the bandits in a finite time.

Nowadays, kidnapping for ransom which involves series of negotiations between the kidnapper(s) and the family of the victim has been on the increase. This is perhaps, due to terrorism, drug addiction, cultism or gangsterism and the high level of corruption and insecurity in some countries. On this note, Okrinya [1], proposed a simple deterministic mathematical model on kidnapping. The model describes the evolution and propagation of kidnapping as a crime in human society and the model features mimic dynamics of an infectious disease. As for the disease transmission models, the threshold parameter that determine the crime propagation called a "crime propagation number", C_{pn} was derived. They showed that kidnap free state exist which is locally and globally asymptotically stable when $C_{pn} < 1$. Also, the analysis reveals that reducing the recruitment rate of kidnappers and increasing the rescue rate without any causality on the part of kidnapped victims will lead to eradication of kidnapping in a society. Furthermore, numerical simulation shows that more than half of the population will become kidnapped victims within a period of twenty years whereas 38% of the population would be attracted to the criminal act of kidnapping [1].

In a similar note, Okrinya and Consul extended the model of kidnapping presented in [2] by incorporating de-radicalizing and rehabilitation of kidnappers as preventive measures. Numerical simulations were carried out on the combination of different levels of kidnappers' recruitment and rehabilitation. In fact, the analysis reveals that increasing the rehabilitation rate of kidnappers is a better and more effective way of ensuring a kidnapping free society [2]. However, the assumptions in both the two models above that, during captivity of kidnapped victims still give birth is not realistic in fact, even kidnapper may not give birth to kidnapper like him not to talk of kidnapped victims who have limited time to be in captivity.

In this paper, we viewed kidnapping as a social crime in which a kidnapper (carrier of the crime) sell the idea to vulnerable humans (especially unemployed youth). On the same vain, other group of individuals suffer for the crime by becoming kidnapped victims. Therefore, the crime of kidnapping affect an individual either by kidnapping or becomes kidnapped victims. So, we consider kidnapping model as a two-strains model in which kidnapped victims are the latent category (see [9, 10, 27] for multi-strains models).

The remaining paper is organized as follows: We give the model description in Section 2. Detailed analysis of the model equilibria is provided in Section 3. Numerical simulation of the model is presented in Section 4 while we give a concluding remarks in Section 5.

2. Model formulation

We consider this crime problem to be similar to a two-strain epidemic disease with a single susceptible (individuals who are vulnerable to become criminal) class but having two criminal classes corresponding to two crime agents (kidnappers with kidnapping propagation mission, K_p and kidnapped victims with adoption mission, K_v). Each strain is to be modelled as a simple *SIS* system such that strain one (K_p) may "super-infect" an individual infected with strain two (K_v), resulting to a new infection in compartment K_p. To formulate the model, we divide the total human population (N(t)) at time t into three compartments: Susceptible population denoted by $S(t)$ kidnappers population by $K_p(t)$, and kidnapped victims' population as $K_v(t)$. Thus

$$N(t) = S(t) + K_p(t) + K_v(t).$$

We now assume that

(i) the interaction between kidnappers and susceptible population is homogenous since nowadays the criminals target ordinary citizen indiscriminately in addition to the rich, elite and expatriates [22, 26],

(ii) the susceptible individuals increase by birth and immigration at a constant rate Λ and become infected by effective social contact with a kidnapper having a kidnapping propagation mission at the rate β_1 with probability $\frac{K_p}{N}$,

(iii) the susceptible may also buy idea of the crime when having contact with a kidnapped victim having kidnapping adoption mission at the rate β_2 with probability $\frac{K_v}{N}$,

(iv) a kidnapper super-infect a kidnapped victim at the rate ρ to become a kidnapper and move to the K_p compartment,

(v) a kidnapper suffers crime related mortality at the rate α_1 due to operation by the security agents in order to rescue kidnapped victim(s) or as a result of the court order if arrested,

(vi) a kidnapped victim suffers crime related mortality at the rate α_2 due to resisting abduction, failure to pay the ransom as demanded by the kidnappers or to pay on time and fear that the victim(s) would identify the criminals if released,

(vii) a kidnapper would become susceptible when he/she quit from the crime for any reason at the rate τ_1,

(viii) a kidnapped victim would become susceptible after he/she gained freedom at the rate τ_2.

Table 1: State variables of the Model

State Variables	Description
$N(t)$	Total human population
$S(t)$	Number of human susceptible to kidnapping
$K_p(t)$	Number of Kidnappers Population
$K_v(t)$	Number of Kidnapped Victims(Kidnappees)

Table 2: Model Parameters and their dimensions values

Parameters	Description	Value	Unit	Source
Λ	Recruitment rate of susceptible	0.0373	day^{-1}	*World Bank*
β_1	contact rate of kidnappers	3.8×10^{-4}	day^{-1}	[2]
β_2	contact rate of kidnapped victims	0.0612	day^{-1}	[2]
μ	Per capital natural death rate	0.0116	day^{-1}	*World Bank*
α_1	Kidnappers' induced death rate	2.35×10^{-9}	$human^{-1}\ day^{-1}$	[2]
α_2	Kidnapped victims induced death rate	1.18×10^{-9}	$human^{-1}\ day^{-1}$	[2]
ρ	Super-infection rate of Kidnappers to Kidnapped victims	0.0311	day^{-1}	*Assumed*
τ_1	Quitting rate of Kidnappers	0.1245	day^{-1}	*Assumed*
τ_2	Rescue/gaining freedom rate of kidnapped victims	0.0136	day^{-1}	[2]

Using the above hypotheses the model is given by the following system of ordinary differential equations and the respective rates of transfer between the three compartments is depicted in Figure 1.

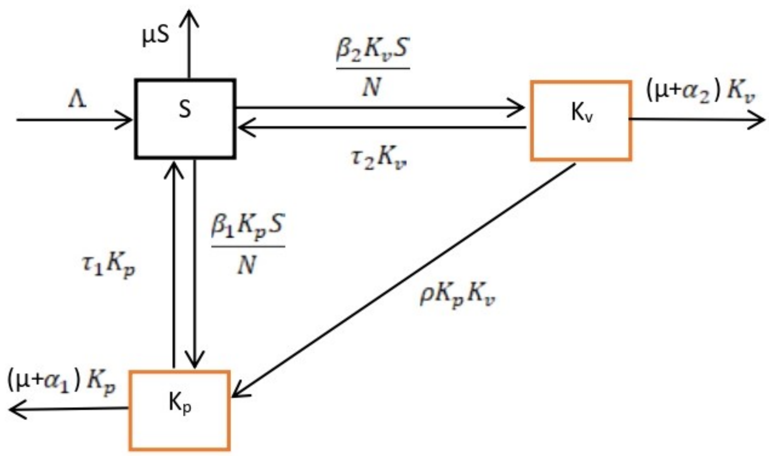

Figure 1: Schematic diagram of the kidnap model (1)

