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Research paper

Rouge wave solutions of a nonlinear pseudo-parabolic physical model through the advance exponential expansion method

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Abstract

In this work, we decide the proliferation of nonlinear voyaging wave answers for the dominant nonlinear pseudo-parabolic physical model through the (1+1)-dimensional Oskolkov equation. With the assistance of the advance -expansion strategy compilation of disguise adaptation an innovative version of interacting analytical solutions regarding, hyperbolic and trigonometric function with some refreshing parameters. We analyze the behavior of these solutions of Oskolkov equations for the specific values of the reared parameters such as rouge wave, multi solution, breather wave bell and kink shape etc. The dynamics nonlinear wave solution is examined and demonstrated in 3-D and 2-D plots with specific values of the perplexing parameters are plotted. The advance -expansion method solid treatment for looking through fundamental nonlinear waves that advance assortment of dynamic models emerges in engineering fields.

Keywords: *Oskolkov Equation; The Advance -Expansion Method; Nonlinear Pseudo-Parabolic Physical Models; Bright and Dark Rouge Wave; Kinky Periodic Wave; Breather Wave.*

1. Introduction

The theoretical examinations of reverberation physical wonders by nonlinear evolution conditions become basic step by step. Since the analytic and explicit traveling wave arrangement of nonlinear evolution equations can be explain different complex wonders in assorted fields of nonlinear science, such as fluid mechanics, nuclear physics, solid-state physics, chemical physics, optical fibre and geochemistry, nonlinear lattices and also in shallow water etc. Numerous researchers arranged through NEEs to build voyaging wave arrangement by actualize a few strategies. The techniques that are entrenched in late writing, for example, extended Kudryashov method [1], Modefied simple equation method [2], New extended (G'/G) expansion method [3], [4], Darboux transformation [5], trial solution method [6], Exp-Function Method [7], Multiple Simplest Equation Method [8]. Nofal applied Simple equation method for nonlinear partial differential equations [10]. Several authors are solved some models by simple equation method [10-13].
Pseudo parabolic model is one kind of partial differential equations in which the time derivative emerged in highest order derivative and they have been misusing for various regions of mathematics and physics for example, for fluid flow in fissured rock, consolidation of clay, shear in second-order fluids, thermodynamics and propagation of long waves of small amplitude These days, much consideration has been paid to examine NEEs, for example, Pseudo parabolic model [14-20]. Note that a totally integrable Pseudo parabolic model gives innovative and explicit different type exact voyaging wave arrangement.
In the present work, we consider the one dimensional Oskolkov condition. Implementing the advance $exp(-\phi(\xi))$ -expansion strategy [21]. We attain the several wave solutions. We utilize numerical recreation to think about the one dimensional Oskolkov condition. We consider (1+1) Dimensional Oskolkov Equation in the accompanying structure

$$U_t - \beta U_{xxt} - \alpha U_{xx} + UU_x = 0. \tag{1}$$

This equation is pseudoparabolic equation and one-dimensional analogue of the oskolkov system

$$(1-\gamma\nabla^2)U_t = \alpha\nabla^2 U - (U \bullet \nabla)U - \nabla^2 p + f, \text{ where } \nabla \bullet U = 0. \tag{2}$$

This system illustrates the dynamics of an incompressible viscoelastic Kelvin-Voigt fluid. It was indicated in [14-20] that the parameter γ can be negative and the negativeness of the parameter γ does not deny the physical meaning of equation (2).

We implemented the advance$exp(-\phi(\xi))$ -expansion strategy to solve equation (1) and obtained new solutions which could not be attained in the past. Mamunur and Bashar found exact and explicit solution from Oskolkov equation with the help of simple equation method

[14], Mamunur applied MSE Schema [15] Faruk applied the tanh-coth strategy for some nonlinear pseudoparabolic conditions to got precise arrangement [16], Turgut Propagation of nonlinear shock waves or the summed up oskolkov condition and its dynamic movements within the sight of an outside intermittent annoyance by actualize unified technique [17] and others creator fathom this model by various predominant strategy [18-20].
The article is set up as pursues: In section 2, the advance $exp(-\phi(\xi))$ -expansion scheme has been talked about. In segment 3, we apply this plan to the nonlinear development conditions raised previously. In section 4, represents Results & Discussion and in section 5, ends are given.

