International Journal of Physical Research, *6 (1) (2018) 25-30*

International Journal of Physical Research

Website: www.sciencepubco.com/index.php/IJPR

doi: 10.14419/ijpr.v6i1.9261

Research paper

New exact solutions of the combined and double combined sinh–cosh–Gordon equations via modified Kudryashov method

Atish Kumar Joardar[1,2*], Dipankar Kumar[3] and K. M. Abdul Al Woadud[4]

[1] *Department of Mathematics, Islamic University, Kushtia–7003, Bangladesh.*
[2] *Institute of Computational Mathematics and Scientific / Engineering Computing, Academy of Mathematics and System Sciences, University of Chinese Academy of Sciences, Beijing, China.*
[3] *Department of Mathematics, Bangabandhu Sheikh Mujibur Rahman Science and Technology University, Gopalganj–8100, Bangladesh.*
[4] *Department of Electrical & Electronic Engineering, Uttara University, Dhaka, Bangladesh.*
[*] *Corresponding author Email: atishjoardar@yahoo.com*

Abstract

The combined and double combined sinh–cosh–Gordon equations are very important to a wide range of various scientific applications that ranges from chemical reactions to water surface gravity waves. In this article, with the assistance of a function transform and Painlevé property, the nonlinear combined and double combined sinh–cosh–Gordon equations turn into ordinary differential equations. Later on, modified Kudryashov method is adopted for investigating new analytical solution of the studied equations. As a consequence, a series of new analytical solutions are accquired and we demonstrated the actual behavior of the achieved solutions of the mentioned equations with the aid of 3D and 2D MATLAB graphs. Finally, we also validate the effectiveness of the modified Kudryashov method for the problem of extracting new exact solutions of the combined and double combined sinh–cosh–Gordon equations with the aid of Maple package program. It is shown that the implemented method is capable to extract new solutions and it can also used to other nonlinear partial differential equation (NLPDE's) arising in mathematical physics or other applied field.

Keywords: *Painlevé property; Combined sinh–cosh–Gordon equation; Double combined sinh–cosh–Gordon equation; Modified Kudryashov method; New exact solutions.*

1. Introduction

Over the last few decades, exact solution of nonlinear partial differential equations (NPDEs) is one of the most significant research topic in the field of applied mathematics, mathematical physics and engineering. As we know, most of the real world problems are nonlinear and its real phenomena may depicts in NPDEs. Generally, its exact solutions are signify not only applied Mathematics but also in different research field like fluid dynamics, plasma physics, quantum field theory, nonlinear optics, signal processing and etc. Though, the procedure for finding exact solution of those equations are generally very complex but the solution of these equation are exists, it gives exact information for describing real phenomena. The concept of travelling wave solution of physical applications such as fluid dynamics, plasma physics, quantum field theory and so on are very popular and significant. Nowadays, the concept of reducing nonlinear partial differential equation into ordinary differential has proved a successful way to generate an analytical solution of nonlinear partial differential equation. According to this concept, many powerful method grown up and used for acquiring exact solutions of NPDEs such as, Homogeneous balance method [1], $G^{'}/G-$expansion method [2], Improved $G^{'}/G-$expansion method [3, 4], Improved $1/G^{'}-$expansion and $(G^{'}/G, 1/G)$ methods [4], Extended tanh function method [5], Extended F–expansion method [6], Jacobi elliptic function method [7], Transformed rational function method [8], Weierstrass elliptic function expansion method [9], Hirota's bilinear method [10], Generalized Kudryashov method [11], Trial equation method [12], Solitary ansatz method [13], Auxiliary equation method [14], Modified Kudryashov method [15, 16, 17], Sine–Gordon equation expansion method [17, 18], Hyperbolic function method [19] and so on. Recently, researchers easily design and applied these method for reducing the computational difficulty using computational software package like Maple, Mathematica, MATLAB etc.

In recent past, many researchers were tried to solve combined and double combined sinh–cosh–Gordon equations with the help of distinct analytical methods. In this portion, we will review some related works about the combined and double combined sinh–cosh–Gordon equations and several analytical approaches. In this regard, Salas and Castillo [20] solved the combined sinh–cosh–Gordon equation through the projective Riccati equation based methods such as Conte's projective Riccati equation method, tanh–coth method, He's exp–function method and sn–ns method. At the end of the paper, authors conclude that sn–ns method more efficient than other applied methods. Gomez and Salas [21] also presented the general projective Riccati equations method to derive exact solutions for the combined sinh–cosh–Gordon equation by means of Painlevé property. Khalique and Magalakwe [22] derived another type of exact solutions of the combined sinh–cosh–Gordon equation by means of the Lie symmetry analysis along with the simplest equation method.

Long Wei [23] find out some new type exact solutions for the combined sinh–cosh–Gordon equation based on the a transformed Painlevé property and the variable separated ODE method. The combined sinh–cosh–Gordon equation is also solved by Jaramillo–Camacho and his co-researchers [24] through the Hamiltonnian systems. Magalakwe et al. [25] studied a generalized double combined sinh–cosh–Gordon equation by employing Lie group method along with the simplest equation method to search new travelling wave solutions. Author also constructed the conservation laws of the above mentioned techniques. Kheiri and Jabbar [26] utilized G'/G-expansion method to find out traveling wave solutions of the combined and double combined sinh–cosh–Gordon equations. Irshad and Mohyud–Din [27] studied some nonlinear differential equations including combined and double combined sinh–cosh–Gordon equations for deriving travelling wave solutions using tanh–coth method. Exact solutions of a generalized double sine–Gordon equation is also obtained by using different analytical techniques such as the tanh method and the variable separated ODE method through Painlevé property [28], Jacobi Amplitude function [29] and exp–function method [30].

