*International Journal of Physical Research*, 12 (2) (2024) 104-115

SPC

**International Journal of Physical Research**

*Website: www.sciencepubco.com/index.php/IJPR*

***Research paper***

# Soliton solutions to the (3+1)-dimensional KP and BA models using advanced exp(-ϕ(ξ))-expansion scheme in mathematical physics

**Md. Mohiuddin Zillu [1] *, Supta Ghosh [1], Anita Biswas [1]**

[1] *Department of Mathematics, European University of Bangladesh (EUB), Dhaka, Bangladesh*
**Corresponding author E-mail: mohiuddineub22@gmail.com*

**Abstract**

In this manuscript, the main motivation is the implement of the advanced exp(-ϕ(ξ)) -expansion method to construct the soliton solution, which contains some controlling parameters of two distinct equations via the Biswas-Arshed (BA) model and the (3+1)- dimensional Kadomtsev-Petviashvili (KP) equation. Here the behaviors of the solutions are presented in graphically under some condition on those parameters. The height of the wave, wave direction, and angle of the obtained wave are formed by substituting the particular values of the considerations over showing figures with control plot. With the collaboration of the advanced exp(-ϕ(ξ))-expansion method, we construct entirely the solitary wave results as well as rogue type soliton, combined singular soliton, kink, singular kink, bright and dark soliton, periodic shape, double periodic shape soliton etc. Therefore, it is remarkable to perceive that the advanced exp(-ϕ(ξ))-expansion method is easy, compatible and powerful mathematical tool to elucidation of exact results to other non-linear equivalences.

***Keywords***: *The BA Model; The (3+1)-Dimensional KP Equation; Advance Exp(-Φ(Ξ))-Expansion Method; Traveling Wave Solution; Rouge Wave Solution; Periodic Solitons.*

## 1. Introduction

Nonlinear partial differential equations (NLPDEs) are a significant topic, have spread widely around the world in many different types of dynamic structures. Many mathematicians and physicists are analyzing dynamic structures. Dynamic structures are a significant part of nonlinear physical simulations and it's used in different fields of science and engineering such as electrical conduction, plasma physics, mathematical natural sciences, fluid mechanics, optical fiber, solid state physics, shallow water wave propagation, mathematical dynamics, mathematical dynamics and many others field [1-10]. Recently, many experts investigated the optical soliton solutions of the NLPDEs and its solutions play a momentous role in visualizing into the internal process of integrated physical phenomena. Many important approaches have been proposed for obtaining optical solutions of NLPDEs such as
the modified polynomial expansion technique [11], the enhanced $(G'/G)$-expansion approach [12], the exp $(-\varphi(\xi))$-expansion technique [13], the generalized Kudryashov approach [14], the new auxiliary equation technique [15], the lie symmetry approach [16], the extended Fan sub-equation technique [17], the complex technique [18], the improved Bernoulli sub-equation approach [19] and so on.
We have considered the two NLPDEs via the BA and (3+1)-dimensional KP model in this manuscript. Many scholars have studied the BA and the (3+1)-dimensional KP models in the last few decays and found many optical solutions. In its continuity, using two distinct schemes in Refs [20, 21], they found the exact soliton solutions as well as the singular and dark solitons of the BA model with Kerr and power law in nonlinearity. The newly $\Phi^6$-model expansion technique applied to the BA model and attained the optical soliton solutions which are represents the dark, bright, singular, rational and periodic wave profile in Ref [22]. In addition, it has been observed that some scholars have found the optical solution of the BA model using the trial solution technique [23], the modified simple equation approach [24], the mapping technique [25], and the extended trial function approach [26], which are dark, bright, singular and periodic type wave profiles. On the other hand, the (3+1)-dimensional KP model was first introduced in 1970 by Soviet physicists Kadomtsev and Petviashvili which narrates the evolution of semi-one-dimensional shallow water waves while the effect of surface tension and viscosity is negligible [27]. After that many authors have studied the different form of KP model in [28, 29]. Recently, one soliton and one resonant soliton solutions have found from the (3+1)-dimensional KP model using consistent tanh expansion in [30]. Using Bilinear method in Ref [31], the KP model have explored the multiple lump solutions via 1-lump wave, 3-lump wave, 6-lump wave and 8-lump waves. In addition, the simplified homogeneous balance method has been applied to the KP model and found the one single soliton and one double soliton solution in [32], and the Hirota bilinear transformation has been applied to the KP equation and obtained the one and two rough wave solutions in Ref [33].
The purpose of the manuscript is to applying the advance $\exp(-\phi(\xi))$–expansion approah to the BA model and the (3+1)-dimensional KP model, and to find some optical soliton solutions namely w-shape, kink shape, periodic soliton solution shape, double periodic shape,

dark soliton shape, combined singular soliton, rogue wave profiles. Base on the above discussion in the previous literature, we can say that some wave profiles of the BA and (3+1)-dimensional KP models are new. Finally, it can be used to perfect water rollers of extended wavelength with softly non-linear repairing forces and regularity distribution and can also be used to model waves in ferromagnetic media, nonlinear optics, optical fiber, plasma physics.

We have divided this article into follows, the literature review, objectives and background are discussed in section one. We talked about the description of the tactic in the section two. The governing equation have represented in the third section. In the fourth section, the propose method applied to the (3 + 1)-dimensional KP and BA model. Graphical and physical explanation have discussed in section five. Finally, conclusion is given in section six.

## 2. The advanced $\exp(-\phi(\xi))$-expansion method

Section 2. is consisting of the summary of advance $\exp(-\phi(\xi))$-expansion method [34, 35]. We consider the NLPDEs which is of the form

$$R\left(U, U_x, U_y, U_t, U_{xx}, U_{xy}, U_{xt}, U_{yy}, U_{yt}, U_{tt}, \dots\dots\right) = 0, \tag{1}$$

Where $U = U(x, y, t)$ is the wave function which is to be determined, Ris a polynomial of $U(x, y, t)$ and its partial derivatives.

