International Journal of Physical Research, *10 (1) (2022) 39-44*

International Journal of Physical Research

Website: www.sciencepubco.com/index.php/IJPR

Research paper

Modified Kudryashov method and its applications to the (2+1)-dimensional cubic Klein-Gordon and (3+1)-dimensional Zakharov-Kuznetsov equations

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Abstract

The exact solutions of nonlinear evolution equations (NLEEs) play a vital role to reveal the central mechanism of complex physical phenomenon. More precisely, in this paper, we acquired new exact solutions to the (2+1)-dimensional cubic Klein–Gordon (cKG) and (3+1)-dimensional Zakharov–Kuznetsov (ZK) equations by using the modified Kudraysov method. As results, a portion of the new accurate voyaging wave answers for the situations above is officially delivered. All arrangements are plotted in the perspective on three-dimensional (3D) and two-dimensional (2D) line shape through the MATLAB programming for exploring the genuine meaning of the concentrated on conditions. The periodic type of solution is created using a modified Kudryashov approach, which is distinct from the other methods investigated.

Keywords: (2+1)-Dimensional Cubic Klein-Gordon Equation; (3+1)-Dimensional Zakharov–Kuznetsov Equation; Modified Kudryashov Method; Traveling Wave Solutions; Symbolic Computation

1. Introduction

Nonlinear phenomena are found in many domains of science and engineering, including fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics, and so on. It is well acknowledged that these complex physical processes can be described using NLEEs. So, study to find exact solutions of NLEEs is very important. A wide collection of researchers is eager to discover effective methods for discovering nonlinear equations analytic and numerical solutions. To find traveling wave solutions to nonlinear scientific phenomena like, the Hirota's bilinear transformation method [1], [2], the tanh-function method [3], the (G′/G)-expansion method [4-12], the Exp-function method [13-16], the homogeneous balance method [17], [18], the F-expansion method [19], the Adomian decomposition method [20], the homotopy perturbation method [21], the extended tanh-method [22], the auxiliary equation method [23], the Jacobi elliptic function method [24], Weierstrass elliptic function method [25], modified Exp-function method [26], the modified simple equation method [27-30], the extended multiple Riccati equations expansion method [31-34] they found many powerful and efficient methods and techniques.
Recently, utilizing computational software packages such as Maple, Mathematica, and MATLAB, researchers were able to readily build and apply these strategies for minimizing computational difficulties. New traveling wave solutions are being investigated, Firstly, we consider the (2+1) dimensional cubic Klein-Gordon equation is given by

$$\psi_{xx} + \psi_{yy} - \psi_{tt} + \alpha\psi + \beta\psi^3 = 0, \tag{1}$$

Where $\psi = \psi(x, y, t)$ and α, β are non zero constants. It's a specific form of nonlinear evolution equation that can be found in a wide range of applications.
Secondly, we consider the (3+1)-dimensional Zakharov–Kuznetsov (ZK) equation is given by

$$\psi_t + a\psi\psi_x + \psi_{xx} + \psi_{yy} + \psi_{zz} = 0, \tag{2}$$

Where $\psi = \psi(x, y, z, t)$.
The major goal of this study is to use an efficient method known as the modified Kudryashov method to generate new accurate traveling wave solutions of the (2+1)-dimensional cubic Klein-Gordon and the (3+1)-dimensional Zakharov–Kuznetsov (ZK) equations [10], [35], [36].

The remainder of this document is organized as follows: The modified Kudryashov method approach is described in Sections 2 and 3. In Section 3, we will use the methods described above to obtain novel accurate traveling wave solutions for the (2+1) dimensional cubic Klein-Gordon equation and the (3+1) dimensional Zakharov–Kuznetsov (ZK) equation. Finally, section 4 concludes with a statement summarizing the findings.