The main purpose of this study is to apply the modified Kudryashov method for solving combined and double combined sinh–cosh–Gordon equations and produce new exact solutions of the above mentioned equations. To this end, the work is organized as follows: In Section 2, we present the algorithm of the modified Kudryashov method. In Section 3, we derived the solutions of combined sinh–cosh–Gordon equation. Next, we extract the exact solution of the double combined sinh–cosh–Gordon equation in Section 4. Finally, Section 5 is devoted to concluding remarks about acquired results and executed method.

2. Basic steps of modified Kudryashov method

The general nonlinear partial differential equation of two independent variables x and t can be written as

$$F\left(u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial t^2}, \frac{\partial^2 u}{\partial x^2}, \ldots\right) = 0, \tag{1}$$

where F is a polynomial of $u(x,t)$ with various partial derivatives. Now we consider the following transformation

$$u(x,t) = f(\xi), \quad \xi = x - ct,$$

where c is a non zero constant. After transformation the Eq. (1) can be written as a non linear ordinary differential equation of the following form

$$G(f, f', f'', \ldots) = 0 \tag{2}$$

where the total derivative $'$ with respect to the new variable ξ. Now consider the solution of the Eq. (2) as

$$f(\xi) = \sum_{i=0}^{m} a_i Q^i(\xi) \tag{3}$$

where $a_i, i = 0,1,2,\ldots,m$ are constants but $a_m \neq 0$ which will be calculated and $Q = Q(\xi)$ is the solution of equation

$$\frac{dQ}{d\xi} = Q(\xi)(Q(\xi)-1)ln(a).$$

If we consider

$$Q(\xi) = \frac{1}{1+da^{\xi}},$$

it will easily verify above equation. where d is an arbitrary constant and $a \neq 0, 1$. We need to consider the balance of higher order derivative and nonlinear terms of Eq. (2) for finding the value of m. Now using Eq. (3) in Eq. (2), we yield a new equation

$$T(Q(\xi)) = 0. \tag{4}$$

by equating like power of $Q(\xi)$ in Eq. (4), we get a system of equation. Now solving the system of equation we get the values of a_i and c. Inserting this values in Eq. (3) which is the solution of Eq. (1).

3. Solution of the combined sinh–cosh–Gordon equation

Consider the combined sinh–cosh–Gordon equation is as follows [20, 21, 22, 23, 24]:

$$u_{tt} - ku_{xx} + \alpha sinh(u) + \beta cosh(u) = 0, \tag{5}$$

where u is the real scalar function of two independent variables x and t. The subscript represent the partial derivatives, α and β are two arbitrary non zero constants which also represent the model parameters. By implementing Painlevé property $u = ln(V)$ or $V = e^u$, then the *sinh* and *cosh* term transformed into the following way:

$$sinh(u) = \frac{V - V^{-1}}{2}, \quad cosh(u) = \frac{V + V^{-1}}{2}, \tag{6}$$

Now using Eq. (5) and Eq. (6) along with $u = ln(V)$, we obtain

$$2V(V''_{tt} - kV''_{xx}) + 2(k(V'_x)^2 - (V'_t)^2) + (\alpha+\beta)V^3 + (\beta-\alpha)V = 0 \tag{7}$$

Now let $V(x,t) = V(\xi)$, where $\xi = x - ct$, putting this in Eq. (7) we get

$$2V(c^2-k)V'' + 2(k-c^2)(V')^2 + (\alpha+\beta)V^3 + (\beta-\alpha)V = 0 \tag{8}$$

By blanching the derivative VV'' and $(V')^2$, we get $m = 2$. So, the finite series is of the form

$$V(\xi) = a_0 + a_1 Q(\xi) + a_2 Q(\xi)^2 \tag{9}$$

putting Eq. (9) with first and second derivative in Eq. (8) and equating the like powers of $Q(\xi)$, we the following system of equation:

$4ln(a)^2c^2a_2^2 - 4ln(a)^2ka_2^2 + \alpha a_2^3 + \beta a_2^3 = 0,$

$8ln(a)^2c^2a_1a_2 - 4ln(a)^2c^2a_2^2 - 8ln(a)^2ka_1a_2 + 4ln(a)^2ka_2^2 + 3\alpha a_1a_2^2 + 3\beta a_1a_2^2 = 0,$

$12ln(a)^2c^2a_0a_2 + 2ln(a)^2c^2a_1^2 - 10in(a)^2c^2a_1a_2 - 12ln(a)^2ka_0a_2 - 2ln(a)^2ka_1^2 + 10ln(a)^2ka_1a_2 + 3\alpha a_0a_2^2 + 3\alpha a_1^2a_2 + 3\beta a_0a_2^2 + 3\beta a_1^2a_2 = 0,$

$4ln(a)^2c^2a_0a_1 - 20ln(a)^2c^2a_0a_2 - 2ln(a)^2c^2a_1^2 + 2ln(a)^2c^2a_1a_2 - 4ln(a)^2ka_0a_1 + 20in(a)^2ka_0a_2 + 2ln(a)^2ka_1^2 - 2ln(a)^2ka_1a_2 + 6\alpha a_0a_1a_2 + \alpha a_1^3 + 6\beta a_0a_1a_2 + \beta a_1^3 = 0,$

