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Research Paper

Application of the Adomian decomposition method and Laplace transform method to solving the convection diffusion-dissipation equation

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Abstract

In this paper, the Adomian decomposition method and Laplace transform method are used to construct the solution of a convection-diffusion-dissipation equation.

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1. Introduction

When investigating the pollution of water table, we meet a lot of phenomena, such as convection, diffusion, dissipation etc. These phenomena are governed by mathematical models. Many of them are complex partial differential equations. To study their behavior we often with the simulations based on numeric solutions. In this paper, we examine the convection-diffusion-dissipation equation; we use the Adomian decomposition method and sometimes the Laplace transform method to get the exact solution.

2. The convection diffusion dissipation equation

The convection-diffusion-dissipation equation in dimension 1 has the following form:

$$\frac{\partial u(x,t)}{\partial t}-\alpha\frac{\partial^2 u(x,t)}{\partial x^2}+v(x)\frac{\partial u(x,t)}{\partial x}+c(x)u(x,t)=f(x,t) \quad (1)$$

Where $u(x,t)$ is the concentration of the pollutant, α is a constant, $v(x)$ is the speed of water, $c(x)$ is the dissipation function, $f(x,t)$ is the function source, x is a space variable and t stands for time.

Remark-1:

$-\alpha\frac{\partial^2 u(x,t)}{\partial x^2}$ is the diffusion term, $v(x)\frac{\partial u(x,t)}{\partial x}$ is the convection term, $c(x)u(x,t)$ and is the dissipation term.

2.1 The convection diffusion dissipation

Let's consider the following initial and boundary value problem:

$$(A):\begin{cases}\dfrac{\partial u(x,t)}{\partial t}-\dfrac{\partial^2 u(x,t)}{\partial x^2}+\dfrac{\partial u(x,t)}{\partial x}+u(x,t)=f(x,t)\\ u(x,0)=\cos(x)\\ u(0,t)=\cos(t)\\ u(10,t)=\cos(10)\cos(t)\end{cases} \quad (2)$$

With

$$f(x,t)=-\cos(x)\sin(t)+2\cos(x)\cos(t)-\sin(x)\cos(t) \quad (3)$$

And $(x,t)\in\Omega=[0,10]\times[0,T]$.

We are going to solve problem (A) using the Adomian decomposition method and the Laplace transform method.

2.1.1. Application of the adomian decomposition method (ADM)

The general theory of the Adomian decomposition method can be found in [1-10].

From (2) and (3), we have:

$$\frac{\partial u(x,t)}{\partial t}=\frac{\partial^2 u(x,t)}{\partial x^2}-\frac{\partial u(x,t)}{\partial x}-u(x,t)-\cos(x)\sin(t)+2\cos(x)\cos(t)-\sin(x)\cos(t) \quad (4)$$

From (4) we have:

$$\begin{cases}u(x,t)=u(x,0)+\int_0^t\dfrac{\partial^2 u(x,s)}{\partial x^2}ds-\int_0^t\dfrac{\partial u(x,s)}{\partial x}ds-\int_0^t u(x,s)ds+\\ \qquad\int_0^t\left(-\cos(x)\sin(s)+2\cos(x)\cos(s)-\sin(x)\cos(s)\right)ds\\ \qquad=\cos(x)\cos(t)+2\cos(x)\sin(t)-\sin(x)\sin(t)+\\ \qquad\int_0^t\dfrac{\partial^2 u(x,s)}{\partial x^2}ds-\int_0^t\dfrac{\partial u(x,s)}{\partial x}ds-\int_0^t u(x,s)ds\end{cases} \quad (5)$$

From (2) and (3) we have:

$$\begin{aligned}&u(10,t)-u(0,t)-\int_0^{10}\frac{\partial^2 u(z,t)}{\partial x^2}dz+\int_0^{10}\frac{\partial u(z,t)}{\partial x}dz+\int_0^{10}u(z,t)dz+\\&\sin(t)\int_0^{10}\cos(z)dz-2\cos(t)\int_0^{10}\cos(z)dz+\cos(t)\int_0^{10}\sin(z)dz=0\end{aligned} \quad (6)$$

$(6)\Leftrightarrow$

$$\begin{aligned}&u(10,t)-u(0,t)-\int_0^{10}\frac{\partial^2 u(z,t)}{\partial x^2}dz+\int_0^{10}\frac{\partial u(z,t)}{\partial x}dz+\int_0^{10}u(z,t)dz+\\&\sin(t)\sin(10)-2\cos(t)\sin(10)+\left(1-\cos(10)\right)\cos(t)=0\end{aligned} \quad (7)$$