2. The advance $exp(-\phi(\xi))$ -expansion method

In this section, we will precise $exp(-\phi(\xi))$ -expansion method step by step. Consider a nonlinear partial differential equation in the following form,

$$R(U, U_{xx}, U_{xz}, U_{xx} \rightleftarrows, U_{xy}, \rightleftarrows U_{xtt}, \ldots \rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\leftrightarrows\leftrightarrows \cdots) = 0. \quad (3)$$

Where $U = U(x, y, z, t)$ is an unknown function, R is a polynomial of U, its different type partial derivatives, in which the nonlinear terms and the highest order derivatives are involved.
Step-1. Now we consider a transformation variable to convert all independent variable into one variable, such as $U(x, t) = u(\xi)$,

$$\xi = kx + ly + mz \pm Vt. \quad (4)$$

By implementing this variable Eq. (4) permits us reducing Eq. (3) in an ODE for $u(x, t) = u(\xi)$

$$P(u, \leftrightarrows u', \leftrightarrows u'', \cdots \leftrightarrows\leftrightarrows\leftrightarrows \cdots \leftrightarrows\leftrightarrows \cdots) = 0. \quad (5)$$

Step-2. Suppose that the solution of ODE Eq. (5) can be expressed by a polynomial in $exp(-\phi(\xi))$ as follows

$$u = \sum_{i=0}^{m} a_i \, exp(-\phi(\xi))^i \, , a_m \neq 0. \quad (6)$$

where the derivative of $\phi(\xi)$ satisfies the ODE in the following form

$$\phi'(\xi) = -\lambda \, exp(-\phi(\xi)) - \mu \, exp(\phi(\xi)). \quad (7)$$

then the solutions of ODE Eq. (7) are
Case I:
Hyperbolic function solution (when $\lambda\mu < 0$):

$$\phi(\xi) = ln\left(\sqrt{\frac{\lambda}{-\mu}} tanh(\sqrt{-\lambda\mu}(\xi + C))\right).$$

And

$$\phi(\xi) = ln\left(\sqrt{\frac{\lambda}{-\mu}} coth(\sqrt{-\lambda\mu}(\xi + C))\right).$$

Case II:
Trigonometric function solution (when $\lambda\mu > 0$):

$$\phi(\xi) = ln\left(\sqrt{\frac{\lambda}{\mu}} tan(\sqrt{\lambda\mu}(\xi + C))\right).$$

And

$$\phi(\xi) = ln\left(-\sqrt{\frac{\lambda}{\mu}} cot(\sqrt{\lambda\mu}(\xi + C))\right).$$

Case III:
when $\mu > 0$ and $\lambda = 0$

$$\phi(\xi) = ln\left(\frac{1}{-\mu(\xi + C)}\right).$$

Case IV:
When $\mu = 0$ and $\lambda \in \Re$
$\phi(\xi) = ln(\lambda(\xi + C))$, where C is integrating constants and $\lambda\mu < 0$ or $\lambda\mu > 0$ depends on sign of μ.
Step-3. By substituting Eq. (6) into Eq.(5) and using the ODE (7), collecting all same order of $exp(\phi(\xi))$ together, then we execute an polynomial form of $exp(\phi(\xi))$. Equating each coefficients of this polynomial to zero, yields a set of algebraic system.
Step-4. Assume the estimation of the constants can be gotten by fathoming the mathematical conditions got in step 4. Substituting the estimations of the constants together with the arrangements of Eq. (7), we will acquire new and far reaching precise traveling wave arrangements of the nonlinear development Eq. (3).

3. Application of the method

In this section we implement the advance$exp(-\phi(\xi))$ -expansion method for (1+1) dimensional Oskolkov equation in the following form:

$$U_t - \beta U_{xxt} - \alpha U_{xx} + UU_x = 0. \tag{8}$$

Where$\beta, \alpha$are arbitrary constants and$U(x,t)$ is an unknown function. Using the traveling wave variable$U(x,t) = U(\xi)$ and$\xi = kx - \omega t$where$k$is a constant and$\omega$is wave speed. Now we renovate the Eq. (8) into the following Ordinary differential equation.