$-6ln(a)^2c^2a_0a_1 + 8ln(a)^2c^2a_0a_2 + 6ln(a)^2ka_0a_1 - 8ln(a)^2ka_0a_2 + 3\alpha a_0^2a_2 + 3\alpha a_0a_1^2 + 3\ beta a_0^2a_2 + 3\beta a_0a_1^2 - \alpha a_2 + \beta a_2 = 0,$

$2ln(a)^2c^2a_0a_1 - 2ln(a)^2ka_0a_1 + 3\alpha a_0^2a_1 + 3\beta a_0^2a_1 - \alpha a_1 + \beta a_1 = 0,$

$\alpha a_0^3 + \beta a_0^3 - \alpha a_0 + \beta a_0 = 0.$

By solving the above systems, we obtain the following different cases:

Case–I:

$$a_0 = \frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta}, \quad a_1 = -\frac{4(\alpha-\beta)}{\sqrt{(\alpha+\beta)(\alpha-\beta)}}$$

$$a_2 = \frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta}, \quad c = \frac{\sqrt{ln(a)^2k - \sqrt{(\alpha+\beta)(\alpha-\beta)}}}{ln(a)}.$$

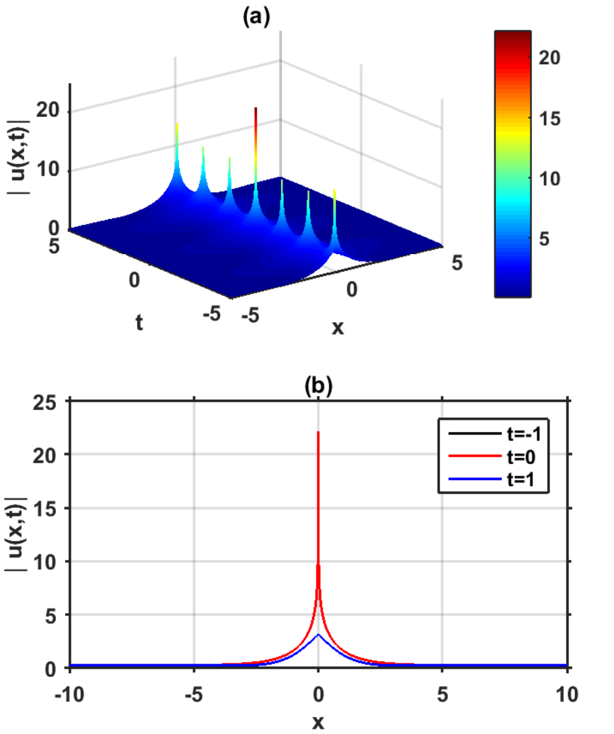

Figure 1: (a) 3D snapshot of the Eq. (10) for the arbitrary parameters $a = 3, \alpha = 2, \beta = 0.5, d = 1$ and $k = -1.5$, and (b) 2D snapshot of (a) at $t = -1, t = 0, t = 1$ respectively.

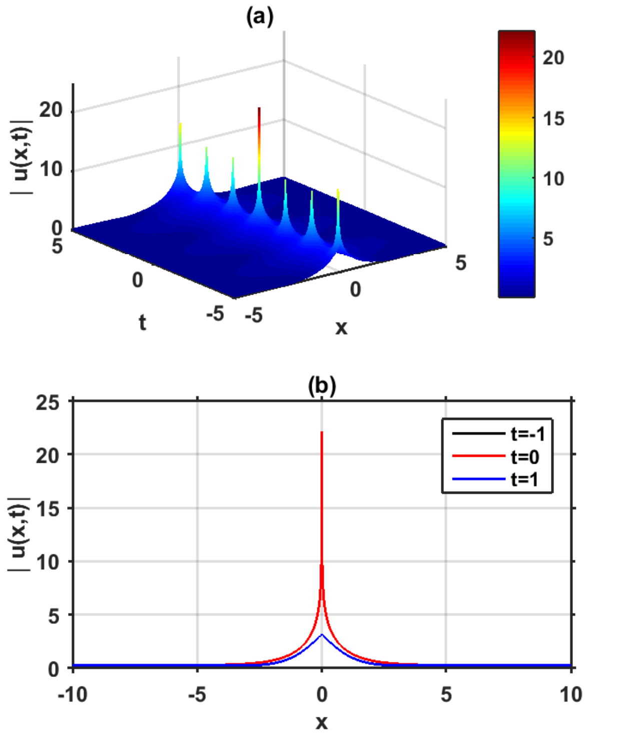

Figure 2: (a) 3D snapshot of the Eq. (11) for the arbitrary parameters $a = 3, \alpha = 2, \beta = 0.5, d = 1$ and $k = -1.5$, and (b) 2D snapshot of (a) at $t = -1, t = 0, t = 1$ respectively.

