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New Periodic and Soliton Solutions of (2 + 1)-Dimensional Soliton Equation

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Abstract

By using the sine-cosine method proposed recently, we give the exact periodic and soliton solutions of the (2 + 1)-dimensional soliton equation in this paper. Many new families of exact traveling wave solutions of the (2 + 1)-dimensional soliton equation are successfully obtained. The computation for the method appears to be easier and faster by general mathematical software.

Keywords: *Sine-cosine method, Soliton equation, Periodic solution, Soliton solution.*

1 Introduction

Many important phenomena and dynamic processes in physics, mechanics, chemistry and biology can be represented by nonlinear partial differential equations. The study of exact solutions of nonlinear evolution equations plays an important role in soliton theory and explicit formulas of nonlinear partial differential equations play an essential role in the nonlinear science. Also, the explicit formulas may provide physical information and help us to understand the mechanism of related physical models. Recently, there have been a multitude of methods presented for solving Nonlinear partial differential equations (NPDEs), for instance, the tanh-method [1–3], the extended tanh method [4–6], the sine-cosine method [7–9], the homogeneous balance method [10], homotopy analysis method [11–14], the $F-$expansion method [15], three-wave method [16–18], extended homoclinic test approach [19, 20], the $(\frac{G'}{G})-$expansion method [21] and the exp-function method [22–24].

In this paper, by means of the Sine-cosine method, we will obtain some Solitary solutions of the following (2 + 1)-dimensional soliton equation given in [25]

$$\begin{aligned} &i\,u_t + u_{xx} + u\,v = 0, \\ &v_t + v_y + (u\,u^*)_x = 0. \end{aligned} \tag{1}$$

where $i = \sqrt{-1}, u(x, y, t)$ is complex function and $v(x, y, t)$ is real function.

2 The Sine-Cosine Method

1. We introduce the wave variable $\xi = x - ct$ into the PDE

$$P(u, u_t, u_x, u_{tt}, u_{xx}, u_{tx}, \ldots) = 0, \tag{2}$$

where $u(x, t)$ is traveling wave solution. This enables us to use the following changes:

$$\frac{\partial}{\partial t} = -c\,\frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial t^2} = c^2\,\frac{\partial^2}{\partial \xi^2}, \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial \xi^2}, \ldots. \tag{3}$$

One can immediately reduce the nonlinear PDE (2) into a nonlinear ODE

$$Q(u, u_\xi, u_{\xi\xi}, u_{\xi\xi\xi}, \ldots) = 0. \tag{4}$$

The ordinary differential equation (4) is then integrated as long as all terms contain derivatives, where we neglect integration constants.

2. The solutions of many nonlinear equations can be expressed in the form

$$u(x, t) = \begin{cases} \lambda\, sin^\beta(\mu\,\xi), & |\xi| \le \dfrac{\pi}{\mu}, \\ 0 & \text{otherwise}, \end{cases} \tag{5}$$

or in the form

$$u(x, t) = \begin{cases} \lambda\, cos^\beta(\mu\,\xi), & |\xi| \le \dfrac{\pi}{2\mu}, \\ 0 & \text{otherwise}, \end{cases} \tag{6}$$

where λ, μ and$\beta \neq 0$ are parameters that will be determined, μ and c are the wave number and the wave speed respectively. We use

$$\begin{aligned} &u(\xi) = \lambda\, sin^{\beta}(\mu\,\xi), \\ &u^n(\xi) = \lambda^n\, sin^{n\,\beta}(\mu\,\xi), \\ &(u^n)_\xi = n\mu\beta\lambda^n \cos(\mu\xi) \sin^{n\beta-1}(\mu\xi), \\ &(u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \sin^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\sin^{n\beta-2}(\mu\xi), \end{aligned} \tag{7}$$

and the derivatives of 6 becoms

$$\begin{aligned} &u(\xi) = \lambda\, cos^{\beta}(\mu\,\xi), \\ &u^n(\xi) = \lambda^n\, cos^{n\,\beta}(\mu\,\xi), \\ &(u^n)_\xi = -n\mu\beta\lambda^n \sin(\mu\xi) \cos^{n\beta-1}(\mu\xi), \\ &(u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \cos^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\cos^{n\beta-2}(\mu\xi), \end{aligned} \tag{8}$$

and so on for other derivatives.

