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Research Paper

On Bayesian estimation from two parameter Bathtub-Shaped lifetime distribution based on progressive first-failure-censored sampling

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Abstract

This paper presents different methods of Bayesian estimation to estimate parameter and reliability function of two parameter bathtub-shaped lifetime distribution based on progressively firrst-failure-censored samples under minimum expected and LINEX loss functions. Comparisons among estimators are investigated through simulation study.

Keywords: *Progressive first-failure-censoring scheme, Empirical Bayesian Estimation, E-Bayesian Estimation, Bathtub-Shaped lifetime distribution .*

1. Introduction

The bathtub shape provides an appropriate conceptual model for the hazard of some electro-mechanical, electronic and mechanical products.In this paper, we discuss the two-parameter lifetime distribution with bathtub-shaped or increasing failure rate function. the cumulative distribution function(CDF), probability density function (PDF), reliability function and corresponding failure rate function of the new two parameter lifetime distribution with bathtub-shaped or increasing failure rate function are given, respectively, by (see Chen [11])

$$F(x) = 1 - e^{\lambda(1-e^{x^{\beta}})}, \qquad x > 0, \quad \lambda, \beta > 0 \tag{1}$$

$$f(x) = \lambda\beta x^{\beta-1} e^{x^{\beta}+\lambda(1-e^{x^{\beta}})}, \qquad x > 0, \quad \lambda, \beta > 0 \tag{2}$$

$$R(x) = e^{\lambda(1-e^{x^{\beta}})}, \tag{3}$$

$$H(x) = \lambda\beta x^{\beta-1} e^{x^{\beta}}. \tag{4}$$

where $\lambda > 0$ is the parameter but it does not affect the shape of the failure rate function, $H(x)$, as in Equation (4) and $\beta > 0$ is the shape parameter. In recent years, the two parameter bathtub-shaped lifetime distribution has been studied by many authors, such as Lee et al. [5] and Wu et al [6]. Also Rastogi et al. [13] consider the problem of estimating unknown parameters, reliability function and hazard function of a two parameter bathtub-shaped distribution on the basis of progressive type-II censored sample by using different symmetric and asymmetric such as LINEX, entropy and squared error loss functions. Sarhan et al. [14] discussed Maximum likelihood and Bayes estimates of the two unknown parameters of two parameter bathtub-shaped lifetime distribution or Selim [15] discussed Bayesian and non-Bayesian estimations problems of the unknown parameters for the two-parameter bathtub-shaped lifetime distribution based on record values.
In this paper, we assume that shape parameter β is known and used different method of Bayesian estimation to estimate parameter λ and reliability function of two parameter bathtub-shaped lifetime distribution under progressively first-failure-censored samples based on different symmetric and asymmetric loss functions. In Section 2, a brief description of progressive first-failure-censored sampling is given. In section 3, based on progressive first-failure-censored samples and gamma prior density we derived likelihood function and posterior densities of parameter λ and reliability function of bathtub-shaped lifetime distribution. In section 4 we derived Bayesian estimators of parameter λ and reliability function of bathtub-shaped lifetime distribution under minimum expected and LINEX loss functions. In section 5 We used empirical Bayesian estimation to estimate the parameter λ and reliability function based on the method of maximum likelihood estimate. E-Bayesian estimation methods for estimation of the parameter and reliability function of bathtub-shaped lifetime distribution are provided in Section 6. Simulation study is provided in Section 7, and finally we conclude the paper in Section 8.

2. A progressive first-failure-censoring scheme

In a life-testing experiment, Suppose that n independent groups with k items within each group are put in a life test. R_1 groups and the group in which the first failure is observed are randomly removed from the test as soon as the first failure ($X^{\mathbf{R}}_{1:m:n:k} = X_1$) has occurred, R_2 groups and the group in which the second failure is observed are randomly removed from the test as soon as the second failure ($X^{\mathbf{R}}_{2:m:n:k} = X_2$) has occurred, and finally $R_m(m \leq n)$ groups and the group in which the m-th failure is observed are randomly removed from the test as soon as the m-th failure ($X^{\mathbf{R}}_{m:m:n:k} = X_m$) has occurred. Then $X_1 < X_2 < \cdots < X_m$ are called progressively first-failure-censored order statistics with the progressive censoring scheme $\mathbf{R}$. It is clear that $n = m + R_1 + R_2 + \cdots R_m$. If the failure times of the $n \times k$ items originally in the test are from a continuous population with distribution function $F(x)$ and probability density function $f(x)$, the joint probability density function for $X_1, X_2, \cdots, X_m$ is given by