Case–I corresponds to the following exact solution of the combined sinh–cosh–Gordon equation is obtained:

$$u_1(x,t) = \ln\left[\frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta} - \frac{4(\alpha-\beta)}{\sqrt{(\alpha+\beta)(\alpha-\beta)}} \times \left(\frac{1}{1+da^{x-\frac{\sqrt{ln(a)^2k-\sqrt{(\alpha+\beta)(\alpha-\beta)}}}{ln(a)}t}}\right) + \frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta} \times \left(\frac{1}{1+da^{x-\frac{\sqrt{ln(a)^2k-\sqrt{(\alpha+\beta)(\alpha-\beta)}}}{ln(a)}t}}\right)^2\right]. \tag{10}$$

Case–II:

$$a_0 = \frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta}, \quad a_1 = -\frac{4(\alpha-\beta)}{\sqrt{(\alpha+\beta)(\alpha-\beta)}}$$

$$a_2 = \frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta}, \quad c = -\frac{\sqrt{ln(a)^2k-\sqrt{(\alpha+\beta)(\alpha-\beta)}}}{ln(a)}.$$

Case–II corresponds to the following exact solution of the combined sinh–cosh–Gordon equation is determined:

$$u_2(x,t) = \ln\left[\frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta} - \frac{4(\alpha-\beta)}{\sqrt{(\alpha+\beta)(\alpha-\beta)}} \times \left(\frac{1}{1+da^{x+\frac{\sqrt{ln(a)^2k-\sqrt{(\alpha+\beta)(\alpha-\beta)}}}{ln(a)}t}}\right) + \frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta} \times \left(\frac{1}{1+da^{x+\frac{\sqrt{ln(a)^2k-\sqrt{(\alpha+\beta)(\alpha-\beta)}}}{ln(a)}t}}\right)^2\right]. \tag{11}$$

Case–III:

$$a_0 = -\frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta}, \quad a_1 = \frac{4(\alpha-\beta)}{\sqrt{(\alpha+\beta)(\alpha-\beta)}}$$

$$a_2 = -\frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta}, \quad c = \frac{\sqrt{ln(a)^2k+\sqrt{(\alpha+\beta)(\alpha-\beta)}}}{ln(a)}.$$

Case–III corresponds to the following exact solution of the combined sinh–cosh–Gordon equation is explored:

$$u_3(x,t) = \ln\left[-\frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta} + \frac{4(\alpha-\beta)}{\sqrt{(\alpha+\beta)(\alpha-\beta)}} \times \left(\frac{1}{1+da^{x-\frac{\sqrt{ln(a)^2k-\sqrt{(\alpha+\beta)(\alpha-\beta)}}}{ln(a)}t}}\right) - \frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta} \times \left(\frac{1}{1+da^{x-\frac{\sqrt{ln(a)^2k+\sqrt{(\alpha+\beta)(\alpha-\beta)}}}{ln(a)}t}}\right)^2\right]. \tag{12}$$

Case–IV:

$$a_0 = -\frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta}, \quad a_1 = \frac{4(\alpha-\beta)}{\sqrt{(\alpha+\beta)(\alpha-\beta)}}$$

$$a_2 = -\frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta}, \quad c = -\frac{\sqrt{ln(a)^2k+\sqrt{(\alpha+\beta)(\alpha-\beta)}}}{ln(a)}.$$

Case–IV corresponds to the following exact solution of the combined

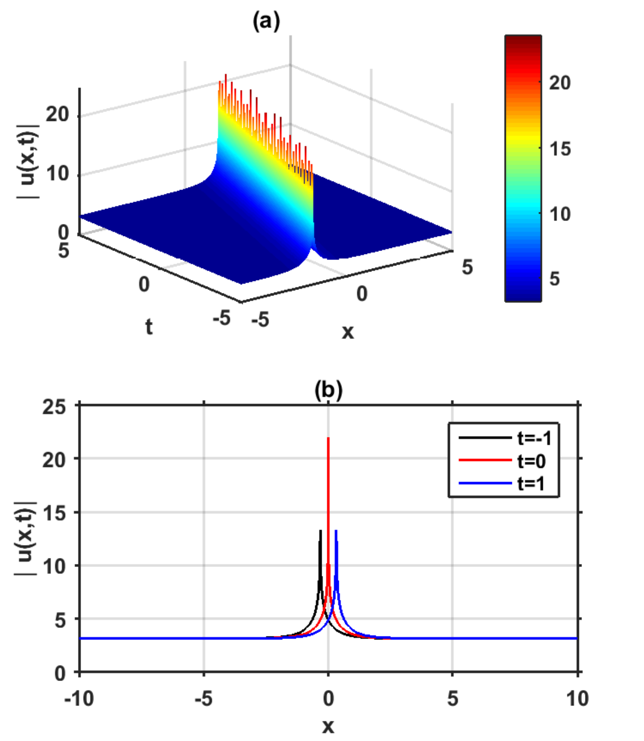

Figure 3: (a) 3D snapshot of the Eq. (12) for the arbitrary parameters $a = 3, \alpha = 2, \beta = 0.5, d = 1$ and $k = -1.5$, and (b) 2D snapshot of (a) at $t = -1, t = 0, t = 1$ respectively.