Step-1. First, we take a conversion variable to change all independent variables into a single variable, such as

$$U(x, t) = u(\eta), \eta = kx + ly \pm Vt. \tag{2}$$

The wave varible mentioned in Eq. 2 turn the NLPDE (1) into an ODE as

$$P(u, u', u'', \cdots\cdots\cdots) = 0. \tag{3}$$

Step-2. According to the advanced $\exp(-\phi(\xi)$-expanssion method, the exact solution of Eq. (3) is assumed to be

$$u = \sum_{i=0}^{m} a_i \exp(-\phi(\xi))^i \tag{4}$$

Where $a_{1,} a_2, a_3 \dots\dots, a_m; a_m \neq 0$, are constants to be determinted. The derivative of $\phi(\xi)$ satisfies the ODE in the succeeding system

$$\phi'(\xi) + A\exp(-\phi(\xi)) + B\exp(\phi(\xi)) = 0, \tag{5}$$

Then, the obtained results of ODE Eq. (5) are of the hyperbolic, trigonometric and the following forms:

Case I: Hyperbolic function solution (when $AB < 0$):

$$\phi(\xi) = \ln\left(\sqrt{\frac{A}{-B}}\tanh(\sqrt{-AB}(\xi + C))\right),$$

And

$$\phi(\xi) = \ln\left(\sqrt{\frac{A}{-B}}\coth(\sqrt{-AB}(\xi + C))\right).$$

Case II: Trigonometric function solution (when $AB > 0$):

$$\phi(\xi) = \ln\left(\sqrt{\frac{A}{B}}\tan(\sqrt{AB}(\xi + C))\right),$$

And

$$\phi(\xi) = \ln\left(-\sqrt{\frac{A}{B}}\cot(\sqrt{AB}(\xi + C))\right).$$

Case III: When $B > 0$ and $A = 0$

$$\phi(\xi) = \ln\left(\frac{1}{-B(\xi + C)}\right),$$

Case IV: When $B = 0$ and $A \in \mathbb{R}$

$$\phi(\xi) = \ln(A(\xi + C)),$$

WhereC is assimilating constants and $AB < 0$ or $AB > 0$ depends on sign of B.

Step-3. On the substitution of Equivalence. (4) into Equivalence. (3) and making use of the Equivalence. (5), bringing together all the similar order of $\exp(\phi(\xi))$, then we acquire a polynomial form of $\exp(\phi(\xi))$. Equating each coefficient of this polynomial to zero, yields a set of algebraic system.

Step-4. Take up the approximation of the constants can be change to by measuring the mathematical terms come to be in step 4. Replacing the approximations of the constants organized with the preparations of Equivalence. (5), we will get new and far-reaching precise traveling wave arrangements of the nonlinear development Equivalence. (1).

## 3. Governing model

### 3.1. The ba model

Recently, Biswas and Arshed [36] proposed a model with Kerr law nonlinearity, namely Biswas and Arshed (BA) model [20-26] is given as

$$iq_t + a_1 q_{xx} + a_2 q_{xt} + i(b_1 q_{xxx} + b_2 q_{xxt})$$

$$= i[\rho(|q|^2 q)_x + T(|q|^2)_x q + \theta|q|^2 q_x]. \quad (6)$$

In Eq. (6), the dependent variable $q(x,t)$ signifies the wave velocity that depends upon spatial $(x)$ and temporal $(t)$ variables. First term portrays temporal evolution. $a_1$ and $a_2$ stand for the coefficient of GVD and spatio-temporal dispersion (STD); $b_1$ and $b_2$ represent third order STD and third order dispersion;$A$ is the effect of self-steepening, $B$ and $\vartheta$are the effect of dispersions.
To start integration process, let

$$q(x,t) = U(\xi)e^{i\eta(x,t)}, \xi = x - vt, \eta(x,t) = -kx + \omega t + N \quad (7)$$

Where$U, v, \eta, k, \omega,$ and N denoted by amplitude portion of the wave, soliton speed, phase component, frequency, wave number and phase constant respectively. Next, put Eq. (7) into Eq. (6), the real part of Eq. (6) has the following form:

$$(a_1 - a_2 v + 3b_1 k - 2b_2 vk - \omega b_2)U''$$

$$-(\omega + a_1 k^2 + b_1 k^3 - a_2 \omega k - b_2 \omega k^2)U = (\rho + \theta)kU^3 \quad (8)$$

And the imaginary part becomes

$$(b_2 vk^2 + 2b_2 \omega k - 3b_1 k^2 - v - 2a_1 k + 2a_2 vk + a_2 \omega)U' + (b_1 - b_2 v)U''' = (3\rho + 2T + \theta)U^2 U' \quad (9)$$

### 3.2. The (3+1) dimensional KP equation

Let us take into account the (3+1) dimensional KP equation [28-33] is in the following form:

$$(U_t + 6UU_x + U_{xxx})_x + 3U_{xx} + 3U_{zz} = 0, \quad (10)$$

The dependent variable $U(x,y,t)$ represents the wave velocity.
Using traveling wave variable $\xi = (\alpha x + \beta y + \gamma z - \omega t)$ to reduce the Eq. (10) becomes

$$\alpha(-\omega U' + 6\alpha UU' + \alpha^3 U''')' + 3(\alpha^2 + \gamma^2)U'' = 0. \quad (11)$$

Eq. (11) is an assimilated equation. Then assimilate two times with the help of $\xi$ and we pursue the assimilating constant to zero. Then, we obtain

$$\alpha^4 U'' + 3\alpha^2 U^2 + (3\alpha^2 + 3\gamma^2 - \alpha\omega)U = 0. \quad (12)$$

Where $U' = \frac{dU}{d\xi}, U'' = \frac{d^2U}{d\xi^2}$.