$$2k^2\omega\beta U'' - 2\alpha k^2 U' - 2\omega U + kU^2 = 0. \tag{9}$$

Where symbolize prime represent the derivative with respect toξ.
Now we compute the balance number of Eq. (9) between the linear term $U''$and the nonlinear termU^2 is$N = 2$. so the solution of the Eq. (9) takes the following form

$$U(\xi) = A_0 + A_1\, exp(-\phi(\xi)) + A_2\, exp(-\phi(\xi))^2. \tag{10}$$

Differential Eq. (10) with respect to$\xi$and substituting the value ofU, U', U'' into the Eq. (9) and equating the coefficients of$e^{i\phi(\xi)}$equal to zero (where$i = 0, \pm 1, \pm 2\ldots\ldots$).
Solving those system of equations, we attain the two sets solutions
Set-1:

$$k = \pm\frac{1}{12}\frac{\sqrt{6}}{\sqrt{\beta\lambda\mu}}, \omega = \pm\frac{1}{10}\frac{\alpha}{\beta\sqrt{-\lambda\mu}}, A_0 = \frac{1}{10}\frac{\alpha\sqrt{6\beta\lambda\mu}}{\beta\sqrt{-\lambda\mu}}, A_1 = \mp\frac{1}{5}\frac{\alpha\sqrt{6\beta\lambda\mu}}{\beta\mu}, A_2 = -\frac{1}{10}\frac{\lambda\alpha\sqrt{6}}{\mu\lambda\sqrt{-\beta}}.$$

Set-2:

$$k = \pm\frac{1}{2\sqrt{-6\beta\lambda\mu}}, \omega = \pm\frac{1}{10}\frac{\alpha}{\sqrt{-\lambda\mu}\beta}, A_0 = \frac{3}{10}\frac{\alpha\sqrt{-6\beta\lambda\mu}}{\sqrt{-\lambda\mu}\beta}, A_1 = \pm\frac{1}{5}\frac{\alpha\sqrt{-6\beta\lambda\mu}}{\beta\mu}, A_2 = -\frac{3}{5}\frac{\lambda\alpha}{\lambda\mu\sqrt{6\beta}}.$$

Case-I: When $\lambda\mu < 0$ we get following hyperbolic solution

Family-1

$$U_{1,2}(x,t) = \frac{1}{10}\frac{\alpha\sqrt{6\beta\mu\lambda}}{\beta\sqrt{-\lambda\mu}} \mp \frac{1}{5}\frac{\alpha\sqrt{\beta\lambda\mu}}{\beta\mu\sqrt{\frac{-\lambda}{\mu}}\, tanh\,(\sqrt{-\lambda\mu}\,(\xi + C)} + \frac{1}{10}\frac{\sqrt{6}\lambda\alpha\mu}{\lambda\mu\sqrt{-\beta}\ tanh(\sqrt{-\lambda\mu}(\xi + C))^2}.$$

$$U_{3,4}(x,t) = \frac{1}{10}\frac{\alpha\sqrt{6\beta\mu\lambda}}{\beta\sqrt{-\lambda\mu}} \mp \frac{1}{5}\frac{\alpha\sqrt{\beta\lambda\mu}}{\beta\mu\sqrt{\frac{-\lambda}{\mu}}\, coth\,(\sqrt{-\lambda\mu}\,(\xi + C)} + \frac{1}{10}\frac{\sqrt{6}\lambda\alpha\mu}{\lambda\mu\sqrt{-\beta}\ coth(\sqrt{-\lambda\mu}(\xi + C))^2}.$$

where,

$$\omega = \pm\frac{1}{10}\frac{\alpha}{\beta\sqrt{-\lambda\mu}} \text{ and } \xi = \pm\frac{1}{12}\frac{\sqrt{6}}{\sqrt{\beta\lambda\mu}}\ x\ \mp\frac{1}{10}\frac{\alpha}{\beta\sqrt{-\lambda\mu}}\ t.$$