$$f(x_1, x_2, \cdots, x_m) = ck^m \prod_{i=1}^{m} f(x_i)\,(1 - F(x_i))^{k(R_i+1)-1} \qquad 0 < x_1 < x_2 < \cdots < x_m < \infty \tag{5}$$

where $c = n(n - R_1 - 1)(n - R_1 - R_2 - 2)\cdots(n - R_1 - R_2 - \cdots - R_{m-1} - m + 1)$. Note that if $k = 1$, the Eq. (5) reduces to the joint probability density function of progressively type II censored order statistics. If $\mathbf{R} = (0, \cdots, 0)$, Eq. (5) reduces to the joint probability density function of first-failure censored order statistics. If $k = 1$ and $R = (0, \cdots, 0)$, then $n = m$ which corresponds to the complete sample, and if $k = 1$ and $\mathbf{R} = (0, \cdots, 0, n - m)$, then type II censored order statistics are obtained (see Wu and kus [1] or Wu and Huang [2]).

3. Likelihood, prior and posterior

Suppose $X_1 < X_2 < \cdots < X_m$ are progressively first-failure-censored sample with progressive censoring scheme $\mathbf{R}$ from the two parameter bathtub-shaped lifetime distribution with CDF and PDF (1), (2). Substituting (1) and (2) into (5), the likelihood function becomes to be proportional to

$$L(\lambda, \mathbf{X}) \propto \lambda^m e^{\lambda k \sum_{i=1}^{m}(R_i+1)(1-e^{x_i^{\beta}})} \tag{6}$$

It is assumed that the parameter $\lambda > 0$ has a conjugate gamma prior distribution with the shape and scale parameters $a > 0$ and $b > 0$, and it has the PDF

$$\pi(\lambda) = \frac{b^a}{\Gamma(a)} \lambda^{a-1} e^{-b\lambda}. \tag{7}$$

Combining (6) and (7), from Bayesian theorem the posterior density function of λ can be written as

$$\pi(\lambda|x) = \frac{(b - Q(x_i))^{m+a}}{\Gamma(m+a)} \lambda^{m+a-1} e^{-\lambda(b-Q(x_i))} \tag{8}$$

where,

$$Q(x_i) = k \sum_{i=1}^{m} (R_i + 1)(1 - e^{x_i^\beta})$$

Substituting $\lambda = \frac{-\ln s}{g(t)}$ into (8), we obtain the posterior probability density function of $S = R(t)$ with fixed $x = t > 0$ as

$$\pi(S|x) = \frac{1}{\Gamma(m+a)} \left(\frac{b - Q(x_i)}{g(t)}\right)^{m+a} (-\ln s)^{m+a-1} s^{\frac{b-Q(x_i)}{g(t)} - 1} \tag{9}$$

where $g(t) = e^{t^\beta} - 1$.

4. Bayesian Estimation

4.1. Bayes estimator under minimum expected loss function

In Bayesian estimation, widely used loss function is a quadratic loss function given by

$$L(\lambda, \hat{\lambda}) = w\left(\hat{\lambda} - \lambda\right)^2$$

If $w = 1$, it reduces to squared error loss function and for $w = \lambda^{-2}$,it becomes

$$L(\lambda, \hat{\lambda}) = \lambda^{-2}\left(\hat{\lambda} - \lambda\right)^2$$

known as minimum expected loss function introduced by Rao Tummala and Sathe [4]. based on Minimum Expected Loss Function The Bayesian estimator of λ is given by

$$\hat{\lambda}_{ME} = \frac{E(\lambda^{-1}|\mathbf{x})}{E(\lambda^{-2}|\mathbf{x})}$$

Therefore we obtain Bayes estimators of parameter λ of bathtub-shaped lifetime distribution as the following form

$$\hat{\lambda}_{ME} = \frac{m + a - 2}{b - Q(x_i)}. \tag{10}$$

Other problems of interest are those of estimating the reliability function $R(t)$. under squared-error loss function, the Bayesian estimators of $R(t)$ are found to be

$$\hat{R}_{ME} = \left[1 - \frac{g(t)}{b - Q(x_i) - g(t)}\right]^{m+a} \tag{11}$$

