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Review paper

The problem of the ultraviolet catastrophe

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Abstract

The study of thermal radiation emitted by a black body founded the quantum theory of radiation. The development of this study revealed discrepancies with the predictions of classical physics, especially the failure of the Rayleigh-Jeans Law for short wavelengths, known as the ultraviolet catastrophe. Within this context, Max Planck suggested that the energy emitted by these bodies be quantized in multiples of , which we know as Planck's constant, developing Planck's Law, which accurately describes the behavior of the blackbody radiation spectrum, intertwining his theory with experimental results. From this, it was possible to prove the limitation of classical physics to explain some phenomena, thus establishing a cradle for quantum physics. This paper addresses these findings and their implications for understanding thermal radiation by analyzing the evolution of the study of blackbody radiation and comparing the different equations proposed over time.

Keywords: *Radiation; Planck; Black Body; Rayleigh-Jeans; Catastrophe.*

1. Introduction

Through the study of the thermal radiation emitted by opaque bodies, he initiated the proposals for the study of the quantum nature of radiation, where part of the radiation incident on an opaque body is reflected and another part absorbed. The radiation absorbed by the body increases the kinetic energy of the constituent atoms, oscillate around their equilibrium positions. The energy absorbed causes the temperature to rise, due to the average translational kinetic energy of the atoms, determining the temperature of the body.
As atoms are also made up of electrons, these electrons are accelerated by oscillations and, according to electromagnetic theory, atoms emit electromagnetic radiation, reducing the kinetic energy of the oscillations, consequently reducing the temperature, where a good radiation absorber is also a good emitter [1].
Planck begins his study of the heat radiation of a black body on the order of hundreds of degrees Celsius, with the analysis of continuous emission and absorption spectra, with the first results of this analysis obtained by Friedrich Paschen in 1984, involving the wavelength in the infrared range and through these observations, Paschen and Wilhelm Wien, Independently, they suggested an equation to fit the experimental curves of the intensity of the emitted radiation.
The article presents the radiation in the cavity and the discussion of the ultraviolet catastrophe by Planck's analysis in a new study [2].

2. Theoretical foundation and discussion

The physics community at different times has been developing work in the search for answers that even favor other areas, such as Newtonian mechanics initially developed by Galileo Galilei (1564-1642) and Isaac Newton (1643-1727), were greatly improved, with more precise works in the observations of the movements of celestial bodies, as well as in the clarity of the electrical and magnetic phenomena of James Maxwell's (1831-1879) electromagnetic theory, showing that light is an electromagnetic wave that propagates in space [3].
William Thomson (1824-1907), known as Lord Kelvin, develops a work on "Nineteenth-century clouds on the dynamic theory of heat and light", where these two clouds became known as the photoelectric effect and the divergence that arises when calculating the power of the radiation emitted by a body due to its temperature, because there is the dissipation of these two clouds, believed that the problem of physics would be solved and closed, but it would be necessary to radically change the conceptual view of classical electromagnetism at the time [4].
Max Planck (1858-1947) at a meeting of the German Physical Society on December 14, 1900, presented a paper with the title: "On the theory of the Law of Energy Distribution of the Normal Spectrum". This article marks the beginning of a revolution in physics, even with little attention from the scientific community of the time. Planck introduces a constant h, called Planck's constant, to explain the observed properties of thermal radiation [5].