Family-2

$$U_{5,6}(x,t) = \frac{3}{10}\frac{\alpha\sqrt{-6\beta\mu\lambda}}{\beta\sqrt{-\lambda\mu}} \pm \frac{1}{5}\frac{\alpha\sqrt{-\beta\lambda\mu}}{\beta\mu\sqrt{\frac{-\lambda}{\mu}}\, tanh\,(\sqrt{-\lambda\mu}\,(\xi + C)} + \frac{3}{5}\frac{\lambda\alpha\mu}{\lambda\mu\sqrt{-6\beta}\ tanh(\sqrt{-\lambda\mu}(\xi + C))^2}.$$

$$U_{7,8}(x,t) = \frac{3}{10}\frac{\alpha\sqrt{-6\beta\mu\lambda}}{\beta\sqrt{-\lambda\mu}} \pm \frac{1}{5}\frac{\alpha\sqrt{-\beta\lambda\mu}}{\beta\mu\sqrt{\frac{-\lambda}{\mu}}\, coth\,(\sqrt{-\lambda\mu}\,(\xi + C)} + \frac{3}{5}\frac{\lambda\alpha\mu}{\lambda\mu\sqrt{-6\beta}\ coth(\sqrt{-\lambda\mu}(\xi + C))^2}.$$

where,

$$\omega = \pm\frac{1}{10}\frac{\alpha}{\sqrt{-\lambda\mu}\beta} \text{ and } \xi == \pm\frac{1}{2\sqrt{-6\beta\lambda\mu}}\ x\ \mp\frac{1}{10}\frac{\alpha}{\sqrt{-\lambda\mu}\beta}\ t.$$

Case-II: When $\lambda\mu > 0$ we get following trigonometric solution

Family-3

$$U_{9,10}(x,t) = \frac{1}{10}\frac{\alpha\sqrt{6\beta\mu\lambda}}{\beta\sqrt{-\lambda\mu}} \mp \frac{1}{5}\frac{\alpha\sqrt{6\beta\lambda\mu}}{\beta\mu\sqrt{\frac{\lambda}{\mu}}\, tan\,(\sqrt{\lambda\mu}\,(\xi + C)} - \frac{1}{10}\frac{\sqrt{6}\lambda\alpha\mu}{\sqrt{\beta\lambda\mu}\sqrt{-\lambda\mu}\ tan(\sqrt{\lambda\mu}(\xi + C))^2}.$$

$$U_{11,12}(x,t) = \frac{1}{10}\frac{\alpha\sqrt{6\beta\mu\lambda}}{\beta\sqrt{-\lambda\mu}} \pm \frac{1}{5}\frac{\alpha\sqrt{6\beta\lambda\mu}}{\beta\mu\sqrt{\frac{\lambda}{\mu}}\,cot\,(\sqrt{\lambda\mu}\,(\xi+C)} - \frac{1}{10}\frac{\sqrt{6}\lambda\alpha\mu}{\sqrt{\beta\lambda\mu}\sqrt{-\lambda\mu}\,cot(\sqrt{\lambda\mu}(\xi+C))^2}.$$

Where,

$$\omega = \pm\frac{1}{10}\frac{\alpha}{\beta\sqrt{-\lambda\mu}} \text{ and } \xi = \pm\frac{1}{12}\frac{\sqrt{6}}{\sqrt{\beta\lambda\mu}}\,x \mp \frac{1}{10}\frac{\alpha}{\beta\sqrt{-\lambda\mu}}\,t.$$

Family-4

$$U_{13,14}(x,t) = \frac{3}{10}\frac{\alpha\sqrt{-6\beta\mu\lambda}}{\beta\sqrt{-\lambda\mu}} \pm \frac{1}{5}\frac{\alpha\sqrt{-6\beta\lambda\mu}}{\beta\mu\sqrt{\frac{\lambda}{\mu}}\,tan\,(\sqrt{\lambda\mu}\,(\xi+C)} - \frac{3}{5}\frac{\lambda\alpha\mu}{\sqrt{-6\beta\lambda\mu}\sqrt{-\lambda\mu}\,tan(\sqrt{\lambda\mu}(\xi+C))^2}.$$

$$U_{15,16}(x,t) = \frac{3}{10}\frac{\alpha\sqrt{-6\beta\mu\lambda}}{\beta\sqrt{-\lambda\mu}} \mp \frac{1}{5}\frac{\alpha\sqrt{-6\beta\lambda\mu}}{\beta\mu\sqrt{\frac{\lambda}{\mu}}\,\cot\,(\sqrt{\lambda\mu}\,(\xi+C)} - \frac{3}{5}\frac{\lambda\alpha\mu}{\sqrt{-6\beta\lambda\mu}\sqrt{-\lambda\mu}\,\cot(\sqrt{\lambda\mu}(\xi+C))^2}.$$

where,

$$\omega = \pm\frac{1}{10}\frac{\alpha}{\sqrt{-\lambda\mu\beta}} \text{ and } \xi == \pm\frac{1}{2\sqrt{-6\beta\lambda\mu}}\,x \mp \frac{1}{10}\frac{\alpha}{\sqrt{-\lambda\mu\beta}}\,t.$$