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \frac{\beta_1 KpS}{N} - \frac{\beta_2 K_v S}{N} + \tau_1 K_p + \tau_2 K_v - \mu S, \\
\frac{dK_p}{dt} &= \frac{\beta_1 KpS}{N} + \rho K_p K_v - (\mu + \alpha_1 + \tau_1) K_p, \\
\frac{dK_v}{dt} &= \frac{\beta_2 K_v S}{N} - (\mu + \tau_2 + \alpha_2) K_v - \rho K_p K_v.
\end{aligned} \tag{1}$$

The three state variables and all the model parameters are non-negatives.
The region

$$\Omega = \left\{ (S, K_p, K_v) \in \mathbb{R}^3_+ : S > 0, K_p \geq 0, K_v \geq 0, S + K_p + K_v < \frac{\Lambda}{\mu} \right\}.$$

is positively invariant and attractive.
Adding the three equations of (1), we have

$$\begin{aligned}
\frac{dN}{dt} &= \Lambda - \mu N - \alpha_1 K_p - \alpha_2 K_v \\
&\leq \Lambda - \mu N
\end{aligned} \tag{2}$$

Thus $N(t) \leq \frac{\Lambda}{\mu}$ if $N(0) \leq \frac{\Lambda}{\mu}$. It follows that Ω is positively invariant. Moreover, if $N(t) \geq \frac{\Lambda}{\mu}$ then $\frac{dN}{dt} < 0$ so that either the solution enters Ω in finite time or $N(t)$ approaches $\frac{\Lambda}{\mu}$ and the kidnap variables (K_p and K_v) approach zero. Hence Ω is attracting and so, all solutions in $\mathbb{R}^3_+$ eventually enter Ω. Therefore it is sufficient to consider the dynamics of system (1) in Ω. Hence the model (1) is considered to be mathematically and criminology well possed in Ω.

Now, using $S=N-K_p-K_v=\mathfrak{S}$ in the second and third equations of (1) and considering the equation of the total population, we obtain

$$\begin{aligned}
\frac{dN}{dt} &= \Lambda-\mu N-\alpha_1 K_p-\alpha_2 K_v,\\
\frac{dK_p}{dt} &= \frac{\beta_1 K_p(N-K_p-K_v)}{N}+\rho K_p K_v-(\mu+\alpha_1+\tau_1)K_p,\\
\frac{dK_v}{dt} &= \frac{\beta_2 K_v(N-K_p-K_v)}{N}-(\mu+\tau_2+\alpha_2)K_v-\rho K_p K_v.
\end{aligned} \tag{3}$$

with domain

$$\Gamma=\left\{(\mathfrak{S},K_p,K_v)\in\mathbb{R}^3_+ : \mathfrak{S}>0, K_p\geq 0, K_v\geq 0, N<\frac{\Lambda}{\mu}\right\}.$$

3. Model analysis

3.1. Kidnap Free Equilibrium and threshold parameters

When there is no kidnapping then there is no kidnapped victims (i. e., $K_p=K_v=0$). Thus, the kidnap-free equilibrium of model (3) is

$$E^0=(N^0,K_p^0,K_v^0)=\left(\frac{\Lambda}{\mu},0,0\right). \tag{4}$$

As for disease transmission models, in order to establish the linear stability of the equilibria, we need a threshold parameter known as the kidnapping propagation number. Such a threshold parameter can be obtained by the next generation operator method [20, 27, 28]. Suppose P is vector that indicate all the kidnapping effects such as kidnappers and kidnapped victims. Thus $P=(K_p,K_v)$ then from (3), it can be seen that:

$$\frac{dP}{dt}=\mathfrak{R}-\mathfrak{I}$$

with

$$\mathfrak{R}=\begin{pmatrix}\frac{\beta_1 K_p(N-K_p-K_v)}{N}\\ \frac{\beta_2 K_v(N-K_p-K_v)}{N}\end{pmatrix}, \mathfrak{I}=\begin{pmatrix}-\rho K_v K_p+(\mu+\alpha_1+\tau_1)K_p\\ (\mu+\tau_2+\alpha_2)K_v+\rho K_p K_v\end{pmatrix}.$$

As in [27], let M be the matrix of the infection terms and T the matrix of transition terms. Then, we have

$$M=\frac{\partial\mathfrak{R}}{\partial P}=\begin{pmatrix}\beta_1 & 0\\ 0 & \beta_2\end{pmatrix},$$

$$T=\frac{\partial\mathfrak{I}}{\partial P}=\begin{pmatrix}\mu+\alpha_1+\tau_1 & 0\\ 0 & \mu+\tau_2+\alpha_2\end{pmatrix}$$

so that

$$MT^{-1}=\begin{pmatrix}\frac{\beta_1}{\mu+\tau_1+\alpha_1} & 0\\ 0 & \frac{\beta_2}{\mu+\tau_2+\alpha_2}\end{pmatrix}.$$

This matrix has the two eigenvalues

$$\mathscr{C}_i=\frac{\beta_i}{\mu+\tau_i+\alpha_i}, \text{ for } i=1,2.$$

Since $M-T$ is reducible then the second and the third equations of (3) decouple near the kidnap-free equilibrium. These two eigenvalues correspond to the crime propagation numbers for each strain. Thus the crime propagation number denoted by $\mathscr{C}_p$ is given by:

$$\mathscr{C}_p=\max_{i\in\{1,2\}}\mathscr{C}_i. \tag{5}$$

We can alternatively interpret this model as that K_p is the sole crime compartment and that K_v is not a crime compartment. Thus the kidnappee-presence equilibrium is given by

$$\begin{aligned}
E^* &= (N^*,K_p^*,K_v^*)\\
&= \left(\frac{\Lambda\mathscr{C}_2}{\mu\mathscr{C}_2+\alpha_2(\mathscr{C}_2-1)},0,\frac{\Lambda(\mathscr{C}_2-1)}{\mu\mathscr{C}_2+\alpha_2(\mathscr{C}_2-1)}\right),
\end{aligned} \tag{6}$$

if and only if $\mathscr{C}_2>1$ with $Q_2=\mu+\alpha_2+\tau_2$.
As in [33], we obtain the invasion threshold associated to this equilibrium point (E^*) as follows

$$\mathscr{C}_{p_{1,2}}=\frac{\mathscr{C}_1}{\mathscr{C}_2}+\frac{\rho\Lambda\mathscr{C}_1(\mathscr{C}_2-1)}{\beta_1[\mu\mathscr{C}_2+\alpha_2(\mathscr{C}_2-1)]}.$$

Another equilibrium point called kidnapper-presence equilibrium is given by

$$\begin{aligned}
E_* &= (N_*,K_{p*},K_{v*})\\
&= \left(\frac{\Lambda\mathscr{C}_1}{\mu\mathscr{C}_1+\alpha_1(\mathscr{C}_1-1)},\frac{\Lambda(\mathscr{C}_1-1)}{\mu\mathscr{C}_1+\alpha_1(\mathscr{C}_1-1)},0\right),
\end{aligned} \tag{7}$$

which exists if and only if $\mathscr{C}_1>1$, where $Q_1=\mu+\alpha_1+\tau_1$. The associated invasion threshold of this equilibrium is

$$\mathscr{C}_{p_{2,1}}=\frac{\mathscr{C}_2}{\mathscr{C}_1}+\frac{\rho\Lambda\mathscr{C}_2(\mathscr{C}_1-1)}{\beta_2[\mu\mathscr{C}_1+\alpha_1(\mathscr{C}_1-1)]}.$$

3.2. Local stability of equilibria

Here, we establish the local asymptotic stability of the three equilibrium points presented in (4), (6) and (7), respectively.

