International Journal of Physical Research, 1 (1) (2013) 21-27

www.sciencepubco.com/index.php/IJPR

Principle of least action and convergence of systems towards state of closure

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Abstract

A natural process is defined as an act, by which a system organizes itself with time. Any natural process drives a system to a state of greater organization. Co-existence of a system in states of maximum organization as well as maximum action forms the core idea of the paper. Organization is a progressive change that gradually converge a system towards a state of closure. To understand this in detail, major influences have been drawn from the Principle of Least Action and, it allows us to see how this, most basic law of physics determines the development of the system towards states with less action *i.e.* organized states. And hence, it is being proposed, that the development of a system towards states of greater organization is cyclic in nature.

Keywords: *action, dead state, organization, cyclic process, complexity, rapidity, Nash Equilibrium, entropy, coherence, directionality*.

1 Introduction

Organization [1, 2] is a progressive change and can be modeled as a part of nature. Nature comprises of open systems. An open system is a continually morphing dynamical system. All natural processes [3–8] occurring in the universe are rooted in physics and have physical explanation. All of the structures in the universe exist, because they are in their state of least action [4–7] or tend towards it. In any system, simple or complex, the system spontaneously calculates which path will use least effort for that process [4]. A system comprises of elements and constraints, both internal as well as external. The internal constraints could be the configurations of the system or the state of elements themselves, whereas, the external constraints are those that define the geometry of the system. The elements apply work on the constraints to modify the organization and minimize the action, which takes finite amount of time, making the reorganization a process [4–8]. Reorganization is a process of optimization. A system thrives to organize itself and in the course of development destroys its previous identity thus, making the process irreversible. The dynamical systems that are present in nature are generally very complex exhibiting various levels of complexity present within themselves. Order implies a state of lesser action hence, greater organization. A complex system with a structure and emergence is said to self-organizing. The process of self-organization of the systems can be called a “Process of achieving a least action state by a system”. It could last billions of years or indefinitely [9].

$$i \xrightarrow{X_t} f_t$$

Fig. 1: Figure representing an evolving system undergoing a natural process (X_t) in time (t) from a state of lesser organization (i) to a state of greater organization (f_t).

The convergence of a system towards a state of greater organization is a coherent act, a summation of the organization of each system elements towards their optimal states of organization [8]. The extent of organization achieved by a system element depends upon its available energy, work potential or its exergy [8, 10] compared to its surrounding media. This energy gradient acts as a driving force enabling a dynamical system to organize itself with continuous passage of time and, to morph towards states of greater organization. In an open system there is always an influx and out flux of energy between the system and surrounding media. So, the action of a single element will not be at minimum, but the sum of the action of all the elements in the system will be at a global minimum. The action of a single element is not maximal as well, because by definition this will destroy the system, so this intermediate state represents an optimum [3–6].

2 Open systems and entropy change

In an open system there is always an influx and out flux of energy between the system and surrounding media, causing the energy of individual system elements to vary continuously. This exchange of energy is accompanied with change in entropy between system and surrounding [3, 4, 5, 10, 11, 12, 13]. The sum total of the entropy generated within the system and the entropy change due to energy exchanged gives the net entropy generated in a natural process.

$$(\partial S)_{gen} = (\partial S)_{int} + (\partial S)_{ex} \quad (1)$$

Where, $(\partial S)_{gen}$ represents total entropy generated [13], $(\partial S)_{int}$ represents total internal entropy of the process and $(\partial S)_{ex}$ represents total exchange of entropy between system and surrounding media.

$$(\partial S)_{ex} = |(\partial S)_{in}| + |(\partial S)_{ex}| \quad (2)$$

Where, $(\partial S)_{ex}$ is the sum total of the influx and out flux of entropy. A natural process is also accompanied by the increase in number of microstates of the system. In words of Statistical Thermodynamics, entropy is simply our lack of knowledge of the actual state of the system [14]. Thus, with increase in time and hence, increase in organization the system elements tend to lose track of their history; motion, trajectories (inherent irreversibilities). The lack of information with increasing organization thus, renders a system towards greater levels of complexity.

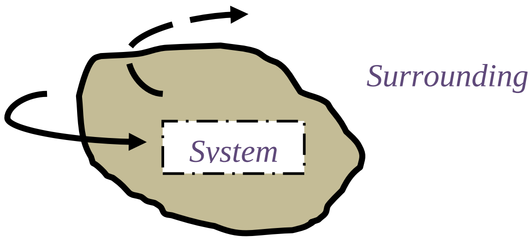

Fig. 2: Figure representing an exchange of energy between a system and the surrounding

3 Transition diagram representation of a complex system

A complex system can be represented in form of a network diagram that, represents the transition states *i.e.*, from initial to final states. Let a system be initially in a state 'i' and make a transition into the t^{th} final state 'f_t' through a natural process X_t causing an increase in the amount of organization, where 't' represents the time elapsed while undergoing the process and $t \in (0, \infty)$.The final state of the system is unknown since the system under consideration is open to surrounding media [3–8] hence; it has been subscripted with 't'. Thus, the final fate of the system can assume any out of the infinite states, $f_t = f_0, f_1, f_2, f_3 \ldots \ldots \ldots f_\infty$

Where, f_0 is same as the initial state 'i'.