Case III & Case IV:
When $\lambda = 0$ the executing value of A_0 and A_2 are undefined. So the solution cannot be obtained. For this purpose this case is rejected. Similarly when $\mu = 0$ the executing value of A_0, A_1 and A_2 are undefined. So the solution cannot be obtained. So this case is also rejected.

4. Results and discussions

4.1. Physical explanation

In this subsection, we talk about the physical portrayal of the got exact and solitary wave answers for the (1+1) dimensional Oskolkov condition by means of advance$exp(-\phi(\xi))$-expansion method. There is various type of traveling wave solutions that one of particular interest in solitary wave theory. For some special values of the physical parameters, we obtain the traveling wave solutions as follows:
Figure 1 represents Dark bell shape solution of the imaginary part of$U_{13}$for the parametric values $\mu = 1, \lambda = 0.4, \beta = 1, \alpha = -2$and $C = 1$ within$-10 \le x, t \le 10$. Figure 2 represents Bright bell shape solution of the imaginary part of$U_{13}$for the parametric values $\mu = 1, \lambda = 0.4, \beta = 1, \alpha = 2, C = 1$ within$-10 \le x, t \le 10$. Figure 3 represents Bright kink shape solution of the absolute value of $U_{15}$for the parametric values $\mu = 3, \lambda = 4, \beta = 1, \alpha = -2, C = 1$ within$-10 \le x, t \le 10$. Figure 4 represents Dark kink shape solution of the absolute value of $U_{15}$for the parametric values $\mu = 3, \lambda = 4, \beta = 1, \alpha = 2, C = 1$ within$-10 \le x, t \le 10$. Figure 5 represents Multi Rouge wave shape solution of the real part of $U_{11}$for the parametric values $\mu = 1, \lambda = 2, \beta = 0.2, \alpha = -1, C = 1$ within$-10 \le x, t \le 10$. Figure 6 represents Rouge wave shape solution of the real part of U_1 for the parametric values $\mu = -1, \lambda = \frac{1}{5}, \beta = 10, \alpha = -25, C = 1$ within$-10 \le x \le 10$ and $-6 \le t \le 4$.

4.2. Graphical explanation

This sub-section represents the graphical representation of the (1+1)-dimensional oskolkov equation. By using mathematical software Maple 18, Contour, 3D and 2D plots of some achieved solutions have been shown in Figure 1 to Figure 6 to envisage the essential instrument of the original equations.

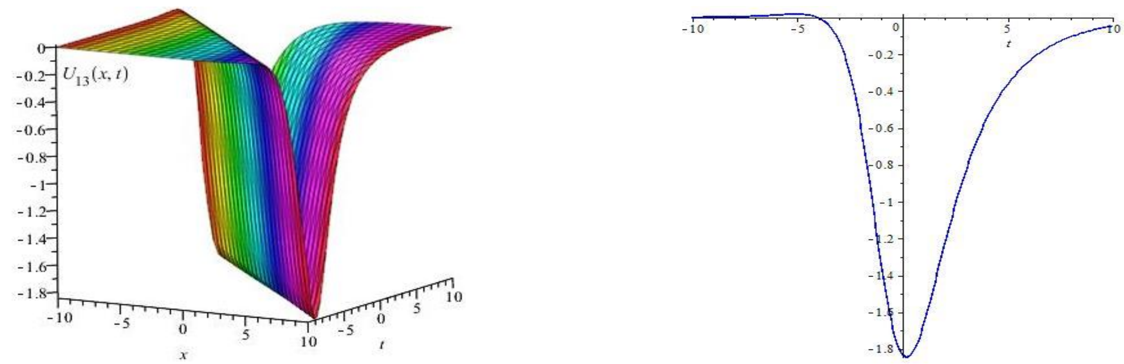

Fig. 1: Dark Bell Shape Solution of U_{13}. the Left Figure Shows the 3D Plot and the Right Figure Shows the 2D Plot For$x = 1$.