2. Outline of the modified kudryashov method

The modified Kudryashov method is a reliable problem-solving methodology that has gotten a lot of attention in the search for new exact solutions to nonlinear differential equations in mathematical physics. To begin, we will offer a quick overview of the modified Kudryashov method approach [10], which is used to find new exact solutions to a nonlinear partial differential equation. We use a non-linear partial differential equation to achieve this goal

$$F(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \dots) = 0, \tag{3}$$

Where the function $u = u(x,t)$ is unknown and F is a polynomial. The main steps are as follows:
Step 1: Introducing the transformation $u(x,t) = U(\xi)$ where $\xi = kx + \omega t$, varies according to Eq. (3). This reduces to the following non-linear ordinary differential equation

$$P(U, U', U'', \dots) = 0, \tag{4}$$

Where P is a polynomial of U and its derivatives, and the superscripts denote the ordinary derivatives with respect to ξ.
Step 2: Let us assume that the solution $U(\xi)$ of the nonlinear Eq. (4) can be written as

$$U(\xi) = A_0 + \sum_{i=1}^{N} A_i Q^i(\xi)\,, A_N \neq 0, \tag{5}$$

Where A_i $(i = 0,1 \dots N)$ are later determined, N is a positive integer that may be obtained using the balancing principle on Eq. (5), and $Q(\xi)$ satisfies the following new equation

$$Q'(\xi) = \left(Q^2(\xi) - Q(\xi)\right)\ln(A), \tag{6}$$

With the exact solution $Q(\xi) = \frac{1}{1+dA^{\xi}}$, where $A \neq 0,1$.
Step 3: We get a system of algebraic equations in parameters A_i $(i = 0,1 \dots N)$, k, and ω by inserting Eq. (5) and Eq. (6) into Eq. (4) and performing basic mathematical operations. By setting the obtained values in Eq. (5), finally generates new exact solutions for the Eq. (3).

3. Mathematical analysis

3.1. The (2+1)-dimensional cubic Klein-Gordon (cKG) equation

By considering the following transformation:

$$\psi(x,y,t) = U(\xi), \xi = x + y - \lambda t \tag{7}$$

Using Eq. (7) into the (2+1) dimensional cubic Klein-Gordon equation (1), which can be reduced to a nonlinear ordinary differential equation as shown below

$$(2 - \lambda^2)U'' + \alpha U + \beta U^3 = 0 \tag{8}$$

N=1 is found using the homogeneous balancing principle. The answer to Eq. (8) then takes the form

$$U(\xi) = A_0 + A_1 Q(\xi) \tag{9}$$

By substituting Eq. (9) along with its second derivative into Eq. (8), and equating the like powers of $Q(\xi)$, then we get in the following system of equations:

$$-2\,(\lambda^2 - 2)\ln(A)^2 A_1 + \beta A_1^3 = 0,$$

$$3\,(\lambda^2 - 2)\ln(A)^2 A_1 + 3\beta A_0 A_1^2 = 0,$$

$$-(\lambda^2 - 2)\ln(A)^2 A_1 + \alpha A_1 + 3\beta A_0^2 A_1 = 0,$$

$$\beta A_0^3 + \alpha A_0 = 0.$$

By solving the above system, we receive the following different solution sets:

Set–I: $\lambda = \frac{\sqrt{2\ln(A)^2 - 2\alpha}}{\ln(A)}, A_0 = \frac{\sqrt{-\alpha\beta}}{\beta}$, and $A_1 = \frac{2\alpha}{\sqrt{-\alpha\beta}}$.

Set-I possesses the following exact solution of the (2+1) dimensional cubic Klein-Gordon equations extracted:

$$\psi_1(x,y,t) = \frac{\sqrt{-\alpha\beta}}{\beta} + \frac{2\alpha}{\sqrt{-\alpha\beta}\left(1+dA^{-\frac{\sqrt{2\ln(A)^2-2\alpha}\,t}{2\ln(A)}+x+y}\right)} \tag{10}$$

Set–II: $\lambda = \frac{\sqrt{2\ln(A)^2-2\alpha}}{\ln(A)}$, $A_0 = -\frac{\sqrt{-\alpha\beta}}{\beta}$, and $A_1 = -\frac{2\alpha}{\sqrt{-\alpha\beta}}$.