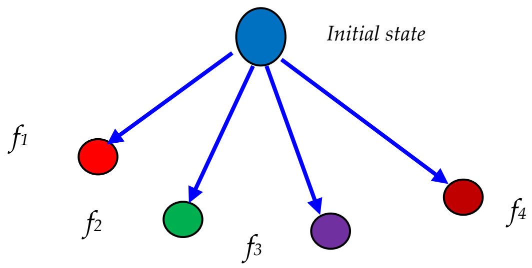

Fig. 3: Figure representing transition diagram for a system

The figure represents the transition diagram for a system from i^{th} state to f^{th} state where, transition from i to f_1 is denoted by the natural process X_1 and so on. Each process can occur in an infinite number of ways, thus, leading the system towards infinite number of final states. But the Principle of Least Action [3, 5, 6, 7, 15] imposes constraint, by causing the system to undergo a specific process out of the available infinite processes. Entropy principle makes certain process thermodynamically favourable over others. According to the Principle of Least Action, any natural process occurs in that way which consumes the least time. The transition of a system from one state to another is a coherent phenomenon of all its constituting elements. The amount of organization present in the system at a later state is always greater than the amount of organization it possessed at an earlier state. Increasing the amount of organization is the driving force behind any natural process. However, the rapidity involved in a natural process depends upon the time each of the system elements takes to organize itself. The time taken by each element to organize itself varies continuously because, the system is always communicating with the surrounding media by exchanging energy and entropy [10–13]. So each system element possesses a set of strategy [16, 17, 18]. Strategies for a system element are its trajectories in phase space obtained from solving its integral equation of motion. The set of strategies for an element is called pure strategy if the system is free from any constraints. Presence of constraints causes the system elements to optimize their strategies in order to follow the least path and organize themselves. Optimization thus, prevents a system element to use its pure strategy. Reorganization of a system or its constituting elements is thus a process of optimization. In presence of constraints, the set of strategies thus, employed by the system elements are their mixed strategies.

3.1 Principle of Least Action for a multi-element system

The Principle for Least Action states, that the actual motion of a conservative dynamical system between two points, occurs in such a manner, that the action has a minimum value in respect to all other paths between the points, which correspond to the same energy [3, 5, 6, 15].
The classical definition of the Principle of Least Action [15] is:

$$I = \int_{t_1}^{t_2} L dt \tag{3}$$

$$L = T - V \tag{4}$$

Where, I is the action of the system, L is the Lagrangian of the system and T and V are the kinetic and the potential energy of the system respectively. The variation of the path is zero for any natural process occurring between two points of time t_1 and t_2, or the nature acts in the simplest way hence, in the shortest possible time. For the motion of the system between time t_1 and t_2, the Lagrangian L has a stationary value for the correct path of motion.

$$\delta I = \delta \left(\int_{t_1}^{t_2} L dt \right) = 0 \tag{5}$$

Eqn. (5) can be summarized as the Hamilton's Principle [15].

For a system consisting of N-elements,

$$L = \sum_{j=1}^{N} L_j \tag{6}$$

$$L_j = T_j - V_j \tag{7}$$

Where, L_j, T_j and V_j represent the Lagrangian, kinetic and potential energies of the j^{th} system elements.

So,

$$I = \sum_{j=1}^{N} I_j = \int_{t_1}^{t_2} \sum_{j=1}^{N} L_j dt = \int_{t_1}^{t_2} \sum_{j=1}^{N} (T_j - V_j) dt \tag{8}$$

Where, I_j is *action* of the j^{th} system element.

3.2 Game-theoretic model of a complex system

Let the set of pure strategies [17, 18] for the j^{th} element, corresponding to the t^{th} final state be given by, $\pi_{f_t}^{j}$

$$\pi_{f_t}^{j} = \left(\pi_{f_0}^{j}, \pi_{f_1}^{j}, \pi_{f_2}^{j}, \dots \dots \dots \dots \right), \forall t \in (0, \infty) \tag{9}$$

Let p_j be a continuous function that maps the set of all n-tuples of pure strategies for each element into real numbers. These sets of real numbers form the set of mixed strategies for each element. Let the set of pure strategies for the j^{th} element, corresponding to the t^{th} final state be given by, $\mu_{f_t}^{j}$

$$so,\ p_j\left(\pi_{f_t}^{j}\right) = \mu_{f_t}^{j}, \forall t \in (0, \infty) \tag{10}$$