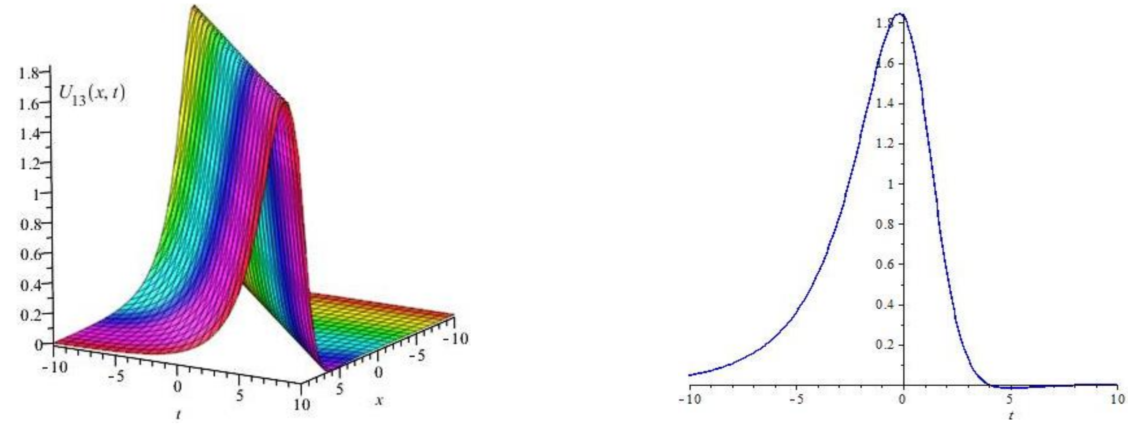

Fig. 2: Bright Bell Shape Solution of U_{13}. the Left Figure Shows the 3D Plot and the Right Figure Shows the 2D Plot For$x = 1$.

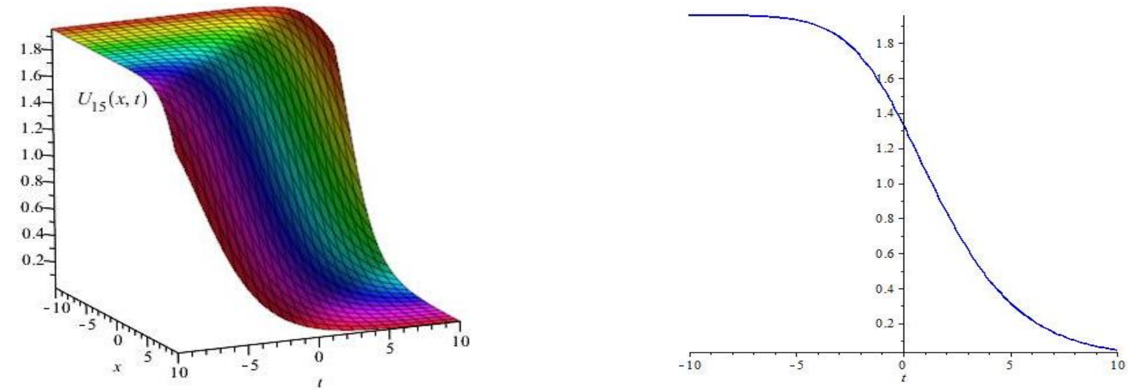

Fig. 3: Bright Kink Shape Solution of U_{15}. the Left Figure Shows the 3D Plot and the Right Figure Shows the 2D Plot For$x = 1$.