Theorem 1. *The kidnap-free equilibrium point (E^0) of model (3) is locally asymptotically stable on Γ if $\mathscr{C}_p < 1$ and is unstable when $\mathscr{C}_p > 1$.*

Proof. Linearizing system (3) around E^0, we have

$$J(E^0) = \begin{pmatrix} -\mu & -\alpha_1 & -\alpha_2 \\ 0 & \beta_1 - Q_1 & 0 \\ 0 & 0 & \beta_2 - Q_2 \end{pmatrix}. \tag{8}$$

Thus, the eigenvalues of this matrix are: $\lambda_1 = -\mu$, $\lambda_2 = -Q_1(1-\mathscr{C}_1)$ and $\lambda_2 = -Q_2(1-\mathscr{C}_2)$ which are negative whenever $\mathscr{C}_1 < 1$ and $\mathscr{C}_2 < 1$. Hence, E^0 is locally asymptotically stable if $\mathscr{C}_1 < 1$ and $\mathscr{C}_2 < 1$ implying that $\mathscr{C}_p < 1$. But, when either $\mathscr{C}_1 > 1$ or $\mathscr{C}_2 > 1$ or both are greater than unity (i.e., $\mathscr{C}_p > 1$), then E^0 is unstable.

Theorem 2. *The kidnappee-presence equilibrium point (E^*) of model (3) is stable locally asymptotically on Γ if $\mathscr{C}_2 > 1$ and $\mathscr{C}_{p_{1,2}} < 1$.*

Proof. Evaluating the Jacobian matrix of system (3) about the equilibrium point, E^*, we obtain

$$J(E^*) = \begin{pmatrix} -\mu & -\alpha_1 & -\alpha_2 \\ 0 & Q_1(\mathscr{C}_{p_{1,2}} - 1) & 0 \\ \frac{\beta_2(\mathscr{C}_2-1)^2}{\mathscr{C}_2^{*2}} & \Phi & -\frac{\beta_2(\mathscr{C}_2-1)}{\mathscr{C}_2} \end{pmatrix}.$$

where $\Phi = -(\mathscr{C}_2 - 1)\frac{Q_2[\mu\mathscr{C}_2+\alpha_2(\mathscr{C}_2-1)]+\rho\Lambda}{\mu\mathscr{C}_2+\alpha_2(\mathscr{C}_2-1)}$ Then it can be seen by inspection that the first eigenvalue of $J(E^*)$ is $\lambda_1 = Q_1(\mathscr{C}_{p_{1,2}} - 1) < 0$ if $\mathscr{C}_{p_{1,2}} < 1$. The remaining two eigenvalues are for the sub-matrix of $J(E^*)$ given by

$$J_0(E^*) = \begin{pmatrix} -\mu & -\alpha_2 \\ \frac{\beta_2(\mathscr{C}_2-1)^2}{\mathscr{C}_2^{*2}} & -\frac{\beta_2(\mathscr{C}_2-1)}{\mathscr{C}_2} \end{pmatrix}.$$

Then the eigenvalues of this matrix will have negative real parts if $\mathrm{tr}(J_0) < 0$ and $\det(J_0) > 0$. But if $\mathscr{C}_2 > 1$

$$\mathrm{tr}(J_0) = -\frac{\mu\mathscr{C}_2 + \beta_2(\mathscr{C}_2 - 1)}{\mathscr{C}_2} < 0 \text{ and } \det(J_0) = \frac{\beta_2(\mathscr{C}_2 - 1)[\mu\mathscr{C}_2 + \alpha_2(\mathscr{C}_2 - 1)}{\mathscr{C}_2} > 0.$$

Thus, the two eigenvalues λ_2 and λ_3 of J_0 have negative real parts and so, the three eigenvalues of $J(E^*)$ will have negative real parts if $\mathscr{C}_2 > 1$ and $\mathscr{C}_{p_{1,2}} < 1$. Hence, the result.

Theorem 3. *If $\mathscr{C}_1 > 1$ and $\mathscr{C}_{p_{2,1}} < 1$, the kidnaper-presence equilibrium point (E_*) of model (3) is locally asymptotically stable on Γ.*

Proof. Evaluating the Jacobian matrix of system (3) about the equilibrium point, E_*, we obtain

$$J(E_*) = \begin{pmatrix} -\mu & -\alpha_1 & -\alpha_2 \\ \frac{\beta_1(\mathscr{C}_1-1)^2}{\mathscr{C}_1^{*2}} & -\frac{\beta_1(\mathscr{C}_1-1)}{\mathscr{C}_1} & \Phi_2 \\ 0 & 0 & Q_2(\mathscr{C}_{p_{2,1}} - 1) \end{pmatrix}.$$

where $\Phi_2 = -(\mathscr{C}_1 - 1)\frac{Q_1[\mu\mathscr{C}_1+\alpha_1(\mathscr{C}_1-1)]-\rho\Lambda}{\mu\mathscr{C}_1+\alpha(\mathscr{C}_1-1)}$ Then it can be seen by inspection that the first eigenvalue of $J(E_*)$ is $\lambda_1 = Q_2(\mathscr{C}_{p_{2,1}} - 1) < 0$ if $\mathscr{C}_{p_{2,1}} < 1$. The remaining two eigenvalues are for the sub-matrix of $J(E_*)$ given by

$$J^0(E_*) = \begin{pmatrix} -\mu & -\alpha_1 \\ \frac{\beta_1(\mathscr{C}_1-1)^2}{\mathscr{C}_1^{*2}} & -\frac{\beta_1(\mathscr{C}_1-1)}{\mathscr{C}_1} \end{pmatrix}.$$

Then the eigenvalues of this matrix will have negative real parts if $\mathrm{tr}(J^0) < 0$ and $\det(J^0) > 0$. But if $\mathscr{C}_1 > 1$

$$\mathrm{tr}(J^0) = -\frac{\mu\mathscr{C}_1 + \beta_1(\mathscr{C}_1 - 1)}{\mathscr{C}_1} < 0 \text{ and } \det(J^0) = \frac{\beta_1(\mathscr{C}_1 - 1)[\mu\mathscr{C}_1 + \alpha_1(\mathscr{C}_1 - 1)}{\mathscr{C}_1} > 0.$$

Thus, the two eigenvalues λ_2 and λ_3 of J^0 have negative real parts and so, the three eigenvalues of $J(E_*)$ will have negative real parts if $\mathscr{C}_1 > 1$ and $\mathscr{C}_{p_{2,1}} < 1$. Hence, the result.