4.2. Bayes estimator under LINEX loss function

The LINEX loss function for λ can be expressed as the following proportional

$$L(\Delta) \propto \exp(k\Delta) - k\Delta - 1; \qquad k \neq 0$$

where $\Delta = (\hat{\lambda} - \lambda)$ and $\hat{\lambda}$ is an estimate of λ. The Bayes estimator of λ, denoted by $\hat{\lambda}_L$ under the LINEX loss function is given by

$$\hat{\lambda_L} = -\frac{1}{k} \ln E[\exp(-k\lambda)]$$

For more details about LINEX loss function see Zellner [12]. Under LINEX loss function, we obtain Bayesian estimator of the parameter λ, and reliability function $R(t)$, as the following forms

$$\hat{\lambda}_L = -\frac{m+a}{\delta} \ln\left[\frac{b-Q(x_i)}{b+\delta-Q(x_i)}\right] \tag{12}$$

$$\hat{R}_{Li} = -\frac{1}{\delta} \ln\left[\sum_{j=0}^{\infty} \frac{(-\delta)^j}{j!}\left(\frac{b-Q(x_i)}{b-Q(x_i)+jg(t)}\right)^{m+a}\right] \tag{13}$$

5. Empirical Bayesian Estimation

We assume that parameter a in the conjugate gamma prior distribution (7) is known and parameter b is unknown In the parametric empirical Bayes method to estimate of the hyperparameter usually used the method of maximum likelihood or a method of moments estimate (Carlin and Louis [7], p.62). Here we estimate the unknown hyperparameter λ based on the method of maximum likelihood estimate. from (6) and (7) the marginl density function of X is

$$f(\mathbf{x}) = \int L(\mathbf{x},\lambda)\pi(\lambda)d\lambda = \int \lambda^m (u(x_n))^\beta \frac{b^a}{\Gamma(a)}\lambda^{a-1}e^{-b\lambda}d\lambda = \frac{\Gamma(n+a)}{\Gamma(a)}\frac{b^a}{(b-\ln u(x_n))^{n+a}}$$

Based on $f(\mathbf{x})$ the MLE of b is

$$\hat{b} = -\frac{aQ(x_i)}{m} \tag{14}$$

5.1. Empirical Bayes Estimation under minimum expected loss function

Substituting $\hat{b}$ into (10), the empirical Bayes estimation of λ under minimum expected loss function is obtained

$$\hat{\lambda}_{EME} = \frac{-m(m+a-2)}{Q(x_i)(a+m)} \tag{15}$$

Similarly, the empirical Bayes estimation of $R(t)$ is given as follows:

$$\hat{R}_{EME} = \left[1+\frac{mg(t)}{(a+m)Q(x_i)+g(t)}\right]^{m+a} \tag{16}$$

5.2. Empirical Estimation under LINEX loss function

Substituting $\hat{b}$ into (12) and (13), the empirical Bayes estimation of λ and reliability function $R(t)$ under LINEX loss function are obtained

$$\hat{\lambda}_{ELI} = -\frac{m+a}{\delta} \ln \frac{(m+a)Q(x_i)}{Q(x_i)(a+m)-m\delta}, \tag{17}$$

$$\hat{R}_{ELI} = -\frac{1}{\delta} \ln\left[\sum_{j=0}^{\infty} \frac{(-\delta)^j}{j!}\left(\frac{(a+m)Q(x_i)}{(a+m)Q(x_i)-mjg(t)}\right)^{m+a}\right]. \tag{18}$$

6. E-Bayesian Estimation

According to Han [8], in the conjugate gamma prior distribution (7), parameters a and b should be selected to guarantee that $\pi(\lambda)$ is a decreasing function of λ. The derivative of $\pi(\lambda)$ with respect to λ is

$$\frac{d\pi(\lambda)}{d\lambda} = \frac{b^a}{\Gamma(a)}\lambda^{a-2}e^{-b\lambda}\left((a-1)-b\lambda\right)$$

since $a > 0, b > 0$, and $\lambda > 0$, it follows $0 < a < 1, b > 0$ due to $\frac{d\pi(\lambda)}{d\lambda} < 0$ and therefore $\pi(\lambda)$ is a decreasing function of λ.