## 4. Applications

### 4.1. For ba model

In this segment, we applied the advanced $exp(-\phi(\xi))$-expansion method for the Eq. (8) and Eq. (9). Balancing the nonlinear terms and heighest order derivative terms we obtain the balance number $m = 2$ for the Eq. (8) and Eq.
So, the solution of the Eq. (8) and Eq. (9) takes the following form:

$$U(\xi) = A_0 + A_1 \exp(-\phi(\xi)) + A_2 \exp(-\phi(\xi))^2 \quad (13)$$

Differentiating the Eq. (13) with respect to $\xi$ and putting the values of $U, U', U''$ and $U'''$ in Eq. (8) and Eq. (9) and equating the coefficient of $e^{i\phi(\xi)}(i = 0, \pm1, \pm2 \ldots\ldots, \pm m)$ equal to zero.
Solving those systems of equivalences, we obtain the results for real part that is Eq. (8) are as follows:
Set one:

$$v=-\frac{1}{2}\frac{k^3b_1-k^2\omega b_2-6kABb_1+2AB\omega b_2+k^2a_1-k\omega a_2-2ABa_1+\omega}{AB\left(2kb_2+a_2\right)}$$

$$A_0=0, A_1=\pm\sqrt{-\frac{-k^3Ab_1+k^2A\omega b_2-k^2Aa_1+kA\omega a_2-A\omega}{k\rho B+kB\theta}}, A_2=0.$$

Case-I: We get the following hyperbolic solutions for $AB<0$, yields
Family-1:

$$q_{1,2}(x,t)=\pm\frac{\sqrt{-\frac{-k^3Ab_1+k^2A\omega b_2-k^2Aa_1+kA\omega a_2-A\omega}{k\rho B+kB\theta}}}{\sqrt{-\frac{A}{B}}\tanh\left(\sqrt{-AB}\,(\xi+C)\right)}*e^{i\eta}$$

$$q_{3,4}(x,t)=\pm\frac{\sqrt{-\frac{-k^3Ab_1+k^2A\omega b_2-k^2Aa_1+kA\omega a_2-A\omega}{k\rho B+kB\theta}}}{\sqrt{-\frac{A}{B}}\coth\left(\sqrt{-AB}\,(\xi+C)\right)}*e^{i\eta}$$

Where

$\xi=x-vt,$

$$v=-\frac{1}{2}\frac{k^3b_1-k^2\omega b_2-6kABb_1+2AB\omega b_2+k^2a_1-k\omega a_2-2ABa_1+\omega}{AB\left(2kb_2+a_2\right)}$$

And

$\eta(x,t)=-kx+\omega t+N.$

Case-II: we get the following trigonometric solutions for $AB>0$, yields
Family-2:

$$q_{5,6}(x,t)=\pm\frac{\sqrt{-\frac{-k^3Ab_1+k^2A\omega b_2-k^2Aa_1+kA\omega a_2-A\omega}{k\rho B+kB\theta}}}{\sqrt{\frac{A}{B}}\tan\left(\sqrt{AB}\,(\xi+C)\right)}*e^{i\eta}$$

$$q_{7,8}(x,t)=\mp\frac{\sqrt{-\frac{-k^3Ab_1+k^2A\omega b_2-k^2Aa_1+kA\omega a_2-A\omega}{k\rho B+kB\theta}}}{\sqrt{\frac{A}{B}}\cot\left(\sqrt{AB}\,(\xi+C)\right)}*e^{i\eta}$$

Where $\xi=x-vt,$

$$v=-\frac{1}{2}\frac{k^3b_1-k^2\omega b_2-6kABb_1+2AB\omega b_2+k^2a_1-k\omega a_2-2ABa_1+\omega}{AB\left(2kb_2+a_2\right)}$$

And

$\eta(x,t)=-kx+\omega t+N$

Case-III and Case IV: When $A=0$ the calculated value of $A_0$ and $A_2$ are undefined. So, the result cannot be determined. For this reason, this case is discarded. Similarly, when $B=0$ the executing value of $A_0, A_1$ and $A_2$ are undefined. So, they cannot be determined. So, this case also discarded.
Again, we obtain the solutions for imaginary part that is Eq. (9) we get following set
Set one:

$$v=\frac{3k^2b_1-2k\omega b_2-2ABb_1+2ka_1-\omega a_2}{k^2b_1-2ABb_2+2ka_2-1}, A_0=0,$$

$$A_1=\pm\sqrt{-\frac{12k^2b_1b_2-12k\omega b_2^2+12ka_1b_2-12ka_2b_1-6\omega a_2b_2+6b_1}{2Tk^2b_2-4TABb_2+3k^2\rho b_2+k^2\theta b_2-6AB\rho b_2-2AB\theta b_2+4Tka_2+6k\rho a_2+2k\theta a_2-2T-3\rho-\theta}A},$$

$$A_2=0$$

Case-I:We get the following hyperbolic solutions when $AB < 0$,
Family-3:

$$q_{9,10}(x,t) = \pm \frac{\Omega}{\sqrt{-\frac{A}{B}\tanh\left(\sqrt{-AB}(\xi+C)\right)}} * e^{i\eta}$$

$$q_{11,12}(x,t) = \pm \frac{\Omega}{\sqrt{-\frac{A}{B}\coth\left(\sqrt{-AB}(\xi+C)\right)}} * e^{i\eta}$$

Where

$$\Omega = \sqrt{-\frac{12k^2b_1b_2 - 12k\omega b_2^2 + 12ka_1b_2 - 12ka_2b_1 - 6\omega a_2b_2 + 6b_1}{2TK^2b_2 - 4TABb_2 + 3k^2\rho b_2 + k^2\theta b_2 - 6AB\rho b_2 - 2AB\theta b_2 \; 4Tka_2 + 6k\rho a_2 + 2k\theta a_2 - 2T - 3\rho - \theta}A}$$

Where

$\xi = x - vt, v = \frac{3k^2b_1 - 2k\omega b_2 - 2ABb_1 + 2ka_1 - \omega a_2}{k^2b_2 - 2ABb_2 + 2ka_2 - 1}$,

And

$\eta(x,t) = -kx + \omega t + N.$

CCase-II: We get following trigonometric solution when $AB > 0$,
FamilFamily-4:

$$q_{13,14}(x,t) = \pm \frac{\Omega}{\sqrt{\frac{A}{B}\tan\left(\sqrt{AB}(\xi+C)\right)}} * e^{i\eta}$$

$$q_{15,16}(x,t) = \mp \frac{\Omega}{\sqrt{\frac{A}{B}\cot\left(\sqrt{AB}(\xi+C)\right)}} * e^{i\eta}$$

Where

$$\Omega = \sqrt{-\frac{12k^2b_1b_2 - 12k\omega b_2^2 + 12ka_1b_2 - 12ka_2b_1 - 6\omega a_2b_2 + 6b_1}{2Tk^2b_2 - 4TABb_2 + 3k^2\rho b_2 + k^2\theta b_2 - 6AB\rho b_2 - 2AB\theta b_2 + 4Tka_2 + 6k\rho a_2 + 2k\theta a_2 - 2T - 3\rho - \theta}A}$$