3.3. Coexistence Equilibria

Equating the right hand side of system (3) to zero, we have

$$\begin{aligned} N &= \frac{\Lambda - \alpha_1 K_p - \alpha_2 K_v}{\mu}, \\ \frac{\beta_1(N - K_p - K_v)}{N} &+ \rho K_v - (\mu + \alpha_1 + \tau_1) = 0, \\ \frac{\beta_2(N - K_p - K_v)}{N} &- (\mu + \tau_2 + \alpha_2) - \rho K_p = 0. \end{aligned} \tag{9}$$

Then from the second and third equations of (9), we obtain using the first equation of (9)

$$\begin{aligned}&(\beta_1+\rho K_v-Q_1)(\Lambda-\alpha_1K_p-\alpha_2K_v)-\mu\beta_1(K_p+K_v)=0,\\&(\beta_2-\rho K_p-Q_2)(\Lambda-\alpha_1K_p-\alpha_2K_v)-\mu\beta_2(K_p+K_v)=0,\end{aligned}\tag{10}$$

with $Q_1=\mu+\tau_1+\alpha_1$ and $Q_2=\mu+\tau_2+\alpha_2$. Eliminating the third terms of (10), we obtain

$$K_v=\frac{\beta_1[Q_2(\mathscr{C}_2-1)-\rho K_p]-\beta_2Q_1(\mathscr{C}_1-1)}{\rho\beta_2}.\tag{11}$$

Using the second equation of (10), we obtain after algebraic manipulations a quadratic equation in terms of K_p.

$$aK_p^2+bK_p+c=0,\tag{12}$$

where

$$\begin{aligned}a&=(\alpha_1\beta_2-\alpha_2\beta_1)\rho^2,\\b&=-\rho\left\{\alpha_2Q_1\beta_2\left(\frac{\mathscr{C}_1}{\mathscr{C}_2}-1\right)+Q_2(\mathscr{C}_2-1)(\alpha_1\beta_2-\alpha_2\beta_1)+\beta_2[\mu(\beta_2-\beta_1)+\rho\Lambda]\right\},\\c&=Q_1Q_2\left(\frac{\mathscr{C}_1}{\mathscr{C}_2}-1\right)[\alpha_2\beta_1(\mathscr{C}_2-1)+\mu\beta_2\mathscr{C}_2]+Q_2\rho\beta_2\Lambda(\mathscr{C}_2-1).\end{aligned}\tag{13}$$

It follows that

(1) If $\alpha_1\beta_2-\alpha_2\beta_1>0$ with either $\mathscr{C}_1<1,\mathscr{C}_2>1$ or $\mathscr{C}_1<\mathscr{C}_2<1$ and $\mathscr{C}_{p_{1,2}}<1,\mathscr{C}_{p_{2,1}}>1$ then the coefficients $a>0,b<0$ and $c<0$ or $a>0,b>0$ and $c<0$ so that (12) has one positive real root $K_p=\frac{-b+\sqrt{b^2+4ac}}{2a}$
(2) If $\alpha_1\beta_2-\alpha_2\beta_1<0,\mathscr{C}_1>\mathscr{C}_2>1$ and $\mathscr{C}_{p_{2,1}}>\mathscr{C}_{p_{1,2}}>1$ then the coefficients $a<0,b>0$ and $c>0$ such that (12) has one positive real root $K_p=\frac{-b+\sqrt{b^2+4ac}}{2a}$
(3) If $\alpha_1\beta_2-\alpha_2\beta_1>0,\mathscr{C}_1<1,\mathscr{C}_2>1$ and $\mathscr{C}_{p_{2,1}}>\mathscr{C}_{p_{1,2}}>1$ then the coefficients $a>0,b<0$ and $c>0$ leading to two positive real roots of (13), $K_{p_{1,2}}=\frac{b\pm\sqrt{b^2-4ac}}{2a}$.

Thus, the only positive real root of (12) with coefficients in equation (13) when it exists in both items (1) and (2) is

$$K_p=\frac{-b+\sqrt{b^2+4ac}}{2a}.$$

Hence, the unique coexistence equilibrium is $\tilde{E}=(\tilde{N},\tilde{K}_p,\tilde{K}_v)$ with

$$\begin{aligned}\tilde{N}&=\frac{\Lambda-\alpha_1\tilde{K}_p-\alpha_2\tilde{K}_v}{\mu},\\\tilde{K}_p&=\frac{-b+\sqrt{b^2+4ac}}{2a},\\\tilde{K}_v&=\frac{\beta_1[Q_2(\mathscr{C}_2-1)-\rho\tilde{K}_p]-\beta_2Q_1(\mathscr{C}_1-1)}{\rho\beta_2}.\end{aligned}$$

For this equilibrium to be biologically feasible we need to show that $\tilde{N}$ and $\tilde{K}_v$ are positive. Suppose $\tilde{N}\geq 0$ then if $\tilde{N}=0$, from the first equation of (9) we have $\Lambda=\alpha_1\tilde{K}_p+\alpha_2\tilde{K}_v$ but $\Lambda\neq\alpha_1\tilde{K}_p+\alpha_2\tilde{K}_v$ and so, $\tilde{N}>0$. Also, suppose $\tilde{K}_v\geq 0$ and if $\tilde{K}_v=0$ then from the second equation of (9) $\tilde{K}_p=\frac{Q_1N(1-\mathscr{C}_1)}{\mathscr{C}_1}$ and the third equation of (9) gives $\tilde{K}_p=\frac{Q_2N(1-\mathscr{C}_2)}{\rho N+\beta_2}$. But $\frac{Q_1N(1-\mathscr{C}_1)}{\mathscr{C}_1}\neq\frac{Q_2N(1-\mathscr{C}_2)}{\rho N+\beta_2}$ indicating that $\tilde{K}_v>0$. On the other hand, if equation (12) with coefficients in (13) has two positive real roots as in item (3), then the two coexistence equilibria of model (3) are $\tilde{E}_{1,2}=(\tilde{N}_{1,2},\tilde{K}_{p_{1,2}},\tilde{K}_{v_{1,2}})$ where

$$\begin{aligned}\tilde{N}_{1,2}&=\frac{\Lambda-\alpha_1\tilde{K}_{p_{1,2}}-\alpha_2\tilde{K}_{v_{1,2}}}{\mu},\\\tilde{K}_{p_{1,2}}&=\frac{b\pm\sqrt{b^2-4ac}}{2a},\\\tilde{K}_{v_{1,2}}&=\frac{\beta_1[Q_2(\mathscr{C}_2-1)-\rho\tilde{K}_{p_{1,2}}]-\beta_2Q_1(\mathscr{C}_1-1)}{\rho\beta_2}.\end{aligned}$$

3.4. Global stability results of equilibria

The local stability of the equilibrium points of model (3) presented in Theorems 1-3 allow us to prove the global stability of these equilibrium points.