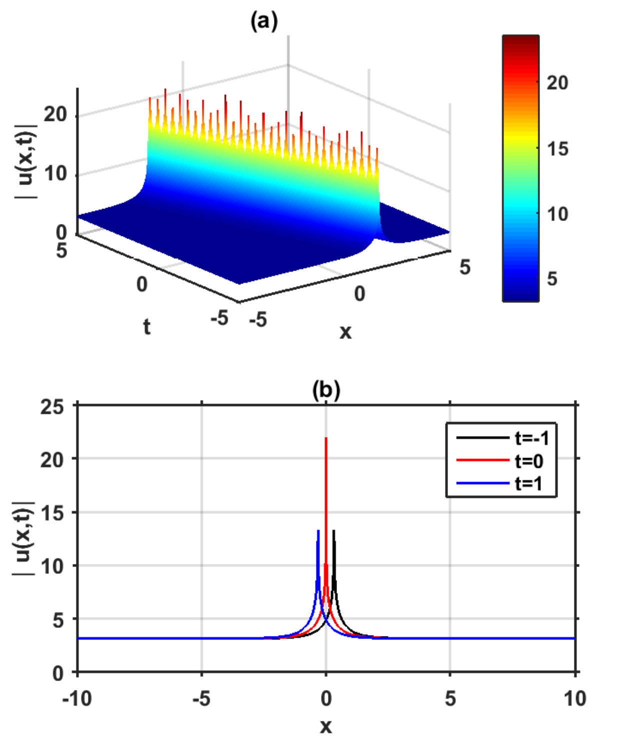

Figure 4: (a) 3D snapshot of the Eq. (13) for the arbitrary parameters $a = 3, \alpha = 2, \beta = 0.5, d = 1$ and $k = -1.5$, and (b) 2D snapshot of (a) at $t = -1, t = 0, t = 1$ respectively.

sinh–cosh–Gordon equation is found:

$$u_4(x,t) = \ln\Bigg[-\frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta} + \frac{4(\alpha-\beta)}{\sqrt{(\alpha+\beta)(\alpha-\beta)}} \times \left(\frac{1}{1+da^{x+\frac{\sqrt{ln(a)^2k-\sqrt{(\alpha+\beta)(\alpha-\beta)}}}{ln(a)}t}}\right) - \frac{\sqrt{(\alpha+\beta)(\alpha-\beta)}}{\alpha+\beta} \times \left(\frac{1}{1+da^{x+\frac{\sqrt{ln(a)^2k+\sqrt{(\alpha+\beta)(\alpha-\beta)}}}{ln(a)}t}}\right)^2\Bigg]. \tag{13}$$

Exact solutions of the combined sinh–cosh–Gordon equation is demonstrated by some MATLAB figures which indicating Figure 1 to Figure 4 by choosing free parameters as $a = 3, \alpha = 2, \beta = 0.5, d = 1$ and $k = -1.5$.

4. Solutions of combined double sinh–cosh–Gordon equation

Consider the double combined sinh–cosh–Gordon equation is as follows [25, 26, 27]:

$$u_{tt} - ku_{xx} + \alpha sinh(u) + \alpha cosh(u) + \beta sinh(2u) + \beta cosh(2u) = 0, \tag{14}$$

where u is the real scalar function of two independent variables x and t. The subscript represent the partial derivatives, α and β are two arbitrary non zero constants which also represent the model parameters. By implementing Painlevé property $u = ln(V)$ or $V = e^u$, then the $sinh$, $sin(2h)$, $cosh$ and $cos(2h)$ term transformed to the following way:

$$\left.\begin{array}{ll} sinh(u) = \frac{V-V^{-1}}{2}, & cosh(u) = \frac{V+V^{-1}}{2}, \\ sinh(2u) = \frac{V^2-V^{-2}}{2}, & cosh(2u) = \frac{V^2+V^{-2}}{2} \end{array}\right\} \tag{15}$$

Now using Eq. (14) and Eq. (15) along with $u = ln(V)$, we obtain

$$V(V''_{tt} - kV''_{xx}) + 2(k(V'_x)^2 - (V'_t)^2) + (\alpha+\beta)V^3 + \beta V^4 = 0 \tag{16}$$

Now let $V(x,t) = V(\xi)$, where $\xi = x - ct$, putting this in Eq. (16) we get

$$V(c^2-k)V'' + (k-c^2)(V')^2 + \alpha V^3 + \beta V^4 = 0 \tag{17}$$

by blanching the derivative VV'' and V^4, we get $m = 1$. So, the finite series is the form

$$V(\xi) = a_0 + a_1 Q(\xi) \tag{18}$$

putting Eq. (18) with first and second derivative in Eq. (17) and equating the like powers of $Q(\xi)$, we get the following system of equation

$$\begin{aligned} 2ln(a)^2c^2a_1^2 - 2ln(a)^2ka_1^2 + 2\beta a_1^4 &= 0, \\ 4ln(a)^2c^2a_0a_1 - 2ln(a)^2c^2a_1^2 - 4ln(a)^2ka_0a_1 + 2ln(a)^2ka_1^2 & \\ +8\beta a_0a_1^3 + 2\alpha a_1^3 &= 0, \\ -6ln(a)^2c^2a_0a_1 + 6ln(a)^2ka_0a_1 + 12\beta a_0^2a_1^2 + 6\alpha a_0a_1^2 &= 0, \\ 2ln(a)^2c^2a_0a_1 - 2ln(a)^2ka_0a_1 + 8\beta a_0^3a_1 + 6\alpha a_0^2a_1 &= 0, \\ 2\beta a_0^4 + 2\alpha a_0^3 &= 0. \end{aligned}$$

By solving the above systems, we received the following different solution sets:

Case–I:

$$a_0 = 0, \quad a_1 = -\frac{\alpha}{\beta}, \quad c = \frac{\sqrt{ln(a)^2k\beta - \alpha^2}}{\sqrt{\beta}\; ln(a)}$$