$\xi = x - vt, v = \frac{3k^2b_1 - 2k\omega b_2 - 2ABb_1 + 2ka_1 - \omega a_2}{k^2b_2 - 2ABb_2 + 2ka_2 - 1}$,

and

$\eta(x,t) = -kx + \omega t + N.$

Case-III: When $A = 0$ the calculated value of $A_0, A_1$ and $A_2$ are undefined. So, the result cannot be determined. For this reason, this case is discarded.
Case-IV: When $B = 0$ and $A \in \mathbb{R}$
Family-5:

$q_{17,18}(x,t) = \pm \frac{\Omega}{(\xi+C)} \times e^{i\eta}$

Where

$$\Omega = \sqrt{-\frac{12k^2b_1b_2 - 12k\omega b_2^2 + 12ka_1b_2 - 12ka_2b_1 - 6\omega a_2b_2 + 6b_1}{2Tk^2b_2 - 4TABb_2 + 3k^2\rho b_2 + k^2\rho b_2 - 6AB\rho b_2 - 2AB\theta b_2 + 4Tka_2 + 6k\rho a_2 + 2k\theta a_2 - 2T - 3\rho - \theta}A}$$

$\xi = x - vt, v = \frac{3k^2b_1 - 2k\omega b_2 - 2ABb_1 + 2ka_1 - \omega a_2}{k^2b_2 - 2ABb_2 + 2ka_2 - 1}$,

And

$\eta(x,t) = -kx + \omega t + N.$

### 4.2. For (3+1)-dimensional KP equation

In this segment, we apply the advance $exp(-\phi(\xi))$-expansion approah for Eq. (12) and since here the nonlinear term is $U^2$ and the highest order derivative is $U''$. So, the balance number is $m = 2$. So, the solution of the Eq. (12) takes the Eq. (13) and differentiating the Eq. (13) with respect to $\xi$ and putting the values of $U$ and $U''$ in Eq. (12) and equating the coefficient of $e^{i\phi(\xi)}(i = 0, \pm1, \pm2....., \pm m)$ equal to zero.Solving those systems of equations, we obtain the solutions for the Eq. (12) are

Set-1:

$$\alpha = \alpha, \omega = -\frac{-4\alpha^4 AB - 3\alpha^2 - 3\gamma^2}{\alpha}, A_0 = -\frac{2}{3}\alpha^2 AB, A_1 = 0, A_2 = -2\alpha^2 A^2$$

Set-2:

$$\alpha = \alpha, \omega = -\frac{4\alpha^4 AB - 3\alpha^2 - 3\gamma^2}{\alpha}, A_0 = -\frac{2}{3}\alpha^2 AB, A_1 = 0, A_2 = -2\alpha^2 A^2$$

Case-I:We get following hyperbolic solutions when $AB < 0$

Family-6:

$$U_{19}(x,t) = -\frac{2}{3}\alpha^2 AB + \frac{2\alpha^2 AB}{\tanh\left(\sqrt{-AB}(\xi + C)\right)^2}$$

$$U_{20}(x,t) = -\frac{2}{3}\alpha^2 AB + \frac{2\alpha^2 AB}{\coth\left(\sqrt{-AB}(\xi + C)\right)^2}$$

Where

$$\xi = (\alpha x + \beta y + \gamma z - \omega t)$$

And

$$\omega = -\frac{-4\alpha^4 A\mu - 3\alpha^2 - 3\gamma^2}{\alpha}.$$

Family-7:

$$U_{21}(x,t) = -2\alpha^2 AB + \frac{2\alpha^2 AB}{\tanh\left(\sqrt{-AB}(\xi + C)\right)^2}$$

$$U_{22}(x,t) = -2\alpha^2 AB + \frac{2\alpha^2 AB}{\coth\left(\sqrt{-AB}(\xi + C)\right)^2}$$

Where

$$\xi = (\alpha x + \beta y + \gamma z - \omega t)$$

And

$$\omega = -\frac{-4\alpha^4 A\mu - 3\alpha^2 - 3\gamma^2}{\alpha}.$$

Case-II: We get following trigonometric solutions when $AB > 0$,

Family-8:

$$U_{23}(x,t) = -\frac{2}{3}\alpha^2 AB - \frac{2\alpha^2 AB}{\tan\left(\sqrt{AB}(\xi + C)\right)^2}$$

$$U_{24}(x,t) = -\frac{2}{3}\alpha^2 AB - \frac{2\alpha^2 AB}{\cot\left(\sqrt{AB}(\xi + C)\right)^2}$$

Family-9:

$$U_{25}(x,t) = -2\alpha^2 AB - \frac{2\alpha^2 AB}{\tan\left(\sqrt{AB}(\xi + C)\right)^2}$$

$$U_{26}(x,t) = -2\alpha^2 AB - \frac{2\alpha^2 AB}{\cot\left(\sqrt{AB}\,(\xi + C)\right)^2}$$

Case-III and Case IV: When $A = 0$, the calculated value of $A_0, A_1$ and $A_2$ are undefined. So, the result cannot be determined. For this reason, this case is discarded. When $B = 0$, the calculated value of $A_0$ and $A_1$ are undefined. So, the result cannot be determined. For this reason, this case is discarded.

## 5. Physical and graphical explanations

In this segment, we will deliberate the physical interpretation and graphical demonstration of the gained exact and solitary wave result of the (3+1) dimensional KP and BA Model. By applying the advanced $exp(-\phi(\xi))$-expansion method, the (3+1) dimensional KP equation and BA models affords the exact traveling wave solutions. The solutions $q_1, q_2, q_3, q_4, q_9,$ $q_{10}, q_{11}, q_{12}, U_{19}, U_{20}, U_{21}, U_{22}$ are all hyperbolic function solutions.The solutions $q_5, q_6, q_7, q_8, q_{13}, q_{14}, q_{15}, q_{16}, U_{23}$. $U_{25}, U_{26}$ are all trigonometric function results and the rational function solutions being $q_{17}, q_{18}$.

