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Research paper

Solitary Pattern Solutions of gBBM Equation using New Iterative Method (NIM)

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Abstract

In this study, we focus on the solitary pattern solutions of the generalized Benjamin-Bona-Mahony equations (gBBM) and propose a new iterative method (NIM) for their numerical solution, given suitable initial conditions. Our proposed NIM approach generates numerical solutions in the form of a convergent power series with computationally simple components. Our results demonstrate that the NIM approach exhibits exceptional performance in terms of accuracy, efficiency, simplicity, stability, and reliability.

Keywords: *New iterative method (NIM), the generalized Benjamin-Bona-Mahony equations (gBBM), Solitary patterns solutions, Numerical methods*

1. Introduction

Nonlinear phenomena manifest in various scientific applications, including plasma physics, solid-state physics, fluid dynamics, and chemical kinetics. The study of solitary waves has garnered significant interest, leading to the use of a broad range of analytical and numerical methods to analyze these models. In physics and engineering, mathematical modeling of many physical systems results in nonlinear ordinary or partial differential equations. It is crucial to have an effective method to analyze these mathematical models that provide solutions that correspond to physical reality. However, common analytic approaches linearize the system or assume that nonlinearities are relatively insignificant, which can significantly impact the solution's conformity to the actual physics of the phenomenon. Thus, seeking exact solutions of nonlinear ordinary or partial differential equations is of utmost importance. Although there are several numerical methods available to solve nonlinear partial differential equations (NPDEs), they all have certain limitations. For example, conventional numerical methods like finite difference and finite element methods rely on domain discretization, which can introduce errors and instability. Consequently, solving NPDEs poses a significant challenge that requires a combination of theoretical and numerical approaches. The quest for new techniques and methods to solve NPDEs will continue to be an active area of research and development across various fields. [1, 2, 3, 4, 5, 6, 7].

The pursuit of precise solutions to nonlinear equations is a fascinating and vital undertaking. Over the years, various techniques have been devised to address nonlinear partial differential equations (NPDEs). These include the pseudospectral method [8], spectral collocation method [9], Adomian decomposition method (ADM) [10, 11], homotopy perturbation method [12, 13, 14], The variational iteration method (VIM) [15, 16, 17], and The differential transformation method (DTM) [18, 19].

Daftardar-Gejji and Jafari [24] have introduced a novel mathematical technique known as the new iterative method (NIM), which is adept at solving linear and nonlinear functional equations. This method has demonstrated exceptional effectiveness in addressing a wide variety of nonlinear equations, including integral equations, algebraic equations, and both fractional and integer order ordinary or partial differential

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equations. NIM is simple to comprehend and execute using computer software and has been observed to produce superior results [25] when compared to well-established methods such as the Adomian Decomposition Method (ADM) [26], the Homotopy Perturbation Method (HPM) [27], and the Variational Iteration Method (VIM) [28].

This paper outlines a dependable algorithm for examining a renowned equation model that describes long waves in a nonlinear dispersive system. The model was originally proposed in [20], and numerous authors have since investigated the existence and uniqueness of solutions for this equation, as documented in publications such as [20, 21, 22].

The equation known as the generalized BBM equation, expressed as

$$u_t + u_x + au^n u_x + u_{xxx} = 0, \quad n \geq 1, \tag{1.1}$$

features a constant parameter a. When $n = 1$, it becomes the BBM equation, also referred to as the regularized long-wave equation (RLW) [23], given by

$$u_t + u_x + auu_x + u_{xxx} = 0. \tag{1.2}$$

The BBM equation describes surface waves in water under certain conditions and serves as an alternative to the KdV equation. On the other hand, for $n = 2$, the generalized BBM equation reduces to the modified BBM (MBBM) equation, which is expressed as

$$u_t + u_x + au^2 u_x + u_{xxx} = 0. \tag{1.3}$$

An important real-world problem is to solve the generalized BBM equations in the form given by (1.1) with initial data $u(x,0) = f(x)$. The primary objective of research in this area is to develop numerical methods to obtain traveling wave solutions using NIM.