$(7)\Leftrightarrow$

$$\begin{aligned}&\cos(10)\cos(t)-\cos(t)-\int_0^{10}\frac{\partial^2 u(z,t)}{\partial x^2}dz+\int_0^{10}\frac{\partial u(z,t)}{\partial x}dz+\int_0^{10}u(z,t)dz+\\&\sin(t)\sin(10)-2\cos(t)\sin(10)+\cos(t)-\cos(t)\cos(10)=0\end{aligned} \quad (8)$$

$(8)\Leftrightarrow$

$$\int_0^{10}\frac{\partial^2 u(z,t)}{\partial x^2}dz-\int_0^{10}\frac{\partial u(z,t)}{\partial x}dz-\int_0^{10}u(z,t)dz-\sin(t)\sin(10)+2\cos(t)\sin(10)=0 \quad (9)$$

From (5) and (9), we obtain:

$$\begin{cases} u(x,t) = \cos(x)\cos(t) + 2\cos(x)\sin(t) - \sin(x)\sin(t) - \\ \qquad \sin(t)\sin(10) + 2\cos(t)\sin(10) + \\ \qquad \int_0^t \frac{\partial^2 u(x,s)}{\partial x^2} ds - \int_0^t \frac{\partial u(x,s)}{\partial x} ds - \int_0^t u(x,s) ds + \\ \qquad \int_0^{10} \frac{\partial^2 u(z,t)}{\partial x^2} dz - \int_0^{10} \frac{\partial u(z,t)}{\partial x} dz - \int_0^{10} u(z,t) dz \end{cases} \tag{10}$$

According to the ADM, we suppose that the solution of (2) has the following form:

$$u(x,t) = \sum_{n=0}^{+\infty} u_n(x,t) \tag{11}$$

From (10) and (11), we construct the following Adomian algorithm:

$$\begin{cases} u_0(x,t) = \cos(x)\cos(t) \\ u_1(x,t) = 2\cos(x)\sin(t) - \sin(x)\sin(t) - \sin(t)\sin(10) + 2\cos(t)\sin(10) + \\ \qquad \int_0^t \frac{\partial^2 u_0(x,s)}{\partial x^2} ds - \int_0^t \frac{\partial u_0(x,s)}{\partial x} ds - \int_0^t u_0(x,s) ds + \\ \qquad \int_0^{10} \frac{\partial^2 u_0(z,t)}{\partial x^2} dz - \int_0^{10} \frac{\partial u_0(z,t)}{\partial x} dz - \int_0^{10} u_0(z,t) dz \\ u_{n+1}(x,t) = \int_0^t \frac{\partial^2 u_n(x,s)}{\partial x^2} ds - \int_0^t \frac{\partial u_n(x,s)}{\partial x} ds - \int_0^t u_n(x,s) ds + \\ \qquad \int_0^{10} \frac{\partial^2 u_n(z,t)}{\partial x^2} dz - \int_0^{10} \frac{\partial u_n(z,t)}{\partial x} dz - \int_0^{10} u_n(z,t) dz \ ; \ n \geq 1 \end{cases} \tag{12}$$

Which gives us:

$$u_1(x,t) = 0 \ \Rightarrow \ u_n(x,t) = 0, \ \forall n \geq 1 \tag{13}$$

Thus the solution of (2) is:

$$u(x,t) = \cos(x)\cos(t) \tag{14}$$

2.1.2. Application of the Laplace transforms method

Let's note the Laplace transform by:

$$U(x,p) = L_P(u(x,t)) = \int_0^{+\infty} u(x,t) e^{-pt} dt \tag{15}$$

Applying the Laplace transform to the equation (2), we obtain:

$$pU(x,p) - u(x,0) - \frac{d^2U(x,p)}{dx^2} + \frac{dU(x,p)}{dx} + U(x,p) = \frac{-1+2p}{p^2+1}\cos(x) - \frac{p}{p^2+1}\sin(x) \tag{16}$$

$(16) \Leftrightarrow$

$$-\frac{d^2U(x,p)}{dx^2} + \frac{dU(x,p)}{dx} + (1+p)U(x,p) = \left(1 + \frac{-1+2p}{p^2+1}\right)\cos(x) - \frac{p}{p^2+1}\sin(x) \tag{17}$$