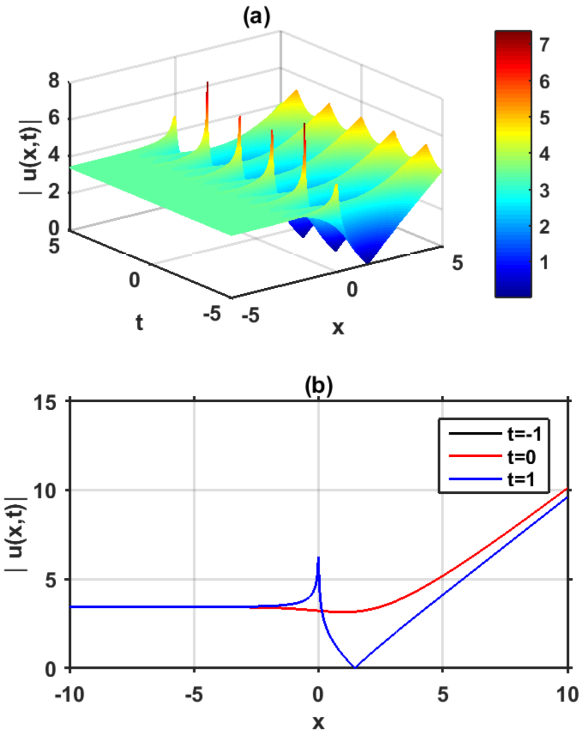

Figure 5: (a) 3D snapshot of the Eq. (19) for the arbitrary parameters $a=3, \alpha=2, \beta=0.5, d=1$ and $k=-1.5$, and (b) 2D snapshot of (a) at $t=-1, t=0, t=1$ respectively.

Case–I corresponds to the following exact solution of the double combined sinh–cosh–Gordon equation is extracted:

$$u_1(x,t)=\ln\left[-\frac{\alpha}{\beta}\left(\frac{1}{1+da^{x-\frac{\sqrt{ln(a)^2k\beta-\alpha^2}}{\sqrt{\beta}\ ln(a)}t}}\right)\right] \tag{19}$$

Case–II:

$$a_0=0,\quad a_1=-\frac{\alpha}{\beta},\quad c=-\frac{\sqrt{ln(a)^2k\beta-\alpha^2}}{\sqrt{\beta}\ ln(a)}$$

Case–II corresponds to the following exact solution of the double combined sinh–cosh–Gordon equation is generated:

$$u_2(x,t)=\ln\left[-\frac{\alpha}{\beta}\left(\frac{1}{1+da^{x+\frac{\sqrt{ln(a)^2k\beta-\alpha^2}}{\sqrt{\beta}\ ln(a)}t}}\right)\right] \tag{20}$$

Case–III:

$$a_0=-\frac{\alpha}{\beta},\quad a_1=\frac{\alpha}{\beta},\quad c=\frac{\sqrt{ln(a)^2k\beta-\alpha^2}}{\sqrt{\beta}\ ln(a)}$$

Case–III corresponds to the following exact solution of the double combined sinh–cosh–Gordon equation is generated:

$$u_3(x,t)=\ln\left[-\frac{\alpha}{\beta}+\frac{\alpha}{\beta}\left(\frac{1}{1+da^{x-\frac{\sqrt{ln(a)^2k\beta-\alpha^2}}{\sqrt{\beta}\ ln(a)}t}}\right)\right] \tag{21}$$

Case–IV:

$$a_0=-\frac{\alpha}{\beta},\quad a_1=\frac{\alpha}{\beta},\quad c=-\frac{\sqrt{ln(a)^2k\beta-\alpha^2}}{\sqrt{\beta}\ ln(a)}$$

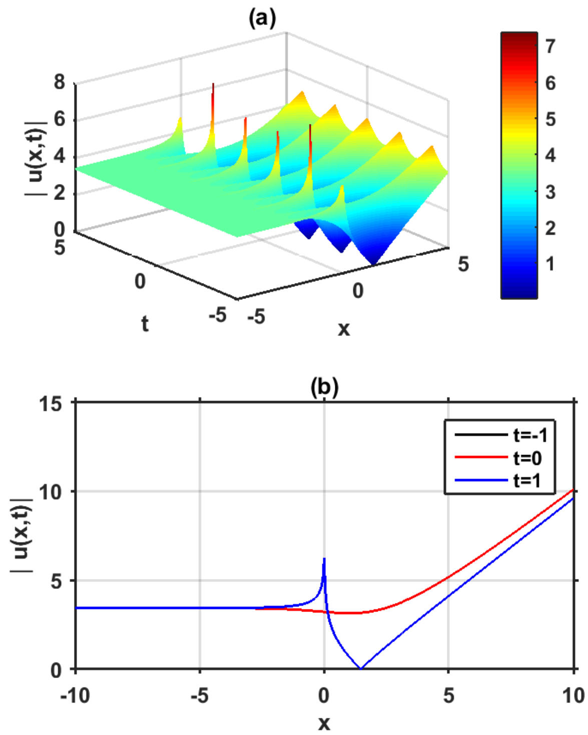

Figure 6: (a) 3D snapshot of the Eq. (20) for the arbitrary parameters $a=3, \alpha=2, \beta=0.5, d=1$ and $k=-1.5$, and (b) 2D snapshot of (a) at $t=-1, t=0, t=1$ respectively.