Although Planck's hypothesis of a quantum of energy had a marked influence on atomic and molecular physics in the twentieth century, his discovery is not linked to the main topics in the field of physics in the period from 1895 to 1900. Planck originally devoted himself to the study of the radiation of the heat of a black body at high temperatures, analyzing the continuous spectra of emission and absorption [2]. The quantum nature of radiation came through the study of thermal radiation emitted by black bodies. When radiation falls on an opaque body, part of it is reflected and part absorbed. The radiation absorbed by the body increases the kinetic energy of the constituent atoms that oscillate over the equilibrium positions, causing the temperature to rise. These atoms contain electrons, being accelerated, emitting electromagnetic radiation, reducing the kinetic energy of the oscillations, and also reducing the temperature. The temperature is constant when the absorption rate is equal to the emission, and the body is in thermal equilibrium [4].
Regardless of body composition, all blackbodies at the same temperature emit thermal radiation with the same spectrum. We cannot confine ourselves to thermodynamic arguments only for this discussion, but we can also analyze other issues, such as the spectral distribution of radiation, specified by the amount $R_T(\nu)$ that is the spectral radiance in a way that $R_T(\nu)d\nu$ describes the energy emitted per unit of time in frequency radiation in the range of ν to $\nu + d\nu$ per unit area of a surface at absolute temperature T [5].
Balfour Stewart (1828-1887) discovered that the ratio of a body's emitting power to absorbing power is a function of the wavelength (λ) of the emitted or absorbed radiation and the absolute temperature T, translated by function $I(\lambda, T)$ and which was also independently discovered by Gustav Kirchhoff (1824-1887) in 1859 [6,7].
Kirchhoff, studying the relationships between the energies emitted and absorbed by bodies, defines two laws of radiation, where the frequency emitted by bodies when they are heated is the sole function of absolute temperature, with the second result being the concept of black body [3].
According to Stefan (1879) [8] and Boltzmann (1884) [9], the search was concentrated at that time to find the function I, being the first step taken by Josef Stefan (1835-1893), where in 1879, he studies the speed at which bodies cool, measuring the areas under the curves of the thermal radiant spectrum, taking his law to $R \propto T^4$, where R is the total intensity of the radiation emitted by a body at a given temperature T, in the form

$$R = \int_0^\infty I(\lambda, T)d\lambda \quad (1)$$

Following this direction, Ludwig Boltzmann (1844-1906) mathematically demonstrated Stefan's law, considering electromagnetic radiation as a gas inside a black body, applying the laws of thermodynamics to this gas, finding the coefficient of proportionality (σ) between R and T^4, known as the Stefan–Boltzmann constant, in the form

$$R = \sigma T^4 \quad (2)$$

Where R is the power radiated per unit area, showing us the rate at which the energy is emitted by the object, and T is the absolute temperature, and $\sigma = 5.6703 \times 10^{-8} W/m^2K^4$ Stefan's constant. The power per unit area radiated by a blackbody depends only on temperature. Equation (2) became known as the Stefan–Boltzmann Law.

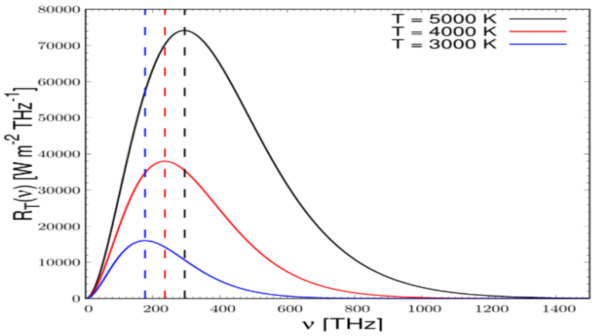

Fig. 1: Spectral Radiance in Wm Thz as A Function of Frequency and Thz (= 10 Hz) for Blackbodies of Three Different Temperatures: 3000 K (Blue), 4000 K (Red), and 5000 K (Black) [10].

Still in the discussion of irradiated power, objects that are not blackbodies radiate at a lower rate than a blackbody at the same temperature, being independent of intrinsic properties, such as surface composition and color, and the effects of these dependencies are given by the permittivity factor ε, dependent on temperature and smaller than unity [4].
This permittivity factor is related to R_{CN}The radiance of the black body and R_S the radiance of the surface, in the form of

$$R_S = \varepsilon\sigma T^4 = \varepsilon R_{CN} \quad (3)$$

The spectral distribution is determined experimentally according to Figure 2.