Set-II possesses to the following exact solution of the (2+1) dimensional cubic Klein-Gordon equations determined:

$$\psi_2(x,y,t) = -\frac{\sqrt{-\alpha\beta}}{\beta} - \frac{2\alpha}{\sqrt{-\alpha\beta}\left(1+dA^{-\frac{\sqrt{2\ln(A)^2-2\alpha}\,t}{2\ln(A)}+x+y}\right)} \tag{11}$$

Set–III: $\lambda = -\frac{\sqrt{2\ln(A)^2-2\alpha}}{\ln(A)}$, $A_0 = \frac{\sqrt{-\alpha\beta}}{\beta}$, and $A_1 = \frac{2\alpha}{\sqrt{-\alpha\beta}}$.

Set-II possesses to the following exact solution of the (2+1) dimensional cubic Klein-Gordon equations determined:

$$\psi_3(x,y,t) = \frac{\sqrt{-\alpha\beta}}{\beta} + \frac{2\alpha}{\sqrt{-\alpha\beta}\left(1+dA^{\frac{\sqrt{2\ln(A)^2-2\alpha}\,t}{2\ln(A)}+x+y}\right)} \tag{12}$$

Set–IV: $\lambda = -\frac{\sqrt{2\ln(A)^2-2\alpha}}{\ln(A)}$, $A_0 = -\frac{\sqrt{-\alpha\beta}}{\beta}$, and $A_1 = -\frac{2\alpha}{\sqrt{-\alpha\beta}}$.

Set-II possesses to the following exact solution of the (2+1) dimensional cubic Klein-Gordon equations determined:

$$\psi_4(x,y,t) = -\frac{\sqrt{-\alpha\beta}}{\beta} - \frac{2\alpha}{\sqrt{-\alpha\beta}\left(1+dA^{\frac{\sqrt{2\ln(A)^2-2\alpha}\,t}{2\ln(A)}+x+y}\right)} \tag{13}$$

Fig.1, Fig.2, Fig.3, and Fig.4 show the three-dimensional (3D) and two-dimensional (2D) plots of Eq. (10), Eq. (11), Eq. (12), and Eq. (13) correspondingly. The plots of Eqs. (10), (11), (12), and (13) appear to have a periodic characteristic that is distinct from others.

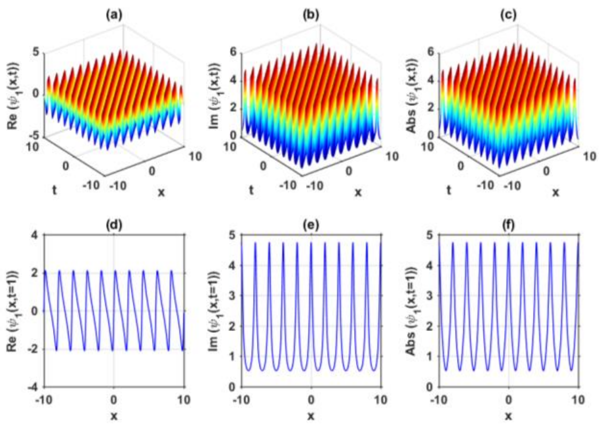

Fig. 1: (a)-(b)-(c) 3D illustrations of the Eq. (10) for the choosing arbitrary parametr $A = -1, \alpha = 5, \beta = 2, y = 0, d = 2$ within $-10 \le x \le 10$, $-10 \le t \le 10$, and (d)-(e)-(f) 2D line illustrations of (a)-(b)-(c) at $t = 1$, respectively.