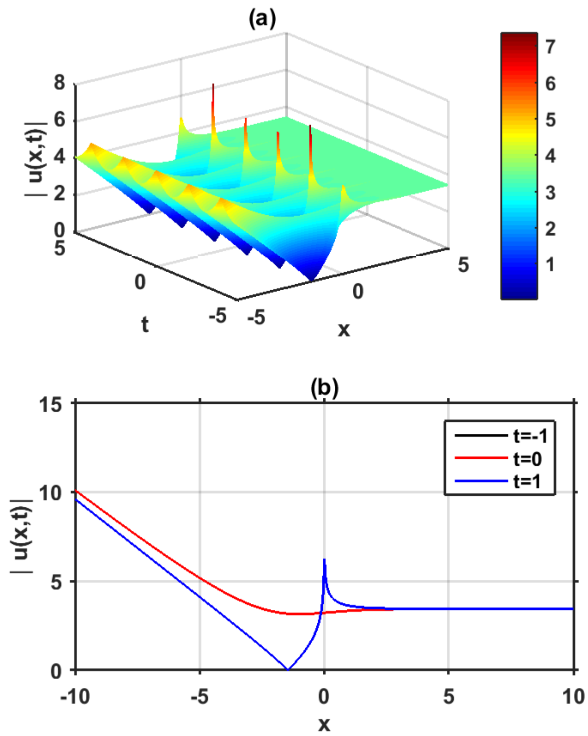

Figure 7: (a) 3D snapshot of the Eq. (21) for the arbitrary parameters $a=3, \alpha=2, \beta=0.5, d=1$ and $k=-1.5$, and (b) 2D snapshot of (a) at $t=-1, t=0, t=1$ respectively.

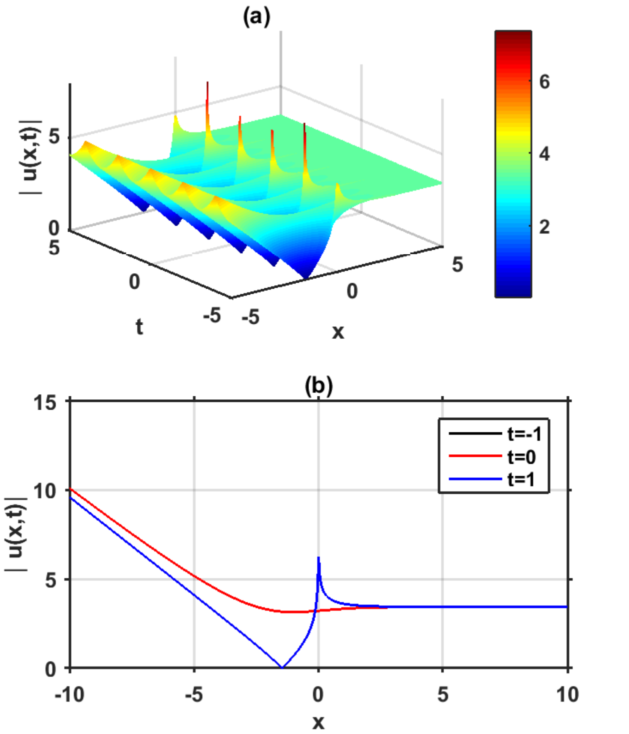

Figure 8: (a) 3D snapshot of the Eq. (22) for the arbitrary parameters $a = 3, \alpha = 2, \beta = 0.5, d = 1$ and $k = -1.5$, and (b) 2D snapshot of (a) at $t = -1, t = 0, t = 1$ respectively.

Case–IV corresponds to the following exact solution of the double combined sinh–cosh–Gordon equation is produced:

$$u_4(x,t) = \ln\left[-\frac{\alpha}{\beta} + \frac{\alpha}{\beta}\left(\frac{1}{1 + da^{x + \frac{\sqrt{ln(a)^2 k\beta - \alpha^2}}{\sqrt{\beta}\ ln(a)}t}}\right)\right] \quad (22)$$

Exact solutions of the double combined sinh–cosh–Gordon equation is demonstrated by some MATLAB figures which indicating Figure 5 to Figure 8 by choosing free parameters as $a = 3, \alpha = 2, \beta = 0.5, d = 1$ and $k = -1.5$.

5. Conclusion

This paper produces exact solution of the combined sinh–cosh–Gordon and double combined sinh–cosh–Gordon equations via modified Kudryashov method. The obtained exact solution is also verified the original equation with the aid of maple package program. We claim that, the derived results are totally new and executed method gives very good consistency as compare to other method. Therefore, the method is robust and concise, and it can also be used to other kinds of nonlinear equations in mathematical physics and others applied field.

References

[1] M. Wang, Y. Zhou, Z. Li, Application of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics, Physics Letters A 216 (1-5) (1996) 67–75.

[2] E.M.E. Zayed, K.A. Gepreel, The (G'/G)-expansion method for finding traveling wave solutions of nonlinear partial differential equations in mathematical physics, Journal of Mathematical Physics 50(1) (2009) 013502

[3] F. Hawlader, D. Kumar, A variety of exact analytical solutions of extended shallow water wave equations via improved (G'/G)-expansion method, International Journal of Physical Research 5(1) (2017) 21–27.