Theorem 4. *If $\mathscr{C}_p<1$, the kidnap-free equilibrium (E^0) is globally asymptotically stable on the domain Γ and is unstable otherwise.*

We define a Lyapunov function by

$$\Theta = K_p + K_v,$$

so that its time derivative along the solutions of (3) is the following.

$$\begin{aligned}
\dot{\Theta} &= \dot{K}_p + \dot{K}_v \\
&= \frac{\beta_1 K_p(N-K_p-K_v)}{N} + \rho K_p K_v - (\mu+\alpha_1+\tau_1)K_p \\
&+ \frac{\beta_2 K_v(N-K_p-K_v)}{N} - (\mu+\tau_2+\alpha_2)K_v - \rho K_p K_v \\
&= \beta_1 K_p + \beta_2 K_v - \left(\frac{K_p+K_v}{N}\right)(\beta_1 K_p + \beta_2 K_v) - Q_1 K_p - Q_2 K_v \\
&= -Q_1 K_p\left(1-\frac{\beta_1}{Q_1}\right) - Q_2 K_v\left(1-\frac{\beta_2}{Q_2}\right) - \left(\frac{K_p+K_v}{N}\right)(\beta_1 K_p + \beta_2 K_v) \\
&= -\left[Q_1 K_p(1-\mathscr{C}_1) + Q_2 K_v(1-\mathscr{C}_2) + \left(\frac{K_p+K_v}{N}\right)(\beta_1 K_p + \beta_2 K_v)\right]
\end{aligned}$$

Thus, $\dot{\Theta} \leq 0$ when $\mathscr{C}_1 < 1, \mathscr{C}_2 < 1$ with $\dot{\Theta} = 0$ if and only if $K_p = K_v = 0$. Using $K_p = K_v = 0$ in system (3) indicates that $N^0 \to \frac{\Lambda}{\mu}$ and K_p, K_v approaches zero as $t \to \infty$. Therefore, Θ is a Lyapunov function on Γ and the set $\{(N,K_p,K_v) \in \Gamma : \Theta = 0\}$ which is the largest invariant set is the singleton set E^0. Hence every solution of system (3) with initial conditions in Γ approaches E^0 as t approaches ∞ whenever $\mathscr{C}_1 < 1, \mathscr{C}_2 < 1$.

Theorem 5. *The kidnappee-presence equilibrium E^* is globally stable asymptotically on Γ when $\mathscr{C}_{p_{1,2}} < 1$ and is unstable if $\mathscr{C}_{p_{2,1}} > 1$.*

Proof. We consider a Lyapunov function

$$H = N - N^* - N^* \ln\left(\frac{N}{N^*}\right) + K_v - K_v^* - K_v^* \ln\left(\frac{K_v}{K_v^*}\right).$$

The Lyapunov derivative of this function along the trajectories of system (3) is

$$\begin{aligned}
\dot{H} &= \left(1-\frac{N^*}{N}\right)\dot{N} + \left(1-\frac{K_v^*}{K_v}\right)\dot{K}_v \\
&= \left(1-\frac{N^*}{N}\right)(\Lambda - \mu N - \alpha_1 K_p - \alpha_2 K_p) \\
&+ \left(1-\frac{K_v^*}{K_v}\right)\left[\frac{\beta_2 K_v(N-K_p-K_v)}{N} - (\mu+\tau_2+\alpha_2)K_v - \rho K_p K_v\right]
\end{aligned} \tag{14}$$

At kidnappee-presence equilibrium, we have the following identities

$$\begin{aligned}
\Lambda &= \mu N^* + \alpha_1 K_p^* + \alpha_2 K_p^* \\
Q_2 &= \frac{\beta_2(N^*-K_p^*-K_v^*)}{N^*} - \rho K_p^*
\end{aligned} \tag{15}$$

Applying these identities, $\dot{H}$ becomes

$$\begin{aligned}
\dot{H} &= \left(1-\frac{N^*}{N}\right)(\mu N^* + \alpha_1 K_p^* + \alpha_2 K_p^* - \mu N - \alpha_1 K_p - \alpha_2 K_p) \\
&+ \left(1-\frac{K_v^*}{K_v}\right)\left[\frac{\beta_2 K_v(N-K_p-K_v)}{N} - \frac{\beta_2 K_v(N^*-K_p^*-K_v^*)}{N^*} + \rho K_p^* K_v - \rho K_p K_v\right] \\
&= \mu N^*\left(2-\frac{N^*}{N}-\frac{N}{N^*}\right) + \alpha_1 K_p^*\left(1-\frac{N^*}{N}\right)\left(1-\frac{K_p}{K_p^*}\right) + \alpha_2 K_v^*\left(1-\frac{N^*}{N}\right)\left(1-\frac{K_v}{K_v^*}\right) \\
&+ \beta_2 K_v\left(1-\frac{K_v^*}{K_v}\right) + \beta_2 K_v^*\left(\frac{K_p+K_v}{N}\right)\left(1-\frac{K_v}{K_v^*}\right) + \rho K_p K_v^*\left(1-\frac{K_v}{K_v^*}\right) \\
&+ \beta_2 K_v^*\left(1-\frac{K_v}{K_v^*}\right) + \beta_2 K_v\left(\frac{K_p^*+K_v^*}{N^*}\right) + \rho K_p K_v\left(1-\frac{K_v^*}{K_v}\right)
\end{aligned}$$

Noting that $N \leq N^*, K_p \leq K_p^*$ and $K_v \leq K_v^*$, we get

$$\begin{aligned}
\dot{H} &\leq \mu N^*\left(2-\frac{N^*}{N}-\frac{N}{N^*}\right) + \rho K_p^* K_v^*\left(2-\frac{K_v^*}{K_v}-\frac{K_v}{K_v^*}\right) \\
&+ \beta_2 K_v^*\left(2-\frac{K_v}{K_v^*}-\frac{K_v^*}{K_v}\right) + \beta_2 K_v^*\left(\frac{K_p^*+K_v^*}{N^*}\right)\left(2-\frac{K_v^*}{K_v}-\frac{K_v}{K_v^*}\right)
\end{aligned}$$

By arithmetic-geometric inequality, we have

$$\left(2-\frac{N^*}{N}-\frac{N}{N^*}\right) \leq 0, \ \left(2-\frac{K_p^*}{K_p}-\frac{K_p}{K_p}\right) \leq 0, \ \left(2-\frac{K_v}{K_v^*}-\frac{K_v^*}{K_v}\right) \leq 0.$$

Thus, $\dot{H} \leq 0$. Then, we conclude this proof using similar argument as in the proof of Theorem 4.