3.We substitute (7) or (8) into the reduced equation obtained above in (4), balance the terms of the cosine functions when (8) is used, or balance the terms of the sine functions when (7) is used, and solving the resulting system of algebraic equations by using the computerized symbolic calculations. We next collect all terms whit same power in $\cos^k(\mu\xi)$ or $\sin^k(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknowns μ, β and λ. We obtained all possible value of the parameters μ, β and λ.

3 The (2+1)-Dimensional Soliton Equation:

In order to seek exact solutions of Eq. (1), we suppose

$$\begin{aligned} &u(x,y,t) = \phi(\xi)\, \exp(i\,\eta), \quad v(x,y,t) = v(\xi), \\ &\eta = k\,x + l\,y + \alpha\,t, \quad \xi = K(x + L\,y - 2k\,t), \end{aligned} \tag{9}$$

where $\phi(\xi)$ and $v(\xi)$ are real functions, k, l, α, K and L are real constants to be determined later. Substituting Eq. (9) into Eq. (1), we have

$$K^2\,\phi^{''}(\xi) - (\alpha + k^2)\,\phi(\xi) + \phi(\xi)\,v(\xi) = 0, \tag{10}$$

$$(L - 2\,k)v^{'}(\xi) + (\phi^2(\xi))^{'} = 0. \tag{11}$$

where prime denotes the differential with respect to ξ. Integrating Eq. (11) with respect to ξ and taking the integration constant as zero yields

$$v(\xi) = \frac{1}{2\,k - L}\,\phi^2(\xi), \qquad if\, L \neq 2k, \tag{12}$$

where c is an integration constant. Substituting Eq. (12) into Eq. (10) yields

$$\phi^{''}(\xi) + A\,\phi(\xi) - B\,\phi^3(\xi) = 0, \tag{13}$$

and

$$A = \frac{-\alpha - k^2}{K^2} \quad , \quad B = \frac{1}{K^2(L - 2k)}$$

Substituting (5) into (13) gives

$$\begin{aligned} &-\mu^2\beta^2\lambda\sin^{\beta}(\mu\xi) + \mu^2\lambda\beta(\beta - 1)\sin^{\beta-2}(\mu\xi) \\ &+A\,\lambda\sin^{\beta}(\mu\xi) - B\,\lambda^3\,\sin^{3\beta}(\mu\xi) = 0. \end{aligned} \tag{14}$$

Equating the exponents and the coefficients of each pair of the sine functions we find the following system of algebraic equations:

$$\begin{aligned} &(\beta - 1) \neq 0, \\ &\beta - 2 = 3\,\beta, \\ &-\mu^2\,\beta^2\,\lambda + A\,\lambda = 0, \\ &\mu^2\,\lambda\,\beta(\beta - 1) - B\,\lambda^3 = 0, \end{aligned} \tag{15}$$

Solving the system (15) yields

$$\beta = -1 \quad , \quad \mu = \pm\sqrt{\frac{-\alpha - k^2}{K^2}} \quad , \quad \lambda = \pm\,\sqrt{2(\alpha + k^2)(2\,k - L)}, \tag{16}$$

where k, l, α, K and L are free parameters. The results (16) give for $\dfrac{\alpha + k^2}{K^2} < 0$, the following periodic solutions:

$$u_{11}(\xi) = \sqrt{2(\alpha + k^2)(2\,k - L)}\,\csc\left(\sqrt{\frac{-\alpha - k^2}{K^2}}\,(\xi)\right)\exp(i\,\eta),$$

$$v_{11}(\xi) = \sqrt{\frac{2(\alpha + k^2)}{2\,k - L}}\,\csc^2\left(\sqrt{\frac{-\alpha - k^2}{K^2}}\,(\xi)\right)$$

where $0<\sqrt{\dfrac{-\alpha-k^2}{K^2}}\,(\xi)<\pi$, $L\neq 2k$, and

$$u_{12}(\xi)=\ \sqrt{2(\alpha+k^2)(2\,k-L)}\ \sec\left(\sqrt{\frac{-\alpha-k^2}{K^2}}\,(\xi)\right)\exp(i\,\eta),$$

$$v_{12}(\xi)=\sqrt{\frac{2(\alpha+k^2)}{2\,k-L}}\ \sec^2\left(\sqrt{\frac{-\alpha-k^2}{K^2}}\,(\xi)\right)$$

and

$$u_{13}(\xi)=-\sqrt{2(\alpha+k^2)(2\,k-L)}\ \sec\left(\sqrt{\frac{-\alpha-k^2}{K^2}}\,(\xi)\right),$$

$$v_{13}(\xi)=\sqrt{\frac{2(\alpha+k^2)}{2\,k-L}}\ \sec^2\left(\sqrt{\frac{-\alpha-k^2}{K^2}}\,(\xi)\right)$$

where $\left|\sqrt{\dfrac{-\alpha-k^2}{K^2}}\,(\xi)\right|<\dfrac{\pi}{2}$ and $L\neq 2k$.