Assuming that a and b are independent with bivariate density function

$$\pi(a,b) = \pi(a)\pi(b),$$

then, the Expectation of the Bayesian estimate of λ (E-Bayesian estimate of λ) can be written as

$$\hat{\lambda}_{EB} = \int\int \hat{\lambda}_{Bayes}\pi(a,b)dadb, \tag{19}$$

where $\hat{\lambda}_{Bayes}$ is the Bayes estimate of λ given by (10) and (12). For more details, see Han [9] or jahen and okasha [10].

Here we condider following prior distribution a and b for obtain the E-Bayesian estimate of parameter λ and reliability function $R(t)$(jahen and okasha)

$$\pi(a,b) = \frac{1}{cB(u,v)}a^{u-1}(1-a)^{v-1}, \quad 0 < a < 1, \quad 0 < b < c. \tag{20}$$

6.1. E-Bayesian Estimation under minimum expected loss function

Based on the minimum expected loss function and independent bivariate prior distribution (20), the E-Bayesian estimation λ takes the form

$$\hat{\lambda}_{E-BME} = \frac{m + \frac{u}{u+v} - 2}{c}\ln\frac{Q(x_i) - c}{Q(x_i)}. \tag{21}$$

Similarly, Based on the minimum expected loss function and independent bivariate prior distribution (20), the E-Bayesian estimation of reliability function $R(t)$ is obtained from (11) as

$$\hat{R}_{E-BME} = \frac{1}{cB(u,v)}\int_0^1\int_0^c a^{u-1}(1-a)^{v-1}\left(1 - \frac{g(t)}{b - Q(x_i) - g(t)}\right)^{m+a} dbda \tag{22}$$

Obtaining a closed form expression for and $\hat{R}_{E-BME}$ is not possible.

6.2. E-Bayesian Estimation under LINEX loss function

Based on the LINEX loss function, the E-Bayesian estimation λ is computed as follows:

$$\hat{\lambda}_{E-BLI} = \frac{1}{c\delta}\left(m + \frac{u}{u+v}\right)\left((Q(x_i) - c)\ln\frac{c - Q(x_i)}{c - Q(x_i) + \delta} + Q(x_i)\ln\frac{\delta - Q(x_i)}{-Q(x_i)} + \delta\ln\frac{c + \delta - Q(x_i)}{\delta - Q(x_i)}\right) \tag{23}$$

. Under LINEX loss function and prior distribution (20), the E-Bayesian estimation of reliability function $R(t)$ is obtained from (13) as

$$\hat{R}_{E-BLI} = -\frac{1}{\delta cB(u,v)}\int_0^1\int_0^c a^{u-1}(1-a)^{v-1}\ln\left(\sum_{j=0}^{\infty}\frac{(-\delta)^j}{j!}\left(\frac{b - Q(x_i)}{b - Q(x_i) + jg(t)}\right)^{m+a}\right) dbda \tag{24}$$

Analytical and numerical computations for the integrals in (24) are very complicated.

Table 1: Prior parameters

	a	b	δ	c	u	v
for estimates of λ	4	1.5	1.5	0.1	0.4	0.9
for estimates of $R(t)$	4	1	0.5	0.1	0.6	0.9