Eqn. (10) is subjected to the constraints;

$$\mu^{j}_{f_t} \geq 0\ \forall j \in (1, N)\ and\ \sum_{t=0}^{\infty} \mu^{j}_{f_t} = 1\ \ \forall t \in (0, \infty) \tag{11}$$

The constraints of the process validate the occurrence of the natural process, since the sum of the mixed strategies for a system element and the probability of occurrence of a process is equal to unity. From the above conditions presented in eqn. (10) and eqn. (11) it can be clearly observed that, p_j is simply a probability density function that operates on the random variable $\pi^{j}_{f_t}$. The mixed strategy for the system at the macroscopic level is denoted by μ_{f_t}.

$$\mu_{f_t} = \sum_{j=1}^{N} \mu^{j}_{f_t}, \forall j \in (1, N) \tag{12}$$

For the occurrence of the phenomena at the macroscopic level, coherence in microscopic level must be existent [14].

$$\sum_{t=0}^{\infty} \mu_{f_t} = \sum_{t=0}^{\infty} \sum_{j=1}^{N} \mu^{j}_{f_t} = 1, \forall t \in (0, \infty) and\ j \in (1, N) \tag{13}$$

A system's mixed strategy is thus a probability distribution. A system's, pay-off represents the amount of organization, (Org) it posses [17, 18]. Let $\rho_j$be the pay-off function that maps the set of mixed strategies for each set of the system elements and in turn generates the pay-off for each system element, which is denoted by;

$$\rho_j\left(\mu^{j}_{f_t}\right) = (Org)^{j}_{f_t} = \frac{1}{I^j} = \frac{1}{\int_0^t L^j dt} \tag{14}$$

The strategy chosen by the system element is that, which tends to maximize the amount of organization present within the system, at a later state. The set of optimal strategy for the *j^th^* system element that maximizes the amount of organization of the system is thus, a process of optimization. The system elements evolve with time and achieve their set of optimal strategies that maximizes organization of the system as a whole. This set of strategies for a j^{th} system element represents its Nash equilibrium strategy [17, 18] profiles, denoted by $\hat{\mu}^{j}_{f_t}$.

$$so, \sum_{j=1}^{N} \rho_j\left(\hat{\mu}^{j}_{f_t}\right) = max(Org)_{f_t}\ at\ a\ particular\ time \tag{15}$$

3.3 Quantifying organization as inverse of action

An organized system tends to have the least value of action. Conversely, lesser is the value of *action* more is the amount of organization present in a system [3–8]. Thus, amount of organization *(Org)* is inversely related to the action of the system *(I)*.

$$(Org) \times (I) = constant \tag{16}$$

Differentiating with respect to time [5, 19],

$$\frac{\partial Org}{\partial t} = -\left(\frac{Org}{I}\right) \times \frac{\partial I}{\partial t} \tag{17}$$

The equation implies that, the rate of increase of organization in a system is equal to the rate of decrease in action of the system multiplied by the ratio of amount of organization to the amount of action possessed by the system at an earlier state.

So, the above equation can be rewritten as;

$$\left(\frac{\partial Org}{\partial t}\right)_{f_t} = -\left(\frac{Org}{I}\right)_i \times \left(\frac{\partial I}{\partial t}\right)_{f_t} \tag{18}$$

The rate of increase of organization is the directionality of a natural process. For any natural process the ratio of organization to action at an earlier state must always be greater than unity. This is because of the fact that a system would have ceased to exist at the earlier state if the ratio became less than unity.

From eqn. (17) we have,

$$\left(\frac{\partial Org}{\partial t}\right)_{f_t} = -\left(\frac{Org}{I}\right)_i \times \left(\frac{\partial I}{\partial t}\right)_{f_t} \tag{19}$$

3.4 An equation for natural process

An equation governing all natural processes can be presented as;

$$X_t = \left(\frac{\partial Org}{\partial t}\right)_{ft} = -\left(\frac{Org}{I}\right)_i \times \left(\frac{\partial I}{\partial t}\right)_{ft} \quad (20)$$

$$X_t = -\alpha\left(\frac{\partial I}{\partial t}\right)_{ft} \quad (21)$$

The above result when modified for a system undergoing a natural process becomes,

$$X_t = -\alpha\left(\frac{\partial I}{\partial t}\right)_{ft} = -\alpha\frac{\partial}{\partial t}\left(\int_0^t \sum_{j=1}^N L^j\, dt\right) \quad (22)$$

The physical significance of this equation is that systems undergoing natural process organize themselves with time and proceed with energy dispersal of its constituting elements.