According to the condition $AB < 0$, the soliton solution $U_{19}$ represents the w-shape wave profile for selecting the free parameters $A = -2, B = 3, \alpha = 0.11, C = \sqrt{-3}, z = 0$ within the displacement $-10 \le x, t \le 10$. The 3D plot wave structure of the solution $U_{19}$ depicted in Fig. 1(a). It can be seen that the wave propagates along with $x$- and $t$- axes. Fig. 1(b) represents the 2D line plot of (a) at $x = -2,0,2$ of the $U_{19}$ within displacement $-5 \le t \le 5$.Fig.1(c) represents density plot of $U_{19}$.For inflection point we need to observe concave up and concave down of our desired sketch.by the observation we find that (-3,0.09) at that point sketch show concave up to concave down by definition of inflection point we define that point.

According to the condition $AB < 0$, imaginary form the of solution $U_{20}$ which represents Kink-Shape with $A = -2, B = 3, \alpha = .2, C = \sqrt{-2}, z = 0$ within the displacements $-10 \le x, t \le 10$. Fig. 2(a) represents 3D plot. Fig. 2(b) indicates the 2D line plot of (a) at $x = -2,0,2$ of the $U_{20}$ within displacement $-5 \le t \le 5$ and Fig.2(c) shows density plot.

In the same way the solution $U_{23}$ is a normal form and the figure indicates in normal system which represents in Fig. 3. It indications the Periodic Soliton solution-Shape type exact traveling wave solution $A = 3, B = 2, \alpha = 10, C = 2, z = 0$ within the displacements $-10 \le x \le 10$ and $-10 \le t \le 10$. Fig. 3(a) show 3D plot. Fig. 3(b) shows the 2D line plot of (a) at $x = -2,0,2$ of the $U_{23}$ within displacement $-5 \le t \le 5$ and Fig.3(c) shows density plot.

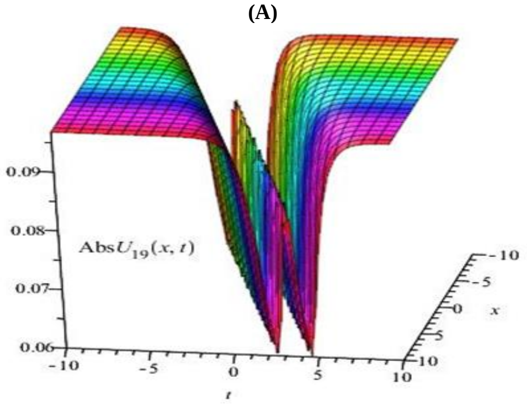
The solution $q_9$ is a complex form and the figure represents an imaginary form which represents in Fig. 4. It spectacles the singular kink-Shape type exact traveling wave solution with $A = -2, B = 3, \omega = 1, b_1 = 1, b_2 = 1, a_1 = 2, a_2 = 1, k = .005, N = 1, C = 1, \theta = 11, p = \sqrt{-2}.2$ within the displacements $-10 \le x, t \le 10$ . Fig. 4(a) show 3D plot. Fig. 4(b) shows the 2D line plot of (a) at $x = -2,0,2$ of the $q_9$ within displacement $-5 \le t \le 5$ and Fig.4(c) shows density plot.

And the solution $q_{11}$ is a complex form and the figure indicates in absolute system which represents in Fig. 5. It shows the Dark Soliton-Shape kind exact traveling wave solution with $A = -2, B = 3, \omega = 3/165, b_1 = 1, b_2 = 1, a_1 = 2, a_2 = 1, k = 1, N = 1, C = 1, \theta = 1$ within the displacements $-10 \le x, t \le 10$ . Fig. 5(a) represents 3D plot. Fig. 5(b) indicates the 2D line plot of (a) at $x = -2,0,2$ of the $q_{11}$ within displacement $-5 \le t \le 5$ and Fig.5(c) indicates density plot.

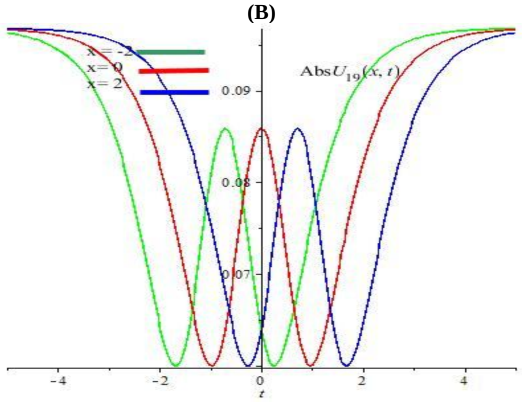
Again the solution $q_{13}$ is a complex form and the figure in imaginary form which represents in Fig. 6. Its expressions the Double periodic-Shape kind exact traveling wave solution with $A = 3, B = 2, \omega = 1, b_1 = 1, b_2 = 1, a_1 = 2, a_2 = 1, k = 1, N = 1, C = 1, \theta = 1, T = 1, p = 1$ within the displacements $-.5 \le x \le 1.5$ and $-.4 \le t \le 1.5$. Fig. 6(a) show 3D plot. Fig. 6(b) indicates the 2D line plot of (a) at $x = -2,0,2$ of the $q_{13}$ within displacement $-5 \le t \le 5$ and Fig.6(c) shows density plot.

Also the solution $q_{17}$ is a complex form and sketch of absolute form and real form represented in Fig. 7 and Fig. 8 respectively. Fig. 7 represent the Combined singular soliton –Shape and Fig. 8 represent rouge kind shape with $A = -2, B = 0, \omega = .003, b_1 = 1, b_2 = 1, a_1 = 2, a_2 = 1, k = 1, N = 1, C = 1, \theta = 1/20, T = 1, p = 2$ within the displacements $-10 \le x, t \le 10$. And also represent 2D line plot for $x = -2,0,2$ within displacement$-5 \le t \le 5$ also represent the density plot.