Theorem 6. *The kidnapper-presence equilibrium E_* is globally stable asymptotically on Γ if $\mathscr{C}_{p_{2,1}} < 1$ and is unstable for $\mathscr{C}_{p_{2,1}} > 1$.*

Proof. We prove this theorem using similar arguments as in the proof of Theorem 5 with following Lyapunov function

$$M = N - N_* - N_* \ln\left(\frac{N}{N_*}\right) + K_p - K_{p_*} - K_{p_*} \ln\left(\frac{K_p}{K_{p_*}}\right).$$

Since there are three cases for the existence of coexistence equilibrium of model (3) as earlier discussed we use the first case to illustrate its global stability result in the following theorem.

Theorem 7. *The coexistence equilibrium $\tilde{E}$ is globally stable asymptotically in the interior of Γ if $\mathscr{C}_{p_{1,2}} < 1$ and $\mathscr{C}_{p_{2,1}} > 1$. It is unstable otherwise.*

Proof. Define a Lyapunov function by

$$G = N - \tilde{N} - \tilde{N} \ln\left(\frac{N}{\tilde{N}}\right) + K_p - \tilde{K_p} - \tilde{K_p} \ln \frac{K_p}{\tilde{K_p}} + K_v - \tilde{K_v} - \tilde{K_v} \ln\left(\frac{K_v}{\tilde{K_v}}\right).$$

The Lyapunov derivative of G along the solutions of system (3) is as follows:

$$\begin{aligned} \dot{G} &= \left(1 - \frac{\tilde{N}}{N}\right)\dot{N} + \left(1 - \frac{\tilde{K_p}}{K_p}\right)\dot{K_p} + \left(1 - \frac{\tilde{K_v}}{K_v}\right)\dot{K_v} \\ &= \left(1 - \frac{\tilde{N}}{N}\right)(\Lambda - \mu N - \alpha_1 K_p - \alpha_2 K_p) \\ &+ \left(1 - \frac{\tilde{K_p}}{K_p}\right)\left[\frac{\beta_1 K_p (N - K_p - K_v)}{N} + \rho K_p K_v - (\mu + \alpha_1 + \tau_1) K_p\right] \\ &+ \left(1 - \frac{\tilde{K_v}}{K_v}\right)\left[\frac{\beta_2 K_v (N - K_p - K_v)}{N} - (\mu + \tau_2 + \alpha_2) K_v - \rho K_p K_v\right] \end{aligned} \tag{16}$$

But at coexistence-equilibrium, we have

$$\begin{aligned} \Lambda &= \mu\tilde{N} + \alpha_1 \tilde{K_p} + \alpha_2 \tilde{K_p} \\ Q_1 &= \frac{\beta_1(\tilde{N} - \tilde{K_p} - \tilde{K_v})}{\tilde{N}} + \rho \tilde{K_v} \\ Q_2 &= \frac{\beta_2(\tilde{N} - \tilde{K_p} - \tilde{K_v})}{\tilde{N}} - \rho \tilde{K_p} \end{aligned} \tag{17}$$

Using equation (17), $\dot{G}$ becomes

$$\begin{aligned} \dot{G} &= \left(1 - \frac{\tilde{N}}{N}\right)(\mu\tilde{N} + \alpha_1\tilde{K_p} + \alpha_2\tilde{K_p} - \mu N - \alpha_1 K_p - \alpha_2 K_p) \\ &+ \left(1 - \frac{\tilde{K_p}}{K_p}\right)\left[\frac{\beta_1 K_p (N - K_p - K_v)}{N} + \rho K_p K_v - \frac{\beta_1 K_p(\tilde{N} - \tilde{K_p} - \tilde{K_v})}{\tilde{N}} - \rho\tilde{K_v} K_p\right] \\ &+ \left(1 - \frac{\tilde{K_v}}{K_v}\right)\left[\frac{\beta_2 K_v (N - K_p - K_v)}{N} - \frac{\beta_2 K_v(\tilde{N} - \tilde{K_p} - \tilde{K_v})}{\tilde{N}} + \rho\tilde{K_p} K_v - \rho K_p K_v\right] \\ &= \mu\tilde{N}\left(2 - \frac{\tilde{N}}{N} - \frac{N}{\tilde{N}}\right) + \alpha_1\tilde{K_p}\left(1 - \frac{\tilde{N}}{N}\right)\left(1 - \frac{K_p}{\tilde{K_p}}\right) + \alpha_2\tilde{K_v}\left(1 - \frac{\tilde{N}}{N}\right)\left(1 - \frac{K_v}{\tilde{K_v}}\right) \\ &+ \beta_1 K_p\left(1 - \frac{\tilde{K_p}}{K_p}\right) + \beta_1\tilde{K_p}\left(\frac{K_p + K_v}{N}\right)\left(1 - \frac{K_p}{\tilde{K_p}}\right) + \beta_2 K_v\left(1 - \frac{\tilde{K_v}}{K_v}\right) \\ &+ \beta_2\tilde{K_v}\left(\frac{K_p + K_v}{N}\right)\left(1 - \frac{K_v}{\tilde{K_v}}\right) + \beta_1\tilde{K_p}\left(1 - \frac{K_p}{\tilde{K_p}}\right) + \beta_1 K_p\left(\frac{\tilde{K_p} + \tilde{K_v}}{\tilde{N}}\right)\left(1 - \frac{\tilde{K_p}}{K_p}\right) \\ &+ \beta_2\tilde{K_v}\left(1 - \frac{K_v}{\tilde{K_v}}\right) + \beta_2 K_v\left(\frac{\tilde{K_p} + \tilde{K_v}}{\tilde{N}}\right)\left(1 - \frac{\tilde{K_v}}{K_v}\right) \\ &\leq \mu\tilde{N}\left(2 - \frac{\tilde{N}}{N} - \frac{N}{\tilde{N}}\right) + \beta_1\tilde{K_p}\left(2 - \frac{\tilde{K_p}}{K_p} - \frac{K_p}{\tilde{K_p}}\right)\left(\frac{\tilde{N} + \tilde{K_p} + \tilde{K_v}}{\tilde{N}}\right) \\ &+ \beta_2\tilde{K_v}\left(2 - \frac{K_v}{\tilde{K_v}} - \frac{\tilde{K_v}}{K_v}\right)\left(\frac{\tilde{N} + \tilde{K_p} + \tilde{K_v}}{\tilde{N}}\right) \end{aligned}$$