$(17) \Leftrightarrow$

$$\frac{d^2U(x,p)}{dx^2} - \frac{dU(x,p)}{dx} - (1+p)U(x,p) = \frac{p}{p^2+1}\sin(x) - \left(\frac{p^2+2p}{p^2+1}\right)\cos(x) \tag{18}$$

We remark that:

$$\begin{cases} U(0,p) = L(u(0,t)) = L(\cos(t)) = \frac{p}{p^2+1} \\ U(10,p) = L(u(10,t)) = \cos(10) L(\cos(t)) = \frac{p}{p^2+1}\cos(10) \end{cases} \tag{19}$$

The solution of (18) is:

$$U(x,p)=c_1 e^{\left(\frac{1+\sqrt{4p+5}}{2}\right)x}+c_2 e^{-\left(\frac{1+\sqrt{4p+5}}{2}\right)x}+\frac{p}{p^2+1}\cos(x) \tag{20}$$

Using the boundary conditions (19), we obtain $c_1=c_2=0$, and $U(x,p)=\frac{p}{p^2+1}\cos(x)$.

Applying the inverse Laplace transform, we obtain:

$$u(x,t)=\cos(t)\cos(x) \tag{21}$$

Remark-2

Both methods give us the same result.

a) The convection-diffusion-dissipation equation with variable coefficients

Let's consider the following initial and boundary value problem:

$$(B):\begin{cases}\frac{\partial u(x,t)}{\partial t}-\frac{1}{2}\frac{\partial^2 u(x,t)}{\partial x^2}+x(5-x)\cos(x)\frac{\partial u(x,t)}{\partial x}+3x^2u(x,t)=f(x,t)\\ u(x,0)=\cos(x)\\ u(0,t)=\cos(t)\\ u(10,t)=\cos(10)\cos(t)\end{cases} \tag{22}$$

With

$$f(x,t)=-\cos(x)\sin(t)+\left(0.5+3x^2\right)\cos(x)\cos(t)-x(5-x)\cos(x)\sin(x)\cos(t) \tag{23}$$

And $(x,t)\in\Omega=[0,10]\times[0,T]$.

We are going to solve problem (B) using the Adomian decomposition method.

From (22), we have:

$$\frac{\partial u(x,t)}{\partial t}=f(x,t)+\frac{1}{2}\frac{\partial^2 u(x,t)}{\partial x^2}-x(5-x)\cos(x)\frac{\partial u(x,t)}{\partial x}-3x^2u(x,t) \tag{24}$$

From (24), we have

$$\begin{cases}u(x,t)=u(x,0)+\int_0^t f(x,s)ds+\frac{1}{2}\int_0^t\frac{\partial^2 u(x,s)}{\partial x^2}ds-\\ \quad x(5-x)\cos(x)\int_0^t\frac{\partial u(x,s)}{\partial x}ds-3x^2\int_0^t u(x,s)ds\\ =\cos(x)+\int_0^t\left(-\cos(x)\sin(s)+\left(0.5+3x^2\right)\cos(x)\cos(s)-x(5-x)\cos(x)\sin(x)\cos(s)\right)ds+\\ \quad\frac{1}{2}\int_0^t\frac{\partial^2 u(x,s)}{\partial x^2}ds-x(5-x)\cos(x)\int_0^t\frac{\partial u(x,s)}{\partial x}ds-3x^2\int_0^t u(x,s)ds\end{cases} \tag{25}$$

From (22), we have:

$$\int_0^{10}\frac{\partial u(x,t)}{\partial t}dx-\frac{1}{2}\int_0^{10}\frac{\partial^2 u(x,t)}{\partial x^2}dx+\int_0^{10}x(5-x)\cos(x)\frac{\partial u(x,t)}{\partial x}dx+\int_0^{10}3x^2u(x,t)dx-\int_0^{10}f(x,t)dx=0 \tag{26}$$

$(26)\Leftrightarrow$

$$\begin{cases}\int_0^{10}\frac{\partial u(x,t)}{\partial t}dx-\frac{1}{2}\int_0^{10}\frac{\partial^2 u(x,t)}{\partial x^2}dx+\int_0^{10}3x^2u(x,t)dx+\sin(t)\sin(10)-\\ 50\cos^2(10)\cos(t)-\int_0^{10}\left(5\cos(x)+x^2\sin(x)-2x\cos(x)-5\sin(x)\right)u(x,t)dx-\\ \left(\frac{1}{2}\sin 10-60\cos 10+294\sin 10\right)\cos(t)+\left(\frac{99}{8}\cos 20-\frac{15}{8}\sin 20+\frac{1}{8}\right)\cos(t)=0\end{cases} \tag{27}$$