2. The new iterative method (NIM)

In this section, the NIM numerical method will be outlined as follows [34, 35, 36, 37]:

$$u = f + L(u) + N(u), \tag{2.1}$$

In the equation above, f is a known function, and L and N are linear and nonlinear operators, respectively.
The NIM solution for Eq. (2.1) has the form

$$u = \sum_{i=0}^{\infty} u_i. \tag{2.2}$$

Since L is linear then

$$L\left(\sum_{i=0}^{\infty} u_i\right) = \sum_{i=0}^{\infty} L(u_i). \tag{2.3}$$

The nonlinear operator N in Eq. (2.1) is decomposed as below

$$\begin{aligned} N\left(\sum_{i=0}^{\infty} u_i\right) &= N(u_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right\}. \\ &= \sum_{i=0}^{\infty} A_i, \end{aligned} \tag{2.4}$$

where

$$\begin{aligned} A_0 &= N(u_0) \\ A_1 &= N(u_0 + u_1) - N(u_0) \\ A_2 &= N(u_0 + u_1 + u_2) - N(u_0 + u_1) \\ &\vdots \\ A_i &= \left\{N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right\}, \quad i \geq 1. \end{aligned}$$

Using Eqs.(2.2), (2.3) and (2.4) in Eq. (2.1), we get

$$\sum_{i=0}^{\infty} u_i = f + \sum_{i=0}^{\infty} L(u_i) + \sum_{i=0}^{\infty} A_i. \tag{2.5}$$

The solution of Eq. (2.1) can be expressed as

$$u = \sum_{i=0}^{\infty} u_i = u_0 + u_1 + u_2 + \ldots + u_n + \ldots, \tag{2.6}$$

where

$$\begin{aligned} u_0 &= f \\ u_1 &= L(u_0) + A_0 \\ u_2 &= L(u_1) + A_1 \\ &\vdots \\ u_n &= L(u_{n-1}) + A_{n-1} \\ &\vdots \end{aligned} \tag{2.7}$$

Algorithm

$$\begin{aligned} INPUT \quad &: \quad Read\ M(Number\ of\ iterations); \\ & \quad Read\ L(u);\ N(u);\ f \\ & \quad Step-1\ : u_{-1} = 0,\ u_0 = f \\ & \quad Step-2 :\ For(n = 0,\ n \leq M, n{+}{+}) \\ & \quad \{ \\ & \quad Step-3 : A_n = f(u_n) - f(u_{n-1}); \\ & \quad Step-4 : u_{n+1} = f + L(u_n) + A_n; \\ & \quad Step-5 : u = u_{n+1} \\ & \quad \}\ end \\ OUTPUT \quad &: \quad u \end{aligned}$$

3. The convergence of the NIM

Theorem 1: For any n and for some real $L > 0$ and $||u_i|| \leq M < \frac{1}{e}, i = 1, 2, \ldots$, if N is $C^{(\infty)}$ in the neighborhood of u_0 and $||N^{(n)}(u_0)|| \leq L$, then $\sum_{n=0}^{\infty} H_n$ is convergent absolutely and $||H_n|| \leq LM^n e^{n-1}(e-1),\ \ n = 1, 2, \ldots$.
Proof: The full details of the proof can be found in [38].

Theorem 2: The series $\sum_{n=0}^{\infty} H_n$ is convergent absolutely if N is $C^{(\infty)}$ and $||N^{(n)}(u_0)|| \leq M \leq e^{-1}, \forall n$.
Proof: The full details of the proof can be found in [38].

4. Numerical results and discussion

In this section, NIM will be employed to reveal the solution of the long wave propagation model in various scenarios.

Case 1

To illustrate the use of NIM in solving the gBBM equation, we will focus on the case where $n = 1$. In this scenario, the solitary wave solution for the BBM equation (1.2) can be obtained using the following initial condition:

$$u(x,0) = \alpha\ \mathrm{sech}^2(\mu x) \tag{4.1}$$

where $\alpha = \frac{(c-1)(n+2)(n+1)}{2a}$ and $\mu = \frac{n}{2}\sqrt{c-1}$.
To solve the long wave propagation model (1.1) with initial condition (4.1) we integrate Equation (1.1) and use Equation (4.1) to get:

$$u = \alpha\ \mathrm{sech}^2(\mu x) - \int_0^t \left(\frac{\partial^3 u}{\partial x \partial x \partial x} + au\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x} \right) dt \tag{4.2}$$