By Arithmetic-geometric inequality, we have

$$\left(2 - \frac{\tilde{N}}{N} - \frac{N}{\tilde{N}}\right) \leq 0, \quad \left(2 - \frac{\tilde{K_p}}{K_p} - \frac{K_p}{\tilde{K_p}}\right) \leq 0, \quad \left(2 - \frac{K_v}{\tilde{K_v}} - \frac{\tilde{K_v}}{K_v}\right) \leq 0.$$

Therefore,

$$\begin{aligned} \dot{G} &\leq \mu\tilde{N}\left(2 - \frac{\tilde{N}}{N} - \frac{N}{\tilde{N}}\right) + \left(\frac{\tilde{N} + \tilde{K_p} + \tilde{K_v}}{\tilde{N}}\right) \\ &\times \left[\beta_1\tilde{K_p}\left(2 - \frac{K_p}{\tilde{K_p}} - \frac{\tilde{K_p}}{K_p}\right) + \beta_2\tilde{K_v}\left(2 - \frac{\tilde{K_v}}{K_v} - \frac{K_v}{\tilde{K_v}}\right)\right] \end{aligned}$$

so that $\dot{G} \leq 0$. We conclude this proof using similar argument as in the proof of Theorem 4.

4. Numerical Simulations

In this section, numerical simulations are conducted using parameter values from published literature and some reasonable estimates to illustrates our results presented in the previous sections. Figure 2(a)–(c) display the global asymptotic stability of the kidnap-free equilibrium, E^0 using parameter values in Table 2 to illustrate the result in Theorem 4. It can be seen from Figures 2(a) and (b) that using different initial conditions, the sub-populations of kidnappers and kidnappees are approaching zero asymptotically, respectively so that $\mathscr{C}_1 = 0.3042$, $\mathscr{C}_2 = 0.4487$ hence $\mathscr{C}_p = 0.4487 < 1$. Similarly, in Figure 2(c), the total population, $N(t)$ approach the respective equilibrium point using different initial values. The Figures 3(a)–(c) illustrate the results in Theorems 5, 6 and 7, respectively. In Figure 3(a),the global asymptotic stability of kidnappee-presence, E^* is shown using different initial conditions and parameter values given in Table 2, $\mathscr{C}_{p_{1,2}} = 0.3037 < 1$. Similarly, in Figure 3(b), the global asymptotic stability of kidnapper-presence, E_* is shown using different initial conditions and parameter values in Table 2 except for $\beta_1 = 0.38, \tau_2 = 0.136$ so that $\mathscr{C}_{p_{2,1}} = 0.3912 < 1$. Figure 3(c) also illustrates the global asymptotic stability for $\tilde{E}$, the coexistence equilibrium using different initial conditions and parameter values in Table 2 except for $\beta_1 = 0.036$ such that $\mathscr{C}_{P_{1,2}} = 0.4115 < 1$ and $\mathscr{C}_{P_{2,1}} = 24.1839 > 1$.

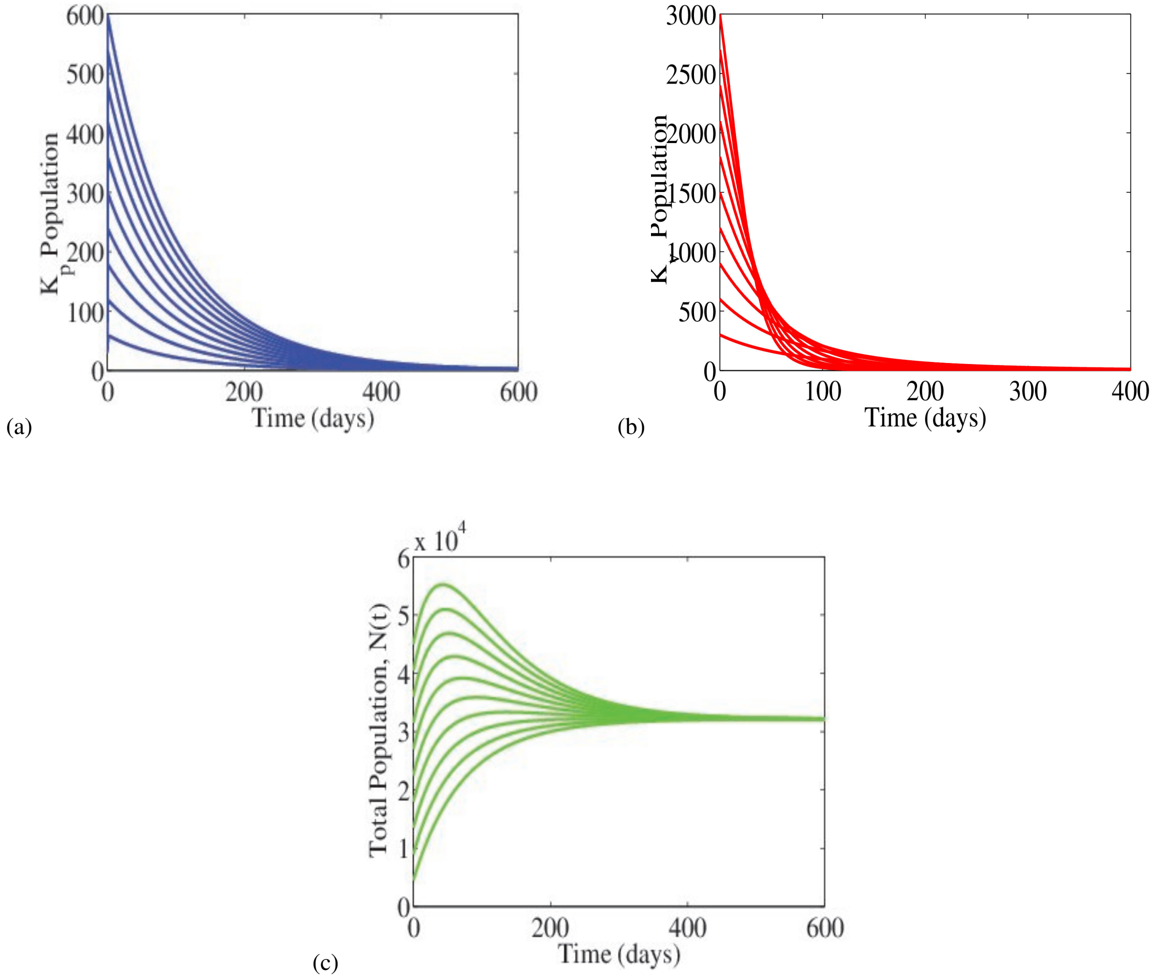

Figure 2: Simulations of model (3), with parameter values as shown in Table 2, to display the result in Theorem 4, the GAS of kidnap-free equilibrium, E^0 showing in (a) kidnappers population $K_p(t)$ (b) kidnappees population $K_v(t)$ and in (c) the total human population $N(t)$, with $\mathscr{C}_1 = 0.3042$, $\mathscr{C}_2 = 0.4487$ so that $\mathscr{C}_p = 0.4487$.