Let r be defined as the rate of a natural process [20],

$$r = \frac{\partial X_t}{\partial t} = -\alpha\frac{\partial}{\partial t}\left(\int_0^t \left(\sum_{j=1}^N \frac{dL^j}{dt}\right) dt\right) \quad (23)$$

Rapidity *(r)* associated with a natural process is a very important tool. Rapidity is also the rate of increase of complexity of a system. It can be used as a tool for comparing between the rates of evolution of two identical systems. Also, rapidity is a decreasing function with respect to time for any natural process. Thus, the rapidity of a system decreases as the system converges towards the state of maximum organization. Rapidity plays a very important role in the existence and evolution of the system as whole. If individual system elements are assigned an unique magnitude of rapidity, denoted as r^j where,

r^j is the *rapidity* associated with the j^{th} system element, then,

$$r \propto \min_{all\ j} r^j \quad (24)$$

The above proposition is a significant and very important result. The rate or rapidity of a system evolving through a natural process is directly proportional to its least rapid constituting element *i.e.* the system element that is least rapid in achieving a state of greater organization governs the overall rapidity of the system

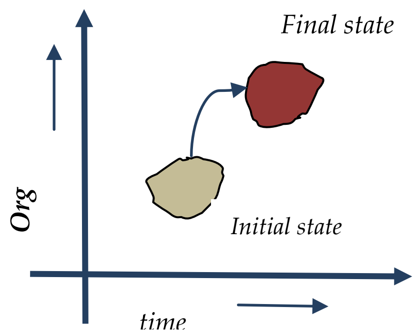

Fig. 4: Figure representing a system undergoing a natural process from lesser organized initial state to greater organized final state

4 Singular states: entropy and organization

A system on achieving the state of maximum organization comes in equilibrium with the surrounding. The amount of action present within a system is a property of the system itself, it is determined the state of its constituting elements and system constraints. When action assumes a null value, the system shrinks to a singular point. Attaining a state of maximum organization implies the disappearance of the energy gradient between the system and its surrounding. Such a system is said to have reached a dead state [10] where all natural processes have ceased to exist. On reaching the dead state the system no longer evolves with time but becomes a static structure. The system would continue to remain at that state for an infinite period of time. A least action state is also a state of least amount of free energy [3–8]. A system's configuration determines the amount of free energy it posses. A system with zero action then must have no free energy, hence, no configuration. This implies that after achieving the dead state the system begins to shrink to a point [19], or more precisely both the processes occur almost simultaneously. Eqn.1 modifies into,

$$(\partial S)_{gen} = (\partial S)_{int} + (\partial S)_{in} - (\partial S)_{out} \quad (25)$$

At equilibrium,$(\partial S)_{gen} = 0$, a necessary condition to morph systems in nature towards the state maximum organization [19]. So, eqn.25 is rewritten as

$$(\partial S)_{gen} = (\partial S)_{int} + (\partial S)_{in} - (\partial S)_{out} = 0 \quad (26)$$

At equilibrium the boundary separating the system and surrounding collapses and the total entropy generated within the system is flushed out to the surrounding. Influx of entropy thus loses its significance.

$$(\partial S)_{gen} = (\partial S)_{int} - (\partial S)_{out} = 0 \quad (27)$$

At equilibrium,

$$X_t = -\alpha \left(\frac{\partial I}{\partial t}\right)_{f_t} = 0 \text{ as,} \left(\frac{\partial Org}{\partial t}\right)_{f_t} = 0 \; and, \frac{\partial^2 Org}{\partial t^2} < 0 \text{ \{condition for maxima\}} \quad (28)$$

This implies that the quantity within the derivative in eqn. 23 assumes a stationary value.

$$X_t = -\alpha \int_0^t \left(\sum_{j=1}^{N} \frac{dL^j}{dt}\right) dt = \tau \frac{\partial S_{int}}{\partial t} = 0 \quad (29)$$

Here, τ is a constant of proportionality.
Hence, at equilibrium it is clearly seen that, all natural processes cease to exist and internal entropy becomes maximum. The system becomes highly organized and exhibit maximum level of complexity. But, on the other hand the free energy of the system becomes maximal and the system instantly disintegrates.

5 Conclusion

Nature in its crude form is very difficult to understand but we must not get carried away by the simplicity of the laws that are thought to govern the nature. As once pointed out by Feynman [21], there is a pleasure in recognizing old things from a new point of view. Action being an extensive property first vanishes causing the system to get highly organized and then causes it to shrink into an infinitesimal point and then re-appears at its maximum magnitude causing the system to become highly unpredictable. Re-organization again starts now in the disintegrated system causing it to develop its levels of complexity. This unpredictable system again tries to achieve the state of least action and the cycle (closure) continues forever. However, this time the course of its development may be entirely different. Every natural process passes through three stages: organization, disintegration and re-organization. Global Complexity occurs thus at the edge of chaos. Thus, organization, disintegration and reorganization collectively compose a cyclic process hence, a closure in formation.

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