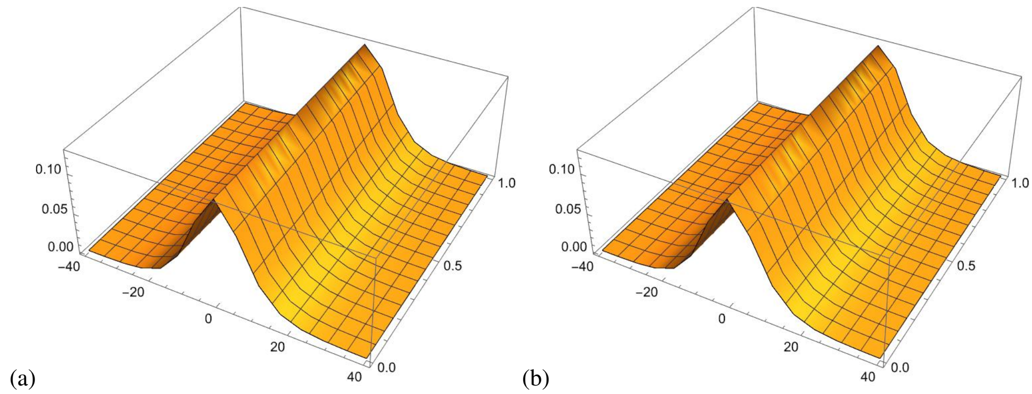

Figure 1: (a) The numerical outcomes of the long wave propagation model (1.1) using one iteration of NIM with $a = 1$ and $c = 1.04225$ are presented for various time periods. (b) The exact solution for the long wave propagation model (1.1) with $a = 1$ and $c = 1.04225$ is demonstrated for various time intervals using one iteration of NIM.

By using Eq. (2.7) we obtain:

$$u_0 = \alpha \operatorname{sech}^2(\mu x), \tag{4.3}$$

$$u_1 = -t(-2a\alpha\mu \tanh(\mu x)\operatorname{sech}^2(\mu x)\left(\alpha\operatorname{sech}^2(\mu x)\right)^n + 16\alpha\mu^3 \tanh(\mu x)\operatorname{sech}^4(\mu x) - 8\alpha\mu^3 \tanh^3(\mu x)\operatorname{sech}^2(\mu x) - 2\alpha\mu \tanh(\mu x)\operatorname{sech}^2(\mu x)), \tag{4.4}$$

So,

$$\sum_{i=0}^{1} u_i = u_0 + u_1. \tag{4.5}$$

Obtaining the remaining components of the repetition formula is a straightforward task that can be accomplished using computer algebra software, such as Mathematica. In order to demonstrate the effectiveness and efficiency of the new iterative method (NIM) when solving the long wave propagation model (1.1) in comparison to the exact solution, we will employ identical parameter values to those used in [39, 40]. The solution of $u(x,t)$ in a closed form is [39]

$$u(x,t) = \alpha\operatorname{sech}^2(\mu(x - ct)) \tag{4.6}$$

Figure 1(a-b) depicts a comparison between the solutions obtained via NIM and the exact solutions for various time intervals.

Case 2

In this case, we will focus on the case where $n = 2$. In this scenario, the solitary wave solution for the BBM equation (1.1) can be obtained using the following initial condition:

$$u(x,0) = \left(\alpha \operatorname{sech}^2(\mu x)\right)^{1/2} \tag{4.7}$$

where $\alpha = \frac{(c-1)(n+2)(n+1)}{2a}$ and $\mu = \frac{n}{2}\sqrt{c-1}$.
To solve the long wave propagation model (1.1) with initial condition (4.7) we integrate Equation (1.1) and use Equation (4.7) to get:

$$u = \left(\alpha \operatorname{sech}^2(\mu x)\right)^{1/2} - \int_0^t \left(\frac{\partial^3 u}{\partial x \partial x \partial x} + au^2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x}\right) dt \tag{4.8}$$

By using Eq. (2.7) we obtain:

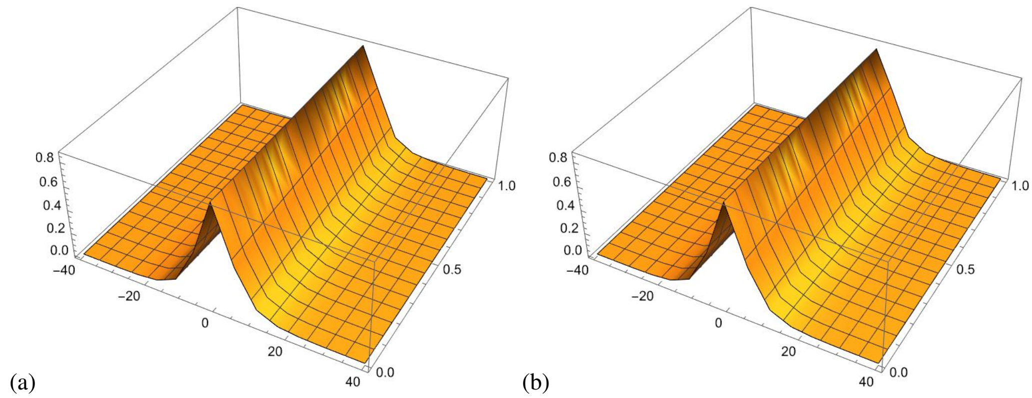

Figure 2: (a) The numerical outcomes of the long wave propagation model (1.1) using one iteration of NIM with $a = 1/2$ and $c = 1.0625$ are presented for various time periods. (b) The exact solution for the long wave propagation model (1.1) with $a = 1/2$ and $c = 1.0625$ is demonstrated for various time intervals using one iteration of NIM.

$$
\begin{aligned}
u_0 &= \left(\alpha\ \mathrm{sech}^2(\mu x)\right)^{1/2}, &(4.9)\\
u_1 &= -t\Big(-\frac{a\alpha^2\mu\tanh(\mu x)\mathrm{sech}^4(\mu x)}{\sqrt{\alpha\mathrm{sech}^2(\mu x)}} - \frac{3\alpha^3\mu^3\tanh^3(\mu x)\mathrm{sech}^6(\mu x)}{\left(\alpha\mathrm{sech}^2(\mu x)\right)^{5/2}}\\
&- \frac{3\alpha^2\mu^3\tanh(\mu x)\mathrm{sech}^6(\mu x)}{\left(\alpha\mathrm{sech}^2(\mu x)\right)^{3/2}} + \frac{6\alpha^2\mu^3\tanh^3(\mu x)\mathrm{sech}^4(\mu x)}{\left(\alpha\mathrm{sech}^2(\mu x)\right)^{3/2}}\\
&- \frac{4\alpha\mu^3\tanh^3(\mu x)\mathrm{sech}^2(\mu x)}{\sqrt{\alpha\mathrm{sech}^2(\mu x)}} + \frac{8\alpha\mu^3\tanh(\mu x)\mathrm{sech}^4(\mu x)}{\sqrt{\alpha\mathrm{sech}^2(\mu x)}}\\
&- \frac{\alpha\mu\tanh(\mu x)\mathrm{sech}^2(\mu x)}{\sqrt{\alpha\mathrm{sech}^2(\mu x)}}\Big). &(4.10)
\end{aligned}
$$

So,

$$\sum_{i=0}^{1} u_i = u_0 + u_1. \tag{4.11}$$

The task of obtaining the remaining components of the repetition formula can be easily accomplished with the use of computer algebra software, such as Mathematica. To showcase the effectiveness and efficiency of the new iterative method (NIM) in solving the long wave propagation model (1.1) in comparison to the exact solution, we will adopt the same parameter values as those used in [39, 40]. According to [39], the solution for $u(x,t)$ in a closed form is as follows:

$$u(x,t) = \sqrt{\alpha\mathrm{sech}^2(\mu(x-ct))} \tag{4.12}$$

Figure 2(a-b) depicts a comparison between the solutions obtained via NIM and the exact solutions for various time intervals.

Case 3

Here, we will focus on the case where $n = 3$. In this scenario, the solitary wave solution for the BBM equation (1.1) can be obtained using the following initial condition:

$$u(x,0) = \left(\alpha\ \mathrm{sech}^2(\mu x)\right)^{1/3} \tag{4.13}$$

where $\alpha = \frac{(c-1)(n+2)(n+1)}{2a}$ and $\mu = \frac{n}{2}\sqrt{c-1}$.
To solve the long wave propagation model (1.1) with initial condition (4.13) we integrate Equation (1.1) and use Equation (4.13) to get:

$$u = \left(\alpha\ \mathrm{sech}^2(\mu x)\right)^{1/3} - \int_0^t \left(\frac{\partial^3 u}{\partial x\partial x\partial x} + au^3\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x}\right) dt \tag{4.14}$$

By using Eq. (2.7) we obtain:

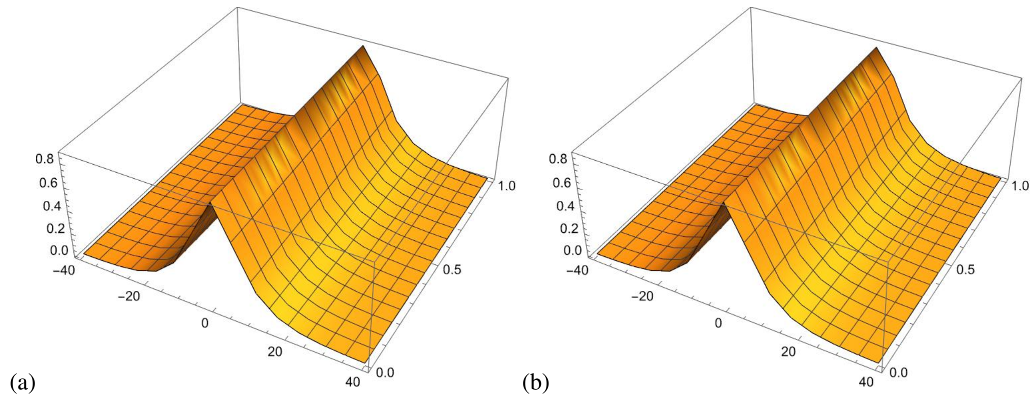

Figure 3: (a) The numerical outcomes of the long wave propagation model (1.1) using one iteration of NIM with $a = 1/3$ and $c = 1.0225$ are presented for various time periods. (b) The exact solution for the long wave propagation model (1.1) with $a = 1/3$ and $c = 1.0225$ is demonstrated for various time intervals using one iteration of NIM.

$$u_0 = \left(\alpha\ \text{sech}^2(\mu x)\right)^{1/3}, \tag{4.15}$$

$$\begin{aligned} u_1 &= -t\Big(-\frac{1}{3}2a\alpha\mu\tanh(\mu x)\text{sech}^2(\mu x)\left(\alpha\text{sech}^2(\mu x)\right)^{\frac{n}{3}-\frac{2}{3}} \\ &- \frac{80\alpha^3\mu^3\tanh^3(\mu x)\text{sech}^6(\mu x)}{27\left(\alpha\text{sech}^2(\mu x)\right)^{8/3}} - \frac{8\alpha^2\mu^3\tanh(\mu x)\text{sech}^6(\mu x)}{3\left(\alpha\text{sech}^2(\mu x)\right)^{5/3}} \\ &+ \frac{16\alpha^2\mu^3\tanh^3(\mu x)\text{sech}^4(\mu x)}{3\left(\alpha\text{sech}^2(\mu x)\right)^{5/3}} - \frac{8\alpha\mu^3\tanh^3(\mu x)\text{sech}^2(\mu x)}{3\left(\alpha\text{sech}^2(\mu x)\right)^{2/3}} \\ &+ \frac{16\alpha\mu^3\tanh(\mu x)\text{sech}^4(\mu x)}{3\left(\alpha\text{sech}^2(\mu x)\right)^{2/3}} - \frac{2\alpha\mu\tanh(\mu x)\text{sech}^2(\mu x)}{3\left(\alpha\text{sech}^2(\mu x)\right)^{2/3}}\Big). \end{aligned} \tag{4.16}$$

So,

$$\sum_{i=0}^{1} u_i = u_0 + u_1. \tag{4.17}$$

Getting the rest of the components for the repetition formula is an easy job that can be done by employing computer algebra software like Mathematica. To showcase the efficiency and effectiveness of the new iterative method (NIM) in solving the long wave propagation model (1.1) compared to the exact solution, we will use the same parameter values utilized in [39, 40]. The closed-form solution of $u(x,t)$ can be found in [39].

$$u(x,t) = \sqrt[3]{\alpha\text{sech}^2(\mu(x-ct))} \tag{4.18}$$

Figure 3(a-b) depicts a comparison between the solutions obtained via NIM and the exact solutions for various time intervals.

5. Conclusions

This paper successfully applies the new iterative method (NIM) to obtain the solitary patterns solution of a gBBM equation, eliminating the need for variable discretization. As a result, the method is not affected by computation round off errors, and large computer memory and time are not required. The exact solution of the equation is closely approximated using only one iteration scheme term. The results indicate that NIM is a powerful mathematical tool for solving gBBM, and it also shows promise for solving other nonlinear equations. The solutions obtained are presented graphically, and the Mathematica Package is used to calculate the series resulting from NIM.

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