However, for $\dfrac{\alpha+k^2}{K^2}>0$, we obtain the soliton solutions

$$u_{21}(\xi)=\ \sqrt{2(\alpha+k^2)(2\,k-L)}\,csch\left(\sqrt{\frac{-\alpha-k^2}{K^2}}\,(\xi)\right)\exp(i\,\eta),$$

$$v_{21}(\xi)=\sqrt{\frac{2(\alpha+k^2)}{2\,k-L}}\,csch^2\left(\sqrt{\frac{-\alpha-k^2}{K^2}}\,(\xi)\right)$$

where $0<\sqrt{\dfrac{-\alpha-k^2}{K^2}}\,(\xi)<\pi$, $L\neq 2k$, and

$$u_{22}(\xi)=\ \sqrt{2(\alpha+k^2)(2\,k-L)}\,sech\left(\sqrt{\frac{-\alpha-k^2}{K^2}}\,(\xi)\right)\exp(i\,\eta),$$

$$v_{22}(\xi)=\sqrt{\frac{2(\alpha+k^2)}{2\,k-L}}\,sech^2\left(\sqrt{\frac{-\alpha-k^2}{K^2}}\,(\xi)\right)$$

and

$$u_{23}(\xi) = -\sqrt{2(\alpha + k^2)(2\,k - L)}\, sech\left(\sqrt{\frac{-\alpha - k^2}{K^2}}\,(\xi)\right),$$

$$v_{23}(\xi) = \sqrt{\frac{2(\alpha + k^2)}{2\,k - L}}\, sech^2\left(\sqrt{\frac{-\alpha - k^2}{K^2}}\,(\xi)\right)$$

where $\left|\sqrt{\frac{-\alpha - k^2}{K^2}}\,(\xi)\right| < \frac{\pi}{2}$ and $L \neq 2k$.

As we know, various other types of exact solutions for the (2+1)-dimensional Soliton equation, such as rational solutions, polynomial solutions and the traveling wave solutions have been obtained by many authors under different approaches [25–27].

4 Conclusion

In this paper, We successfully obtained exact and explicit analytic solutions to the (2+1)-dimensional Soliton equation via the sine-cosine approach. Some of these results are in agreement with the results reported by others in the literature, and new results are formally developed in this work. It is shown that the algorithm can be also applied to other NLPDEs in mathematical physics.

References

[1] A.M. Wazwaz, The tanh method: solitons and periodic solutions for the Dodd-Bullough-Tzikhailov and the Tzitzeica-Dodd-Bullough equations, Chaos, Solitons and Fractals 25 (2005) 55-63.

[2] A.M. Wazwaz, "The tanh method and the sine-cosine method for solving the KP-MEW equation", *Int. J. Comput. Math.*, Vol.82, No.2, (2005), pp.235-246.

[3] A.M. Wazwaz, "The tanh method for travelling wave solutions of nonlinear equations", *Appl. Math. Comput.*, Vol.154, No.3, (2004), pp.713-723.

[4] A.M. Wazwaz, "The extended tanh method for new solitons solutions for many forms of the fifth-order KdV equations", *Appl. Math. Comput.*, Vol.184, No.2, (2007), pp.1002-1014.

[5] A.M. Wazwaz, The extended tanh method for abundant solitary wave solutions of nonlinear wave equations, Appl. Math. Comput 187 (2007), 1131-1142.

[6] A.M. Wazwaz, "New solitons and kinks solutions to the Sharma-Tasso-Olver equation", *Appl. Math. Comput.*, Vol.188, No.2, (2007), pp.1205-1213.

[7] E. Yusufoğlu, A. Bekir, "Solitons and periodic solutions of coupled nonlinear evolution equations by using sine-cosine method", *Int. J. Comput. Math.*, Vol.83, No.12, (2006), pp.915-924.