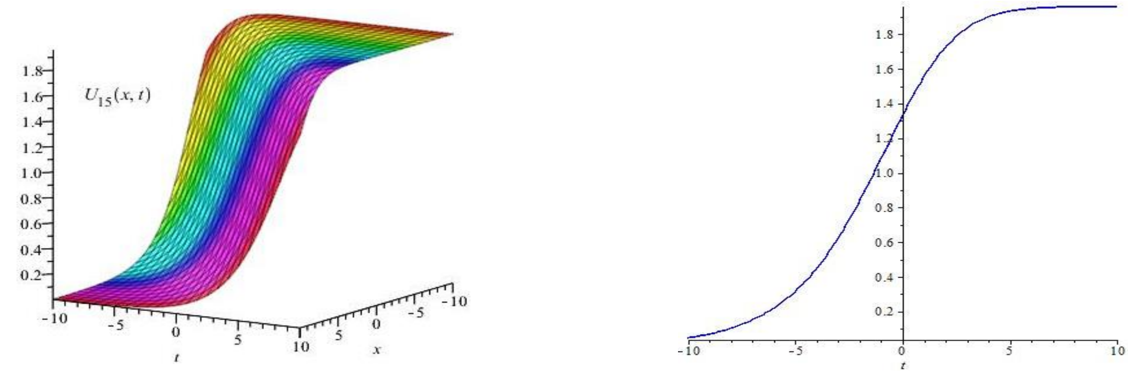

Fig. 4: Dark Kink Shape Solution of U_{15}. the Left Figure Shows the 3D Plot and the Right Figure Shows the 2D Plot For$x = 1$.

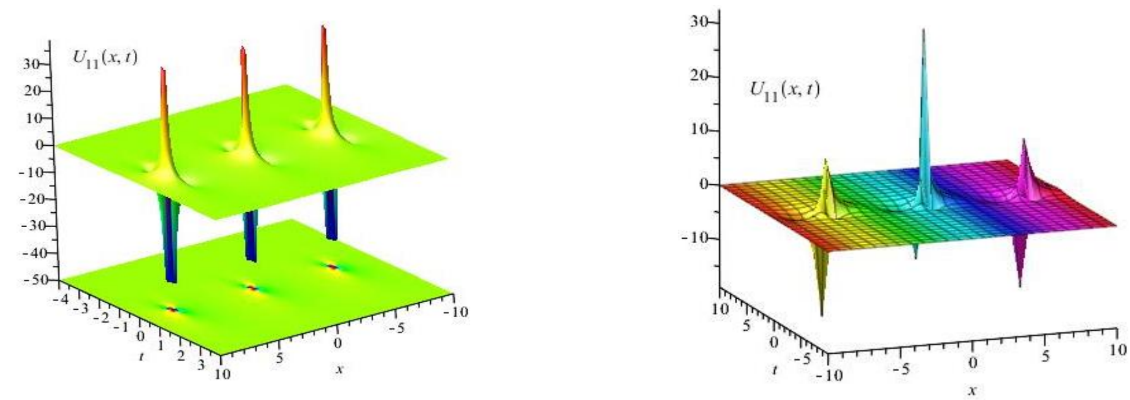

Fig. 5: Multi Rouge Wave Shape Solution of U_{11}. the Left Figure Shows the Contour Plot and the Right Figure Shows the 3D Plot.

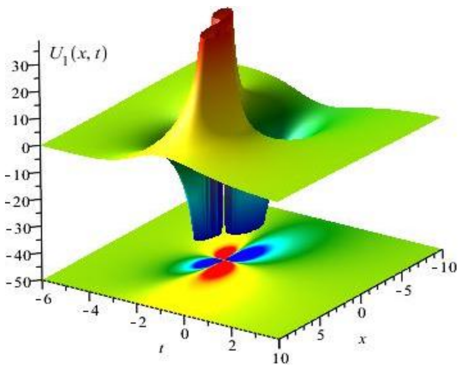

Fig. 6: Rouge Wave Shape Solution of U_1.

5. Conclusion

In this segment, we have seen that two kinds of traveling wave arrangements as far as hyperbolic and trigonometric capacities for the (1+1)-dimensional Oskolkov equation is effectively discovered by utilizing the advance$exp(-\phi(\xi))$ -expansion method. From our outcomes got in this paper, we finish up the advance$exp(-\phi(\xi))$ -expansion method strategy is amazing, powerful and helpful. The exhibition of this technique is dependable, basic and gives numerous new arrangements. As an outcomes, the progressed - extension technique shows a significant method to discover novel voyaging wave arrangements as far as capacity from which we can fabricate exceptionally Rouge wave arrangement, solitary and periodic wave arrangement. The got arrangements in this paper uncover that the technique is a powerful and effectively material of defining more definite voyaging wave arrangements than others strategy for the nonlinear advancement conditions emerging in numerical physical science.

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Competing interests

The authors declare that they have no competing interests.

Author's contributions

All authors contributed equally to this work. All authors read and approved the final manuscript.

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