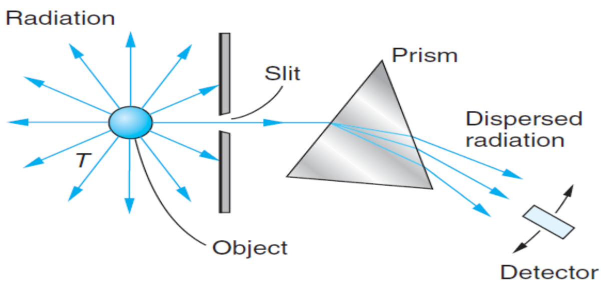

Fig. 2: Scattering of the Light Emitted by the T-Temperature Object Passing Through the Slit According to Its Lengths, with the Prism Showing the Light Emitted in the Visible Region [11].

The explanation of the curve of the distribution of energy emitted by the black body at a given temperature T as a function of wavelength λ is that it needed to be solved. There was a difficulty with this resolution, and Wien in 1894 presented, in a theoretical way, a discussion in which the wavelength at the spectral radiance peak was a single function of the absolute temperature T, with

$$\lambda = \frac{2{,}898}{T}(\mathrm{mm.K}) \tag{4}$$

where he stated that the curve was the same regardless of the material that constituted the cavity, arriving at the expression for the spectral distribution of energy in the form

$$R_\lambda = \frac{c_1}{\lambda^5}\cdot\frac{1}{e^{\frac{c_2}{\lambda T}}} \tag{5}$$

The R_λ spectral radiance and the constants c_1 and c_2, determined experimentally, fitting better to the curve, but Wien's expression is fit only for high frequencies or short wavelengths [3].
In 1986, Wien obtained this equation when he considered that thermal radiation resulted from the vibration of molecular oscillators, and that the intensity of this radiation was proportional to the number of these oscillators, and that Louis Carl Paschen (1865-1940) also obtained this same expression, but being applied to high frequencies [12-14].
For John Strutt Rayleigh (1842-1919), when considering the intensity of thermal radiation proportional to the normal vibration tones of molecular oscillators, a new expression was found in 1900 in the form

$$R_\lambda = \frac{c'_1}{\lambda^4}\cdot\frac{1}{e^{\frac{c_2}{\lambda T}}} \tag{6}$$

With arguments for the entropy of molecular oscillators. However, Max Planck (1858-1947) rewrote Wien's equation by presenting it to the Berlin Physical Society in 1900, whereby making the interpolation between these two formulas, he arrived at the expression

$$R_\lambda = \frac{c_1}{\lambda^5\left(\exp\left(\frac{c_2}{\lambda T}\right)-1\right)} \tag{7}$$

Where it was reduced to $\lambda T \ll 1$ (Wien) and $\lambda T \gg 1$ (Rayleigh), because Heinrich Rubens (1865-1922) and Ferdinand Kurlbaum (1857-1927) showed that Wien's equation failed for $\lambda T \gg 1$, where they fit Rayleigh's equations [15], [16].
The radiated power R is discussed as falling on a small hole connected to a cavity of energy U, this energy per unit volume of radiation inside the cavity, being related in the way $R = \frac{1}{4}cU$ in which using the power in the form of the spectral distribution function $R(\lambda)$ the portion $u(\lambda)d\lambda$ is the fraction of energy per unit volume inside the cavity between λ and $\lambda + d\lambda$, staying in shape

$$R(\lambda) = \frac{1}{4cu(\lambda)} \tag{8}$$

With $u(\lambda)$ determined by the electromagnetic field oscillation in the cavity with the average energy per mode, in which the number of oscillation modes $n(\lambda)$ per unit volume, does not depend on the shape of the cavity, but relates to the shape

$$n(\lambda) = 8\pi n\lambda^{-4} \tag{9}$$

Referring to the average oscillation energy to KT, with K the Boltzmann constant, obtaining the spectral distribution of energy density in the form