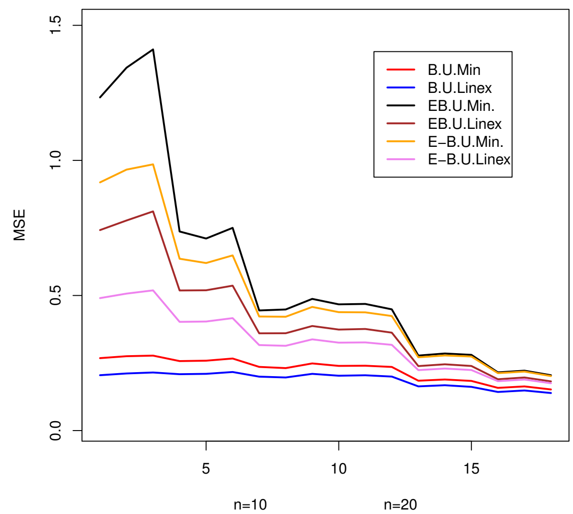

Figure 1: MSE for different estimators of parameter λ

7. Simulation study

Applying the algorithms of Balakrishnan and Sandhu [3], we have generated 2000 different progressive first-failure-censored samples from bathtub-shaped lifetime distribution with parameters ($\lambda = \beta =$) and different size n, m, k and different censoring secheme $\mathbf{R} = (R_1, \cdots, R_m)$. The true value of $R(t)$ in $t = 0.2$ is obtained $R(0.2) = 0.9235971$. We used the above sample and prior parameters which displayed in Table 1, and compute the Bayesian estimators, empirical Bayes estimators and E-Bayesian estimators of parameter λ and reliability function $R(t)$ respectively, using (10), (12), (15), (17), (21), (23), (11), (13), (16), (18), and obtain the means and the MSEs (Mean Squared Error) for generated 2000 different progressive first-failure-censored samples. The results are summarized in Tables 2, 3, and figures 1, 2, 3 and 4.

8. Conclusions

Based on the results shown in Table 2, one can conclude Bayesian estimation methods give relatively more accurate estimators as compared with the Empirical Bayes estimation methods or E-Bayesian estimation methods. While the figures 1 and 2, shows that the Bayes estimates of parameter λ under the LINEX loss function have the smallest MSE's as compared with the estimates under minimum expected loss function, empirical Bayes estimators and E-Bayesian estimators under minimum expected and LINEX loss functions. Table 3, shows that the empirical Bayes estimators of reliability function $R(t)$ under the LINEX loss function more accurate estimators as compared with the other estimators, but according to figures 3, 4 one can conclude the Bayes estimates of reliability function $R(t)$ under the LINEX loss function have the smallest estimated MSE's as compared with the Bayes estimates under minimum expected loss function and Empirical Bayes estimators under minimum expected and LINEX loss functions. Also as sample size n increases, Bayes estimators of reliability function $R(t)$ under the LINEX loss function and Empirical Bayes estimators under minimum expected and LINEX loss functions are Approximately equal. In all above cases it is immediate to note that MSE's decrease as sample size n and m increases, and generally when k increases the MSE's of all estimators increases.

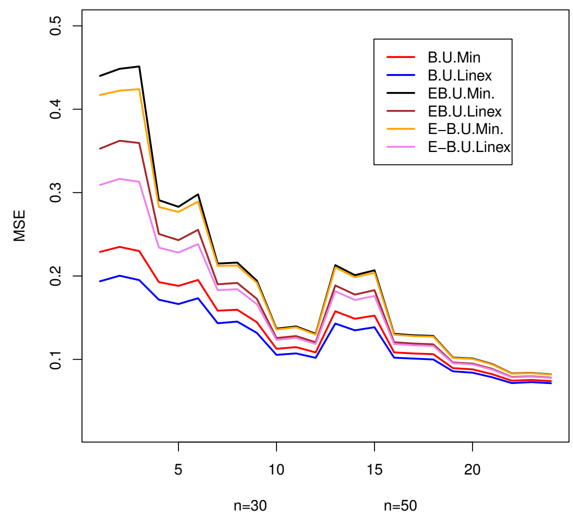

Figure 2: MSE for different estimators of parameter λ

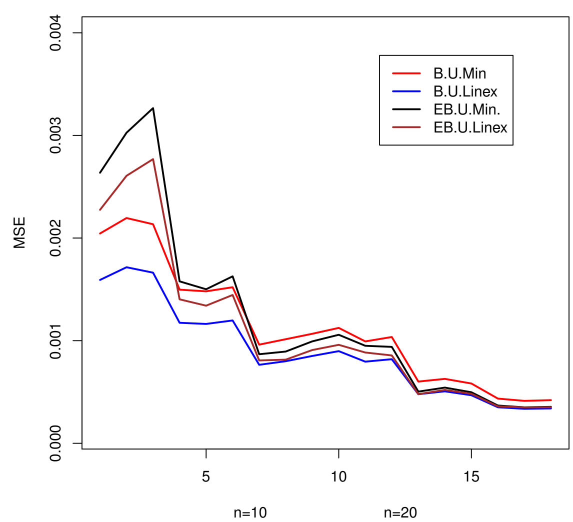

Figure 3: MSE for different estimators of reliability function $R(t)$

Table 2: Averaged values for estimates of the parameter λ.