**(A)**




**(B)**



**(C)**

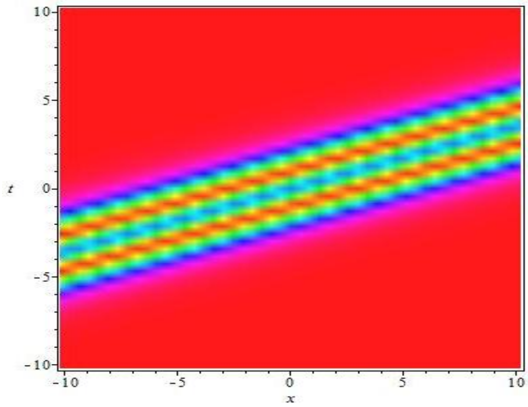
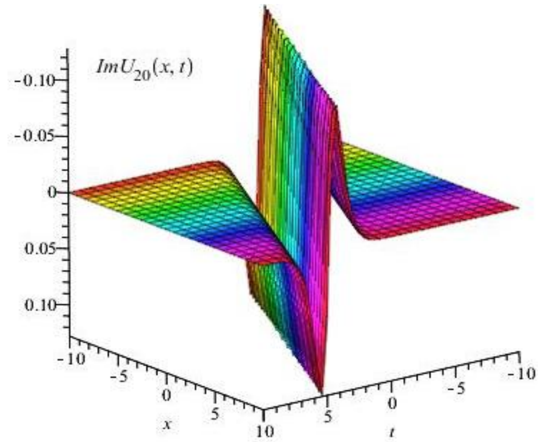
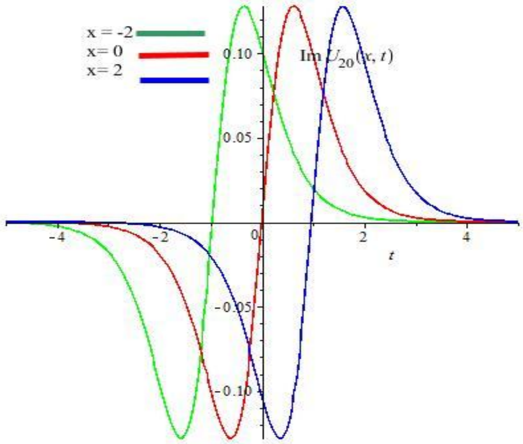


**Fig. 1:** 3D , 2D and Density Wave Structure of the Solution $U_{19}$.

**(A)**




**(B)**




**(C)**

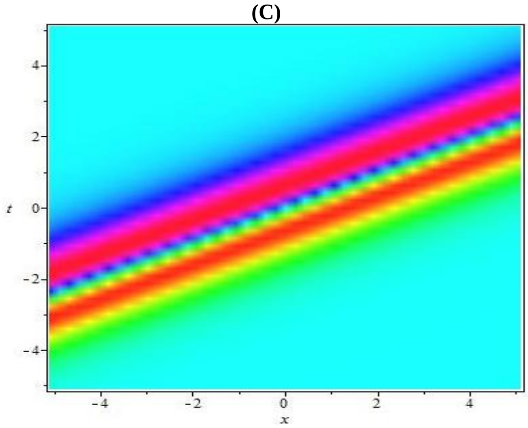
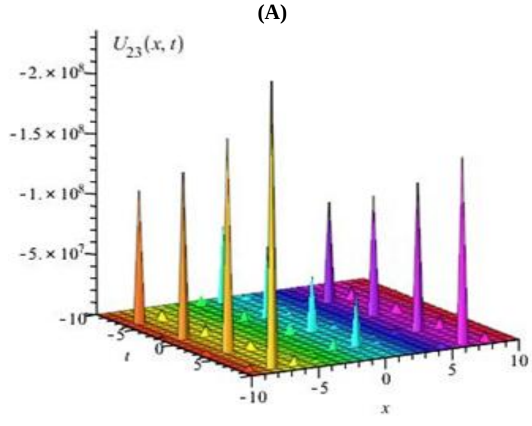
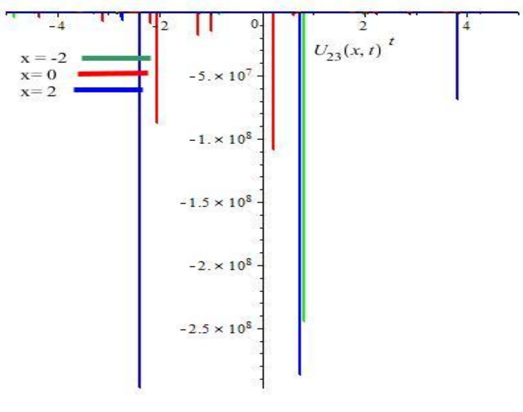