From (25) and (27), we obtain:

$$\begin{cases} u(x,t) = \cos(x)\cos(t) + \left(\frac{1}{2} + 3x^2\right)\cos(x)\sin(t) - x(5-x)\cos(x)\sin(x)\sin(t) + \\ \frac{1}{2}\int_0^t \frac{\partial^2 u(x,s)}{\partial x^2}ds - \left(5x - x^2\right)\cos(x)\int_0^t \frac{\partial u(x,s)}{\partial x}ds - 3x^2\int_0^t u(x,s)ds + \\ \int_0^{10} \frac{\partial u(x,t)}{\partial t}dx - \frac{1}{2}\int_0^{10} \frac{\partial^2 u(x,t)}{\partial x^2}dx - \int_0^{10} 3x^2 u(x,t)dx + +\sin(t)\sin(10) \\ -50\cos^2(10)\cos(t) - \int_0^{10}\left(5\cos x + x^2\sin x - 2x\cos x - 5x\sin x\right)u(x,t)dx - \\ \left(\frac{1}{2}\sin 10 + 60\cos 10 + 294\sin 10\right)\cos(t) + \left(\frac{99}{8}\cos 20 - \frac{15}{8}\sin 20 + \frac{1}{8}\right)\cos(t) \end{cases} \quad (28)$$

According to the ADM, we suppose that the solution off (22) has the following form:

$$u(x,t) = \sum_{n=0}^{+\infty} u_n(x,t) \quad (29)$$

From (28) and (29), we construct the following Adomian algorithm:

$$\begin{cases} u_0(x,t) = \cos(x)\cos(t) \\ u_1(x,t) = \left(\frac{1}{2} + 3x^2\right)\cos(x)\sin(t) - x(5-x)\cos(x)\sin(x)\sin(t) + \sin(t)\sin(10) - \\ 50\cos^2(10)\cos(t) - \int_0^{10}\left(5\cos x + x^2\sin x - 2x\cos x - 5x\sin x\right)u_0(x,t)dx - \\ \left(\frac{1}{2}\sin 10 + 60\cos 10 + 294\sin 10\right)\cos(t) + \left(\frac{99}{8}\cos 20 - \frac{15}{8}\sin 20 + \frac{1}{8}\right)\cos(t) + \\ \frac{1}{2}\int_0^t \frac{\partial^2 u_0(x,s)}{\partial x^2}ds - \left(5x - x^2\right)\cos(x)\int_0^t \frac{\partial u_0(x,s)}{\partial x}ds - 3x^2\int_0^t u_0(x,s)ds + \\ \int_0^{10} \frac{\partial u_0(x,t)}{\partial t}dx - \frac{1}{2}\int_0^{10} \frac{\partial^2 u_0(x,t)}{\partial x^2}dx - \int_0^{10} 3x^2 u_0(x,t)dx \\ u_{n+1}(x,t) = \frac{1}{2}\int_0^t \frac{\partial^2 u_n(x,s)}{\partial x^2}ds - \left(5x - x^2\right)\cos(x)\int_0^t \frac{\partial u_n(x,s)}{\partial x}ds - 3x^2\int_0^t u_n(x,s)ds + \\ \int_0^{10} \frac{\partial u_n(x,t)}{\partial t}dx - \frac{1}{2}\int_0^{10} \frac{\partial^2 u_n(x,t)}{\partial x^2}dx - \int_0^{10} 3x^2 u_n(x,t)dx - \\ \int_0^{10}\left(5\cos x + x^2\sin x - 2x\cos x - 5x\sin x\right)u_n(x,t)dx \ ; \ n \geq 1 \end{cases} \quad (30)$$

Which gives us:

$$u_1(x,t) = 0 \Rightarrow u_n(x,t) = 0, \ \forall n \geq 1 \quad (31)$$

Thus the solution of (22) is:

$$u(x,t) = \cos(x)\cos(t) \quad (32)$$

3. Conclusion

Through these examples, we showed again the usefulness of the Adomian decomposition method, in the search of an approximate solution of an equation and this method gave us the exact solution.

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