n	m	k	$\mathbf{R}=(R_1,...,R_m)$	$\hat{\lambda}_{ME}$	$\hat{\lambda}_{L}$	$\hat{\lambda}_{EME}$	$\hat{\lambda}_{ELI}$	$\hat{\lambda}_{E-BME}$	$\hat{\lambda}_{E-BLI}$
10	5	1	(5,0,0,0,0)	1.871828	2.006320	1.919919	1.999702	1.582412	1.841578
		3		1.875971	2.009581	1.935179	2.009738	1.593847	1.848868
		6		1.890754	2.022845	1.966642	2.035284	1.618714	1.869175
	7	1	(2,1,0*5)	1.925583	2.026798	1.905310	1.984977	1.731613	1.897927
		3		1.929062	2.029930	1.910352	1.989916	1.736282	1.902275
		6		1.925478	2.026146	1.908643	1.986906	1.734480	1.899145
	10	1	(0,...,0)	1.943456	2.018982	1.885488	1.955556	1.805343	1.915682
		3		1.959967	2.034663	1.906348	1.975081	1.825084	1.934046
		6		1.945886	2.020692	1.892360	1.960400	1.811628	1.919760
20	10	1	(5,2,2,1,0*6)	1.956458	2.031007	1.903850	1.971958	1.822617	1.930862
		3		1.965889	2.039793	1.916135	1.983390	1.834244	1.941558
		6		1.950859	2.025925	1.895195	1.964520	1.814515	1.924065
	15	1	(5,0,...,0)	1.971041	2.024554	1.907372	1.960794	1.876985	1.947531
		3		1.976647	2.029730	1.914570	1.967389	1.883991	1.953853
		6		1.963621	2.017541	1.898724	1.952513	1.868526	1.939502
	20	1	(0,...,0)	1.969550	2.011589	1.910076	1.952853	1.896985	1.949154
		3		1.981481	2.022897	1.923939	1.965989	1.910663	1.962009
		6		1.986857	2.028373	1.928878	1.971202	1.915586	1.967239
30	10	1	(5*4,0,...,0)	1.946533	2.022190	1.888272	1.958511	1.808007	1.918613
		3		1.957004	2.031685	1.902962	1.971829	1.821874	1.930935
		6		1.957070	2.032015	1.902530	1.971488	1.821453	1.930697
	15	1	(5*3,0,...,0)	1.983027	2.035669	1.922531	1.974772	1.891748	1.960956
		3		1.964471	2.018178	1.900135	1.953721	1.869895	1.940623
		6		1.983668	2.036203	1.923753	1.975724	1.892917	1.961828
	20	1	(5,5,0,...,0)	1.971456	2.013408	1.912223	1.954919	1.899107	1.951182
		3		1.987876	2.029150	1.930743	1.972687	1.917399	1.968626
		6		1.978818	2.020864	1.919154	1.962148	1.906008	1.958415
	30	1	(0,...,0)	1.981112	2.010524	1.933288	1.963559	1.931408	1.965721
		3		1.980798	2.010179	1.933070	1.963294	1.931186	1.965447
		6		1.992298	2.021482	1.945069	1.975132	1.943146	1.977227
50	20	1	(5*6,0*14)	1.970682	2.012678	1.911257	1.954037	1.898157	1.950325
		3		1.968875	2.011197	1.908400	1.951635	1.895367	1.948049
		6		1.980048	2.021829	1.921316	1.963890	1.908111	1.960049
	30	1	(5*4,0*26)	1.997010	2.026062	1.950142	1.980064	1.948197	1.982123
		3		1.997607	2.026667	1.950708	1.980649	1.948764	1.982709
		6		1.989471	2.018777	1.941903	1.972107	1.939999	1.974236
	40	1	(5*2,0*38)	2.001631	2.023932	1.963415	1.986356	1.964709	1.989845
		3		1.994897	2.017371	1.956256	1.979373	1.957565	1.982893
		6		1.993730	2.016323	1.954784	1.978041	1.956105	1.981584
	50	1	(0,...,0)	1.992724	2.011083	1.960029	1.978864	1.962448	1.982681
		3		2.003110	2.021269	1.970894	1.989524	1.973303	1.993317
		6		2.002490	2.020679	1.970198	1.988864	1.972610	1.992661