**Fig. 2:** 3D, 2D and Density Structure of the Complex Part of Solution $U_{20}$.

**(A)**




**(B)**



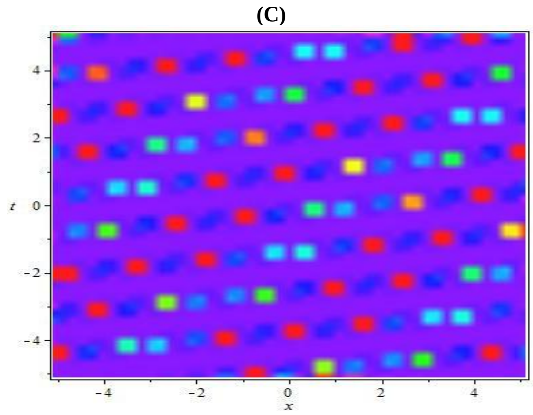



**Fig. 3:** 3D, 2D and Density Wave Structure of the Solution $U_{23}$..

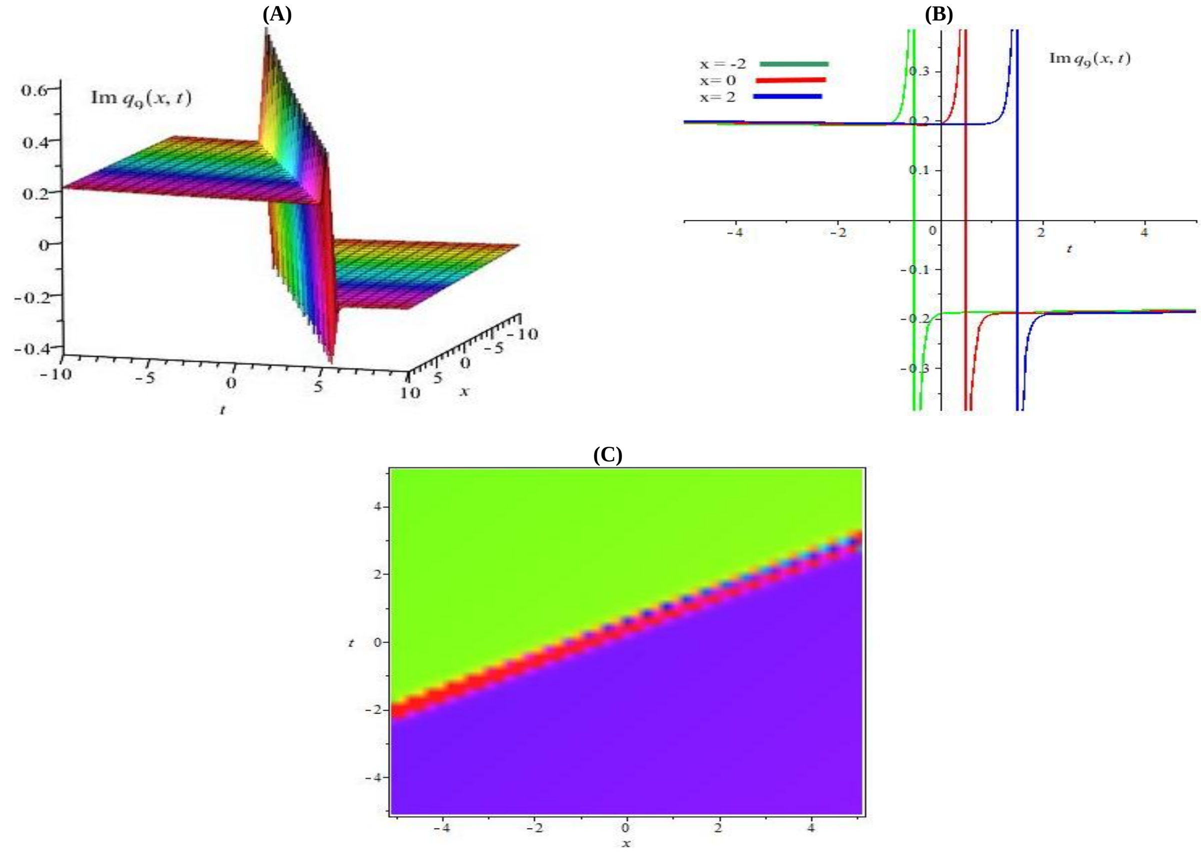
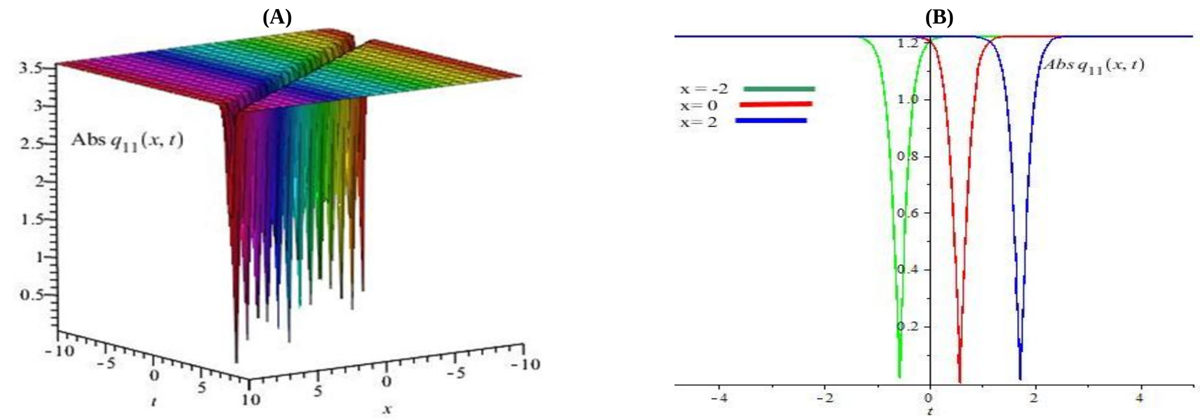



**Fig. 4:** 3D, 2D and Density Structure of the Complex Part of Solution$q_9$.



**(C)**

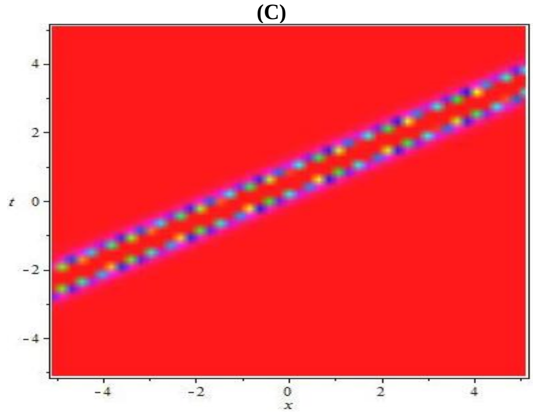
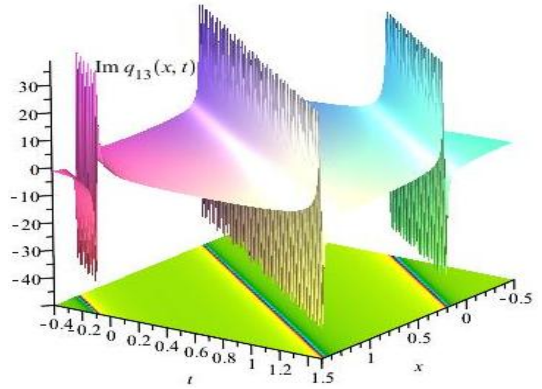
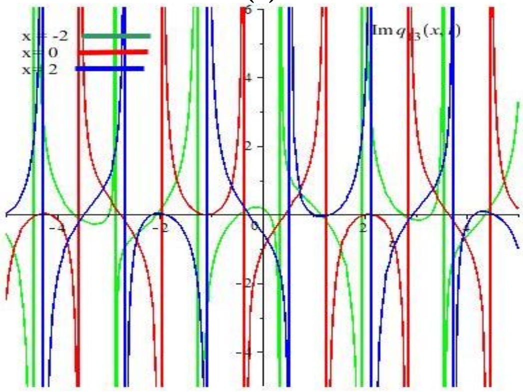