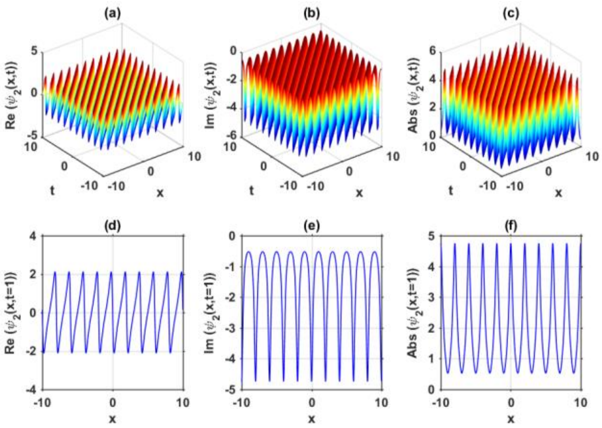

Fig. 2: (a)-(b)-(c) 3D illustrations of the Eq. (11) for the choosing arbitrary parameters $A = -1, \alpha = 5, \beta = 2, y = 0, d = 2$ within $-10 \le x \le 10$, $-10 \le t \le 10$, and (d)-(e)-(f) 2D line illustrations of (a)-(b)-(c) at $t = 1$, respectively.

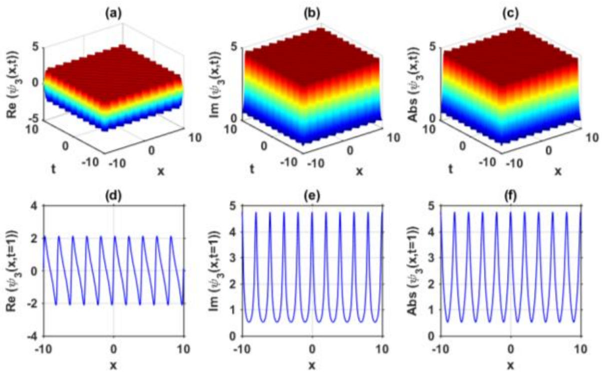

Fig. 3: (a)-(b)-(c) 3D illustrations of the Eq. (12) for the choosing arbitrary parameters $A = -1, \alpha = 5, \beta = 2, y = 0, d = 2$ within $-10 \leq x \leq 10$, $-10 \leq t \leq 10$, and (d)-(e)-(f) 2D line illustrations of (a)-(b)-(c) at $t = 1$, respectively.

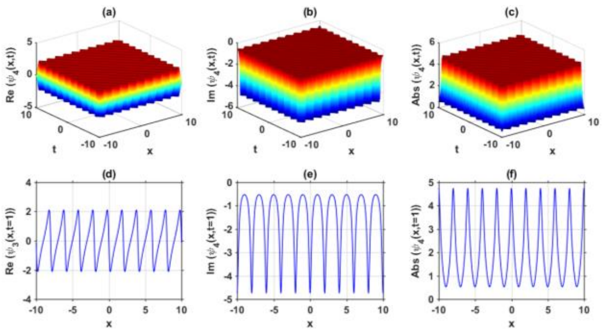

Fig. 4: (a)-(b)-(c) 3D illustrations of the Eq. (13) for the choosing arbitrary parameters $A = -1, \alpha = 5, \beta = 2, y = 0, d = 2$ within $-10 \leq x \leq 10, -10 \leq t \leq 10$, and (d)-(e)-(f) 2D line illustrations of (a)-(b)-(c) at $t = 1$, respectively.

3.2. The (3+1)-dimensional Zakharov –Kuznetsov (ZK) equation

By introducing the following transformation:

$$\psi(x, y, z, t) = U(\xi), \xi = x + y + z - \lambda t \tag{14}$$

By employing the transformation Eq. (14) into the (3+1)-dimensional Zakharov–Kuznetsov (ZK) equation Eq. (2) can be converted to a nonlinear ordinary differential equation as follows

$$-\lambda u + \frac{1}{2} a u^2 + 3u' = 0 \tag{15}$$

N=1 is found using the homogeneous balancing principle. Then, the solution of Eq. (15) takes the form

$$U(\xi) = A_0 + A_1 Q(\xi) \tag{16}$$

By substituting Eq. (16) along with its first derivative into Eq. (15), and equating the like powers of (ξ) , then we get in the following system of equations:

$$\frac{1}{2} A_1\left(a\, A_1 + 6\, ln(A)\right) = 0,$$

$$A_1\left(a\, A_0 - \lambda - 3\, ln(A)\right) = 0,$$

$$\frac{1}{2} A_0 (a\, A_0 - 2\lambda) = 0.$$

By solving the above system, we receive the following different solution sets:

Set–I: $\lambda = -3ln(A), A_0 = 0, A_1 = -\frac{6\, ln(A)}{a}$

Set-I corresponds the following exact solution of the (3+1)-dimensional Zakharov–Kuznetsov (ZK) equations derived:

$$\psi_1(x, y, z, t) = \frac{6 \ln(A)}{a \left(1 + dA^{3\ln(A)t + x + y + z}\right)} \tag{17}$$

Set-II: $\lambda = 3\ln(A), A_0 = \frac{6\ln(A)}{a}, A_1 = -\frac{6\ln(A)}{a}$

Set-II corresponds the following exact solution of of the (3+1)-dimensional Zakharov–Kuznetsov (ZK) equations explored:

$$\psi_2(x,y,z,t) = \frac{6\ln(A)}{a} - \frac{6\ln(A)}{a\left(1+dA^{-3\ln(A)t+x+y+z}\right)} \tag{18}$$

In Fig.5 and Fig.6, the three-dimensional (3D) and two-dimensional (2D) plots of Eq. (17) and Eq. (18) are shown, respectively. The plots appear to have a periodic activity that is distinct from others.

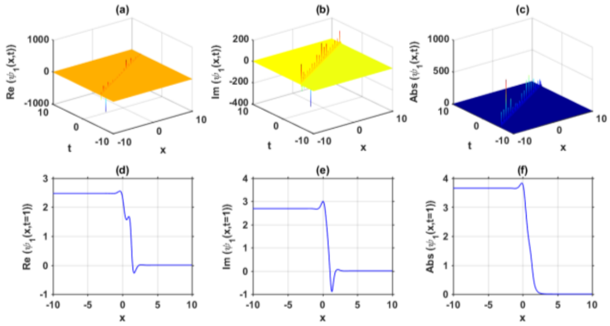

Fig. 5: (a)-(b)-(c) 3D illustrations of the Eq. (18) for the choosing arbitrary parameters $A=-18, a=-7, y=0, z=0, d=8$ within $-10 \le x \le 10, -10 \le t \le 10$, and (d)-(e)-(f) 2D line illustrations of (a)-(b)-(c) at $y=0, z=0, t=1$, respectively.

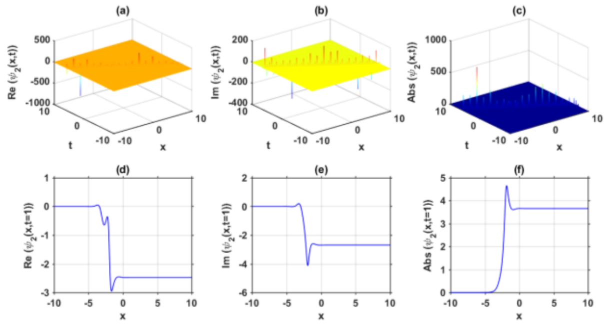

Fig. 6: (a)-(b)-(c) 3D illustrations of the Eq. (17) for the choosing arbitrary parameters $A=-18, a=-7, y=0, z=0, d=8$ within $-10 \le x \le 10, -10 \le t \le 10$, and (d)-(e)-(f) 2D Line Illustrations of (a)-(b)-(c) at $y=0, z=0, t=1$, respectively.

4. Conclusions

The modified Kudraysov method is used to solve the (2+1)-D cKG and (3+1)-D ZK equations in this study as a robust strategy. A number of new accurate traveling wave solutions for the nonlinear equations are explicitly derived as a result of this work. It should be noted that the validity of the results presented in this study is tested by re-entering each new solution into its original equation.

Acknowledgment

The authors would like to express their gratitude to Md. Moiful Alam and Mohammed Khurshed Alam of Uttara University's Department of Electrical and Electronic Engineering for their helpful ideas in the preparation of the manuscript.

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