Table 3: Averaged values for estimates of reliability function $R(t)$

n	m	k	$\mathbf{R}=(R_1,...,R_m)$	$\hat{R}_{ME}$	$\hat{R}_L$	$\hat{R}_{EME}$	$\hat{R}_{ELI}$
10	5	1	(5,0,0,0,0)	0.8909059	0.8982645	0.9051733	0.9125776
		3		0.8897451	0.8971451	0.9030546	0.9105621
		6		0.8901883	0.8975713	0.9035326	0.9110755
	7	1	(2,1,0*5)	0.8964211	0.9032041	0.9092949	0.9160919
		3		0.8966601	0.9034401	0.9096409	0.9164238
		6		0.8963549	0.9031429	0.9091343	0.9159407
	10	1	(0,...,0)	0.9033478	0.9096739	0.9141569	0.9205263
		3		0.9019640	0.9082915	0.9126717	0.9190375
		6		0.9019728	0.9083093	0.9125187	0.9189002
20	10	1	(5,5,0*8)	0.9012217	0.9075638	0.9116302	0.9180175
		3		0.9034319	0.9097635	0.9141250	0.9205076
		6		0.9018968	0.9082276	0.9125352	0.9189076
	15	1	(5,0,...,0)	0.9079865	0.9139739	0.9160856	0.9221470
		3		0.9081530	0.9141458	0.9162027	0.9222708
		6		0.9087300	0.9147227	0.9168541	0.9229230
	20	1	(0,...,0)	0.9109119	0.9167426	0.9173221	0.9232341
		3		0.9115435	0.9173803	0.9179813	0.9239004
		6		0.9112657	0.9170995	0.9176946	0.9236103
30	10	1	(5*4,0,...,0)	0.9028701	0.9091974	0.9136361	0.9200047
		3		0.9015599	0.9078982	0.9120589	0.9184405
		6		0.9024728	0.9088025	0.9131752	0.9195460
	15	1	(5*3,0,...,0)	0.9084077	0.9143994	0.9164982	0.9225663
		3		0.9079183	0.9139054	0.9160118	0.9220730
		6		0.9076882	0.9136773	0.9157232	0.9217868
	20	1	(5,5,0,...,0)	0.9113908	0.9172250	0.9178342	0.9237504
		3		0.9115603	0.9173989	0.9179826	0.9239033
		6		0.9115230	0.9173601	0.9179560	0.9238752
	30	1	(0,...,0)	0.9146747	0.9203677	0.9192032	0.9249719
		3		0.9145144	0.9202046	0.9190430	0.9248087
		6		0.9152099	0.9209103	0.9197556	0.9255324
50	20	1	(5*6,0*14)	0.9113908	0.9172250	0.9178342	0.9237504
		3		0.9115230	0.9173601	0.9179560	0.9238752
		6		0.9113972	0.9172298	0.9178589	0.9237735
	30	1	(5*4,0*26)	0.9145422	0.9202347	0.9190545	0.9248226
		3		0.9145598	0.9202504	0.9190927	0.9248588
		6		0.9138723	0.9195531	0.9183854	0.9241410
	40	1	(5*2,0*38)	0.9161004	0.9217220	0.9195912	0.9252785
		3		0.9159366	0.9215568	0.9194149	0.9251004
		6		0.9162100	0.9218342	0.9196972	0.9253871
	50	1	(0,...,0)	0.9171047	0.9226884	0.9199385	0.9255793
		3		0.9172774	0.9228650	0.9201088	0.9257536
		6		0.9170120	0.9225943	0.9198421	0.9254814

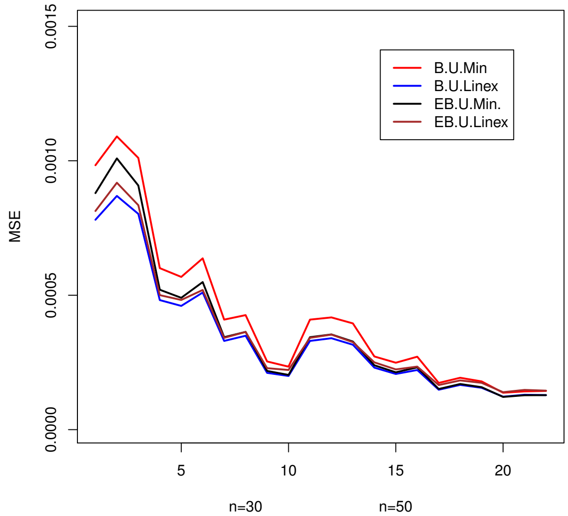

Figure 4: MSE for different estimators of reliability function $R(t)$

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