**Fig. 5:** 3D, 2D and Density Wave Structure of the Solution$|q_{11}|$..

**(A)**




**(B)**




**(C)**

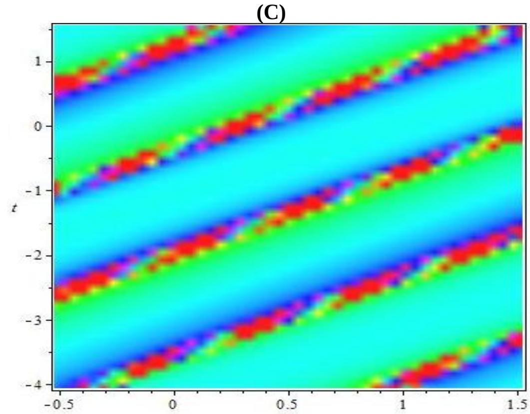
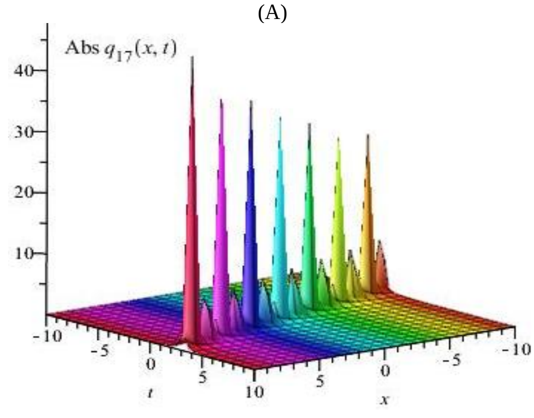
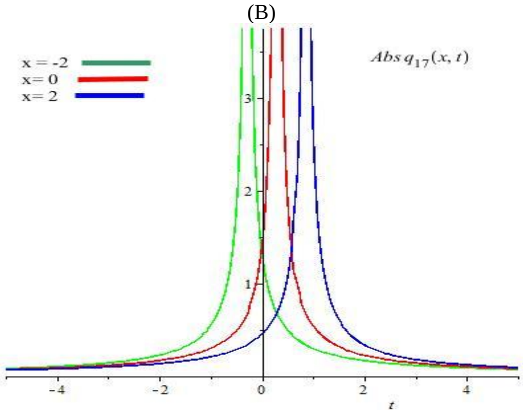



**Fig. 6:** 3D, 2D and Density Structure of the Complex Part of Solution$q_{13}$..

(A)




(B)



(C)

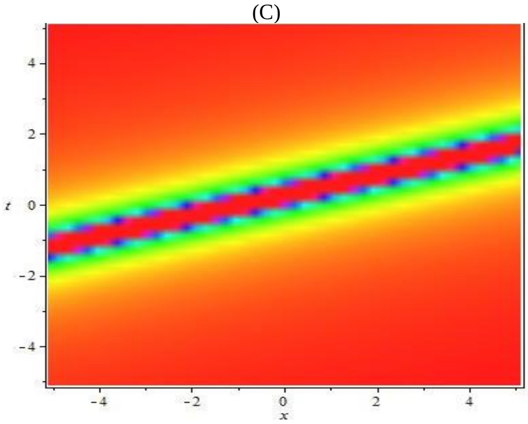
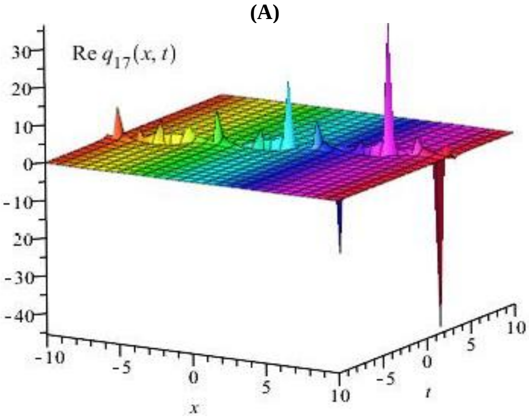
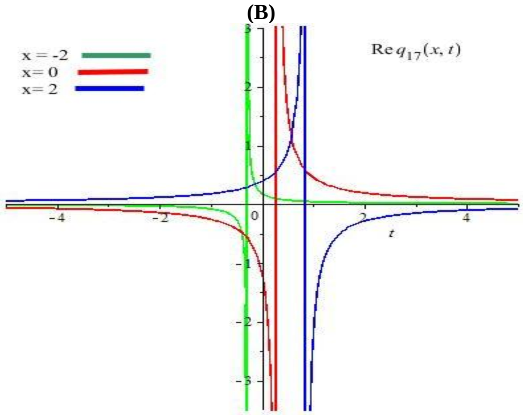


**Fig. 7:** 3D, 2D and Density Wave Structure of the Solution$|q_{17}|$..

**(A)**




**(B)**




**(C)**

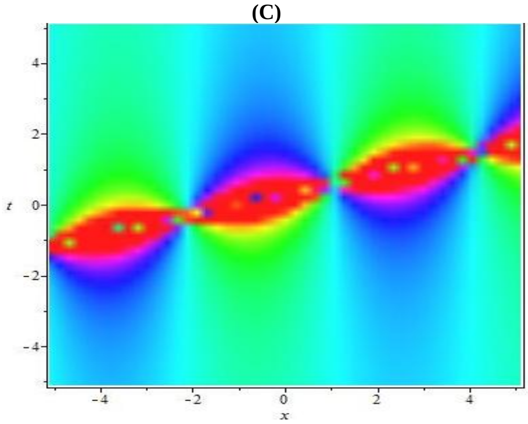


**Fig. 8:** 3D, 2D and Density Wave Structure of the Real Part of Solution $q_{17}$..

## 6. Conclusion

In this work, the advanced $exp(-\phi(\xi))$-expansion method are explored successfully and constructing the significant shape solution with controlling parameters. These solutions are elaborated systematically as well as graphically with 3D, 2D and density plot. Finally, it is found that the advanced $exp(-\phi(\xi))$-expansion method to BAM and KP equation and such typically solutions might be beneficial to analyze and characterize many nonlinear phenomena in nonlinear optic, quantum field theory, solid state physics. This method offers solutions with free parameters that might be important to explain some intricate nonlinear physical phenomena. The obtained solutions in this paper reveal that the method is a very effective and easily applicable.

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