5. Conclusion

We propose a new deterministic mathematical model for the dynamics of kidnapping in a community. The problem is considered like a multi-strain communicable disease such that the kidnapping propagation mission by kidnappers is one strain while the kidnapping adoption mission for the kidnapped victim is the second strain. The presented model is found to exhibit four equilibrium points including the kidnap-free state, kidnappee-presence equilibrium, kidnapper-presence equilibrium and coexistence equilibrium. The dynamics of the proposed model is determined by a threshold parameter called crime propagation number, $\mathscr{C}_p$ which is the maximum of the crime numbers $\mathscr{C}_i$ for $i = 1,2$ associated with two strains and two other threshold parameters associated with individual strains, $\mathscr{C}_{p_{1,2}}$ and $\mathscr{C}_{p_{2,1}}$. We carried out the stability analysis of the model equilibrium points and show that

(i) The kidnap-free equilibrium is locally and globally asymptotically stable if $\mathscr{C}_p < 1$.
(ii) If $\mathscr{C}_2 > 1$ and $\mathscr{C}_{p_{1,2}} < 1$, the kidnappee-presence equilibrium is stable locally and globally
(iii) For $\mathscr{C}_1 > 1$ and $\mathscr{C}_{p_{2,1}} < 1$,the kidnapper-presence equilibrium is asymptotically stable locally and globally

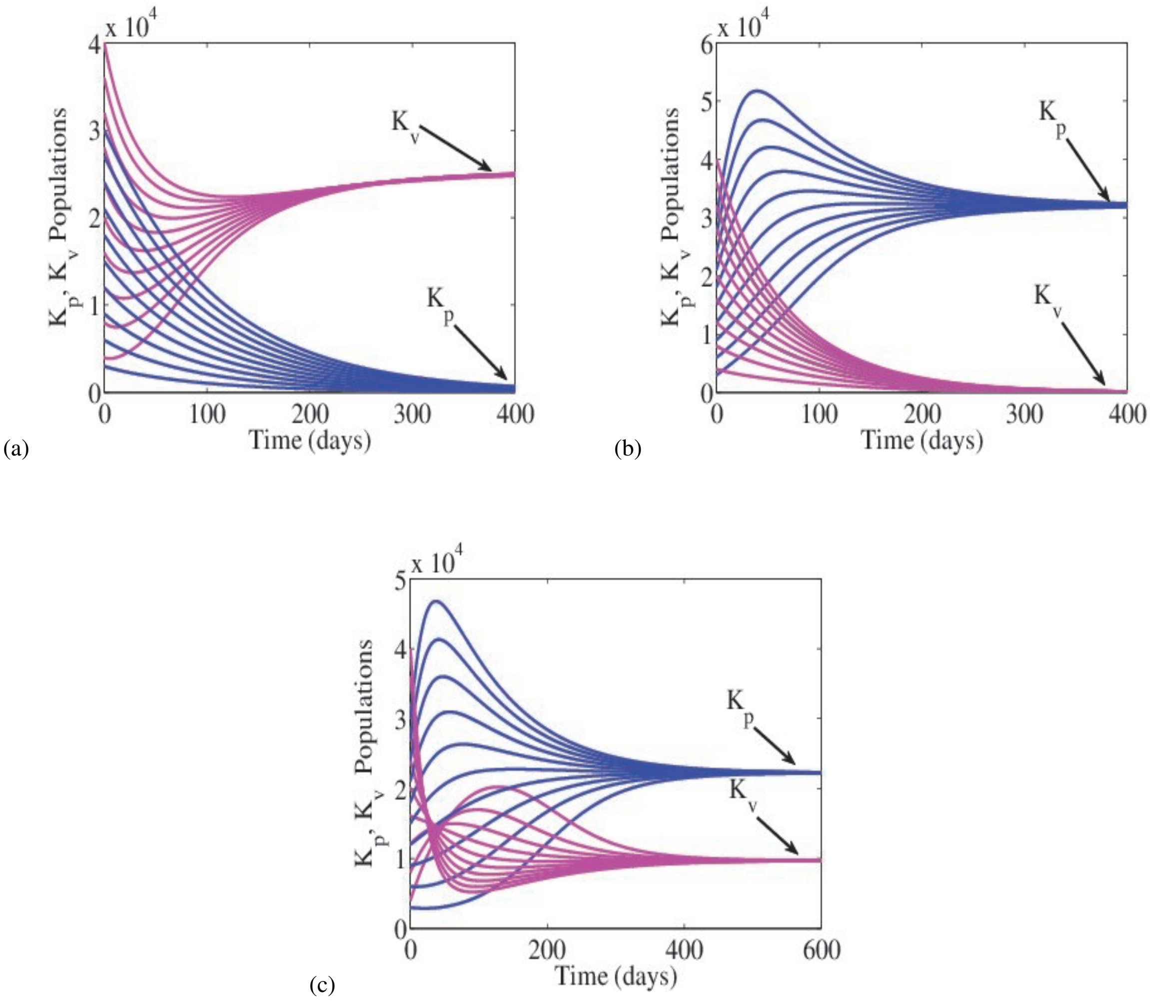

Figure 3: Simulations of model (3), with parameter values as shown in Table 2, displaying the results in Theorems 5, 6 and 7 showing GAS (a) for E^* with $\mathscr{C}_2 = 2.4286, \mathscr{C}_{p_{1,2}} = 0.3037 < 1$, (b) for E_* with $\beta_1 = 0.38, \tau_2 = 0.136, \mathscr{C}_1 = 2.7921, \mathscr{C}_{p_{2,1}} = 0.3912 < 1$ and (c) for $\tilde{E}$ with $\beta_1 = 0.036, \mathscr{C}_1 = 0.2645, \mathscr{C}_2 = 2.4286, \mathscr{C}_{P_{1,2}} = 0.4115 < 1$ and $\mathscr{C}_{P_{2,1}} = 24.1839 > 1$.

(iv) The coexistence equilibrium is globally stable asymptotically when $\mathscr{C}_{p_{1,2}} < 1$ and $\mathscr{C}_{p_{2,1}} > 1$.

Furthermore, these theoretical global stability results are established using numerical simulations in Figures 2 and 3. In fact, the numerical results indicate that the super-infection scenario destabilizes the coexistence equilibrium for which each strain can dominate if its threshold parameter is larger than one and the other strain cannot invade its equilibrium, see Figure 3. For further research, kidnap quitters for any reasons can be incorporated in the model to serve like a recovery in disease transmission models. Rehabilitation as a control measure may also be captured in the model to assess its positive impact on the spread of the menace.

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