[4] M.M.A. Khater, D. Kumar, Implementation of three reliable methods for finding the exact solutions of (2+1) dimensional generalized fractional evolution equations, Optical and Quantum Electronics 49(12), (2017) 427.

[5] E. Fan, Extended tanh–function method and its applications to nonlinear equations, Physics Letters A 277(4) (2000) 212–218.

[6] M.A. Abdou, The extended F–expansion method and its application for a class of nonlinear evolution equations, Chaos, Solitons & Fractals 31(1) (2007) 95–104.

[7] S. Liu, Z. Fu, S. Liu, Q. Zhao, Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations, Physics Letters A 289(1) (2001) 69–74.

[8] W.-X. Ma, J.-H. Lee, A transformed rational function method and exact solutions to the (3+1) dimensional Jimbo–Miwa equation, Chaos, Solitons & Fractals 42(3) (2009) 1356–1363.

[9] Y. Chen, Z. Yan, The Weierstrass elliptic function expansion method and its applications in nonlinear wave equations, Chaos, Solitons & Fractals 29(4) (2006) 948–964.

[10] A.-M. Wazwaz, Multiple–soliton solutions for the KP equation by Hirota's bilinear method and by the tanh–coth method, Applied Mathematics and Computation 190(1) (2007) 633–640.

[11] M.M.A. Khater, D. Kumar, New exact solutions for the time fractional coupled BoussinesqBurger equation and approximate long water wave equation in shallow water, Journal of Ocean Engineering and Science 2(3) (2017) 223–228.

[12] C.-S. Liu, Trial equation method to nonlinear evolution equations with rank inhomogeneous: mathematical discussions and its applications, Communications in Theoretical Physics 45 (2006) 219–223.

[13] N.A. Kudryashov, M.B. Soukharev, Popular Ansatz methods and Solitary wave solutions of the Kuramoto–Sivashinsky equation, Regular and Chaotic Dynamics 14(3) (2009) 407–419.

[14] A. Akbulut, M. Kaplan, Auxiliary equation method for time-fractional differential equations with conformable derivative. Computers & Mathematics with Applications, (2017). https://doi.org/10.1016/j.camwa.2017.10.016

[15] K. Hosseini, F. Samadani, D. Kumar, M. Faridi, New optical solitons of cubic–quartic nonlinear Schrdinger equation,Optik, 157 (2018) 1101–1105.

[16] D. Kumar, A.R. Seadawy, A. K. Joardar, Modified Kudryashov method via new exact solutions for some conformable fractional differential equations arising in mathematical biology, Chinese Journal of Physics, 56(1)(2018) 75–85.

[17] K. Hosseini, D. Kumar, M. Kaplan, E.Y. Bejarbaneh, New Exact Traveling Wave Solutions of the Unstable Nonlinear Schrdinger Equations, Communications in Theoretical Physics, 68(6) (2017) 761.

[18] D. Kumar, K. Hosseini, F. Samadani, The sine–Gordon expansion method to look for the traveling wave solutions of the Tzitzica type equations in nonlinear optics, Optik, 149 (2017) 439–446.

[19] F. Xie, Z. Yan, H. Zhang, Explicit and exact traveling wave solutions of Whitham–Broer–Kaup shallow water equations, Physics Letters A 285 (2001) 76–80.

[20] A.H. Salas, J.E. Castillo H, New exact solutions to sinh–cosh–Gordon equation by using techniques based on projective Riccati equations, Computers & Mathematics with Applications 61 (2) (2011) 470–481.

[21] C. A. Gómez, A.H. Salas, New exact solutions for the combined sinh–cosh–Gordon equation, Lecturas Matemáticas 27 (2006) 87–93.

[22] C. M. Khalique, G. Magalakwe, Combined sinh–cosh–Gordon equation: symmetry reductions, exact solutions and conservation laws, Quaestiones Mathematicae 37(2) (2014) 199–214.

[23] L. Wei, Exact solutions to a combined sinh–cosh–Gordon equation, Communications in Theoretical Physics 54(4) (2010) 599.

[24] M. Ramirez, A. Magnolia, C. de Indias, V.P.J. Jaramillo-Camacho, R.D.O. -Ortiz, Solutions for the Combined sinh–cosh–Gordon Equation, International Journal of Mathematical Analysis, 9(24) (2015) 1159-1163.

[25] G. Magalakwe, B. Muatjetjeja, C.M. Khalique, Exact solutions and conservation laws for a generalized double combined sinh–cosh–Gordon equation, Mediterranean Journal of Mathematics 13(5) (2016) 3221–3233.

[26] H. Kheiri, A. Jabbari, The (G'/G)-expansion method for solving the combined and the double combined sinh–cosh–Gordon Equations, Acta universitatis apulensis 22 (2010) 185–194.

[27] A. Irshad, S.T. Mohyud–Din, Tanh–Coth method for nonlinear differential equations, Studies in Nonlinear Sciences 3(1) (2012) 24–48.

[28] A.-M. Wazwaz, Exact solutions to the double sinh–Gordon equation by the tanh method and a variable separated ODE method, Computers & Mathematics with Applications 50(10-12) (2005) 1685–1696.

[29] Y. Sun, New exact traveling wave solutions for double sine–Gordon equation, Applied Mathematics and Computation 258 (2015) 100–104.

[30] G. Magalakwe, C.M. Khalique, New exact solutions for a generalized double sinh–Gordon equation, Abstract and Applied Analysis (2013).