$$u(\lambda) = KTn(\lambda) \tag{10}$$

Known as the Rayleigh-Jeans Law, with wavelengths according to the experimental results, but for short wavelengths, the law diverges to zero, but the experimental data point to the law tends to zero when the length tends to zero. This discrepancy became known as an ultraviolet catastrophe [4].
In 1877, Planck used the probabilistic interpretation presented by Boltzmann for the treatment of the entropy of molecular oscillators of frequency ν, admitting the hypothesis that the energy of the oscillators varied slightly in shape $\varepsilon = h\nu$, believing it to be only a calculation device, which could reach zero for h. It would be necessary for the value of h to have a finite value for the results to be compatible with the experiments, where it presents the value for this constant $h = 6.55.10^{-27}$ ergs, where it later became known as Planck's constant [12].
In a system of molecules at equilibrium, the Boltzmann function is related to the Maxwell distribution function for that energy, and this distribution function tells us about the energies in the form of an average value in the form.

$$\bar{\varepsilon} = \frac{\int_{\infty}^{\infty}\varepsilon P(\varepsilon)d\varepsilon}{\int_0^{\infty}P(\varepsilon)d\varepsilon} \tag{11}$$

The $P(\varepsilon)$a special form of the Boltzmann distribution in the form

$$P(\varepsilon) = \frac{e^{-\frac{\varepsilon}{kT}}}{kT} \tag{12}$$

By integrating equation (8) over all possible energies, the average value of the energy in the form of the

$$\bar{\varepsilon}(\nu) = \frac{h\nu}{e^{\frac{h\nu}{kT}}-1} \tag{13}$$

Planck subsequently adjusts the value for the experimental data close to the currently accepted value for $h = 6.63.10^{-34}$ joule.s. To obtain the energy density of the blackbody spectrum, he used the result of equation (10), in the form

$$\rho_T(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}d\nu \tag{14}$$

Which results for the Planck blackbody spectrum, confirming the experimental results. Planck's consideration of the Boltzmann distribution was for the treatment of electromagnetic standing wave energy, treating it as discrete [5].
By considering the number of oscillators per unit volume in the interval between λ and $\lambda + d\lambda$, we obtain the distribution of energy density in the cavity in the shape

$$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{\frac{hc}{\lambda kT}}-1} \tag{15}$$

This expression is Planck's Law, and for large values of λ, we are left with the exponential term of equation (12) in the form [4].

$$e^{\frac{hc}{\lambda kT}} - 1 \approx \frac{hc}{\lambda kT} \tag{16}$$

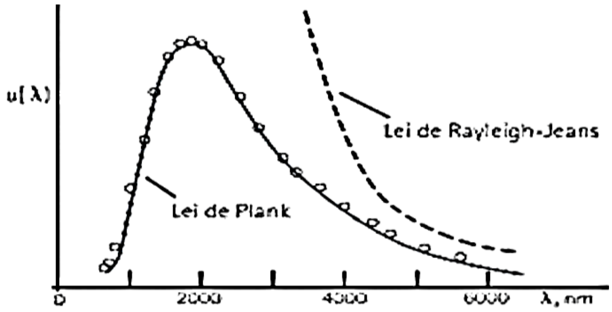

Fig. 3: Comparative Graph of Planck's Law and the Rayleigh-Jeans Law [4].

3. Conclusion

We used the mathematical development of Wien and Rayleigh-Jeans' theory, exploring its limitations that are given by classical physics for high frequencies, which led to a milestone called the ultraviolet catastrophe. From this, we approach the mathematical and experimental foundation presented by Planck with the quantum perspective of the phenomenon, which proved its validity within the study of blackbody radiation.
Planck's theory, with the introduction of quantum physics, was successfully accompanied by experimental results consistent with the theorists, solving the problem of the ultraviolet catastrophe and from there opening a path to a new vision of physics, quantum physics.
We seek to provide a brief and detailed understanding of the evolution of the physical models that describe thermal radiation and its implications for the development of modern physics.

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