[8] F. Taşcan, A. Bekir, Analytic solutions of the $(2 + 1)$-dimensional nonlinear evolution equations using the sine-cosine method, Appl. Math. Comput 215 (2009), 3134-3139.

[9] S. Arbabi Mohammad-Abadi, "Analytic solutions of the Kadomtsev-Petviashvili equation with power law nonlinearity using the sine-cosine method", *American Journal of Computational and Applied Mathematics.*, Vol.1, No.1, (2011), pp.1-4.

[10] Z. Xiqiang, W. Limin, S. Weijun, "The repeated homogeneous balance method and its applications to nonlinear partial differential equations", *Chaos, Solitons and Fractals* , Vol.28, No.2, (2006), 448-453.

[11] S.J. Liao, On the homotopy analysis method for nonlinear problems, Appl. Math. Comput 147 (2004), 499-513.

[12] S.J. Liao, A new branch of solutions of boundary-layer flows over an impermeable stretched plate, Int. J. Heat Mass Transfer 48 (2005), 2529-2539.

[13] M.T. Darvishi, F. Khani, A series solution of the foam drainage equation, Comput. Math. Appl 58 (2009), 360-368.

[14] F. Khani, M.T. Darvishi, R.S.R. Gorla, " Analytical investigation for cooling turbine disks with a non-Newtonian viscoelastic fluid", *Appl. Math. Comput.*, Vol.61, No.7, (2011), pp.1728-1738.

[15] E. Fan, Z. Jian, " Applications of the Jacobi elliptic function method to special-type nonlinear equations", *Phys. Lett. A.*, Vol.305, No.6, (2002), pp.383-392.

[16] M.T. Darvishi, Maliheh Najafi, Mohammad Najafi, " Exact three-wave solutions for high nonlinear form of Benjamin-Bona-Mahony-Burgers equations", *International Journal of Computational and Mathematical Sciences.*, Vol.6, No.3, (2010), pp.127-131.

[17] M.T. Darvishi, Maliheh Najafi, Mohammad Najafi, " New exact solutions for the (3+1)-dimensional breaking soliton equation", *International Journal of Computational and Mathematical Sciences.*, Vol.6, No.2, (2010), pp.134-137.

[18] S.Arbabi Mohammad-Abadi, Maliheh Najafi , Extend three-wave method for the (3+1)-Dimensional Soliton Equation, World Academy of Science, Engineer-ing and Technology, 80 (2011), 1342-1345.

[19] M.T. Darvishi, Maliheh Najafi, Mohammad Najafi, "New application of EHTA for the generalized (2+1)-dimensional nonlinear evolution equations", *International Journal of Computational and Mathematical Sciences.*, Vol.6, No.3, (2010), pp.132-138.

[20] M.T. Darvishi, Mohammad Najafi, "Some complexiton type solutions of the (3+1)-dimensional Jimbo-Miwa equations", *International Journal of Computational and Mathematical Sciences.*, Vol.6, No.1, (2012), pp.25-27.

[21] M.T. Darvishi, Maliheh Najafi, Mohammad Najafi , Traveling wave solutions for the (3+1)-dimensionalbreaking soliton equation by $(\frac{G'}{G})$-expansion method and modified F-expansion method, World Academy of Science, Engineer-ing and Technology, 88 (2012), 37-42.

[22] F. Khani, M.T. Darvishi, A. Farmani, L. Kavitha , New exact solutions of coupled (2+1)-dimensional nonlinear system of Schrödinger equations, ANZIAM Journal, 52 (2010), 110-121.

[23] X.H. Wu, J.H. He, "Exp-function method and its application to nonlinear equations", *Chaos, Solitons and Fractals.*, Vol.38, No.3, (2008), pp.903-910.

[24] A. R. Adem, C. M. Khalique, A. Biswas , Solutions of KadomtsevPetviashvili equation with power law nonlinearity in 1+3 dimensions, Math. Meth. Appl. Sci, 34 (2011), 532-543.

[25] C. Ye, W. Zhang, New explicit solutions for (2 + 1)-dimensional soliton equation, Chaos Solitons Fractals 44 (2011) 1063-1069.

[26] K. Porsezian, Painlev analysis of new higher-dimensional soliton equation, J Math Phys 38 (1997) 4675-4679.

[27] Z. Y. Yan, Extended Jacobian elliptic function algorithm with symbolic computation to construct new doubly-periodic solutions of nonlinear differential equations, Comput. Phys. Commun. 148 (2002) 30-42.