

# Designing anti-reflection coatings for optical surface

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## Abstract

In this paper, we have analyzed the optical properties of single and double layer antireflection coatings with the help of influential characteristics matrix formulation method in the visible electromagnetic spectrum. We have studied deeply for the glass coated with MgF<sub>2</sub> as a single layer coating and the spectrum analysis of double layer coatings of glass with [MgF<sub>2</sub>-Al<sub>2</sub>O<sub>3</sub>]. With the aid of the algebraic mathematics and MatLab program, we have developed antireflection coating designs which leads to an enhancement of material's ability for transmission spectrum through it. According to the result we have obtained, it clearly seen that the maximum transmission power for single layer and double layer coatings are about 97.2% which occurs at wavelength of 450 nm and 100% occurs at 324 nm, respectively. It is also observed that the minimum value recorded in transmission spectra for double layer antireflection coating is 99.83% at a wavelength of 418 nm (for normal incidence) and 400 nm (for  $\theta=60^\circ$  angle of incident). In broadly speaking, our fundamental work shows double layer antireflection coating is more convenient and feasible than single layer antireflection coating.

**Keywords:** Anti-Reflection; Wavelength; Normal Incidence; Reflectance; Optical Surface.

## 1. Introduction

Anti-reflection coating has been studied for different purpose by different authors [1-8]. Different applications of antireflection coatings have been studied by different scholars in advances of energy conservation. Ulrike Schulz et al. discussed about the antireflection coating for plastics optics [1]. One of the fundamental applications of antireflection coatings is the fabrication of photovoltaic module in which the semiconductor materials has been used. In photovoltaic module or solar cell development, experts pay their attention in to effective energy converting process methods to have versatile energy source through the designed solar cell. Some studies show that until now a day, there need to be an improvement of such materials design for better energy conservation. Feng Zhan et al. analyzed the reflectivity of broad band solar cell. According to their work, it was found that antireflection coating is optimal, convenient and feasible to optical design [6]. In common language, the efficiency of the solar panels will be reduced due to dust materials besides the material's proficiency.

Regular cleaning of solar panels is our daily experience to enhance the amount of solar energy [3-7] delivered by the solar cell for different purposes at home, schools, offices and soon, but in the mean time when we make these actions may leads to scratches on the material that may damage it. Since regular cleaning is not scientific way to enhance our energy source, we need a scientific method in order to overcome such problems in utilization use of energy in more advanced manner. To impend, such non-profitable actions and to make sure that sustainable energy can have, we need to improve the property of the glass surface.

One of the best ways to alter the glass surface property is coating process which leads another very significant property through semiconductor materials. To do this, Anti-reflection coatings is the selectively fit to achieve the desired result. Many authors have studied about anti-reflection coating of a certain materials in some wavelength range. The single anti-reflection coating is one of the cases for material design [9-10].

In this paper, we considered the situation of single and double antireflection coatings. We explore optical nature of the considered system by applying analytical approaches to formulate mathematical expression for its optical characterizations. Basically, the potential transmission and reflectance has been determined in accordance with specific wavelength. The transmittance and reflection spectra for the considered system have been drawn using the licensed MatLab software. Moreover, we have insightfully shown that the angle of incidence effect of the power transmissions and reflection in our analysis of optical properties.

## 2. Model and schematic diagram

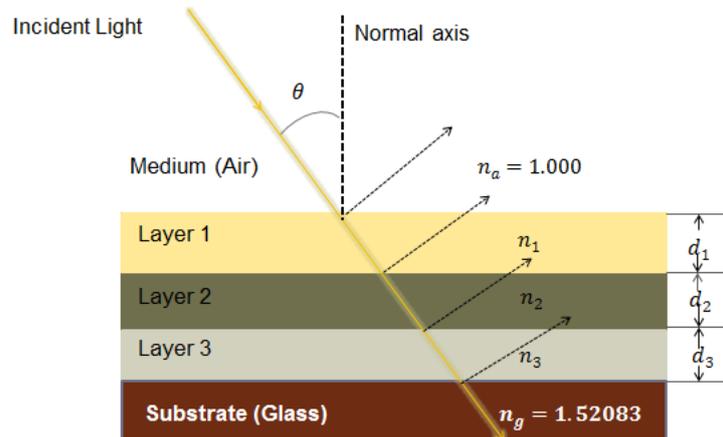
We considered the cascade form of triple anti-reflection coating. The Thickness of the  $j^{\text{th}}$  layer as depicted in figure 1 with  $j = 1, 2, 3$  is determined according to the quarter wave theory. That is given by the relation,  $d_j = \lambda / 4n_j$  in which  $\lambda$  is the wavelength used to generate thickness elements and  $n_j$  is the refractive index of the  $j^{\text{th}}$  layer at a given wavelength. We ignore the extinction coefficients of each sur-

face by considering the system layers are not absorbing materials. By applying optical interference matrices, we can formulate and determine numerical values for reflectance and transmittance of the system.

Let's now start with the characteristics matrix formulation for the system [10-11]. Then the matrix notation that describes the case is written as:

$$\begin{pmatrix} E \\ B \end{pmatrix} = \prod_{j=1}^3 \begin{pmatrix} \cos \sigma_j & \frac{i \sin \sigma_j}{n_j} \\ \frac{i n_j \sin \sigma_j}{\cos \sigma_j} & \cos \sigma_j \end{pmatrix} \begin{pmatrix} n_a \\ n_g \end{pmatrix} \tag{1}$$

Where E and B are the tangential electric and magnetic fields, respectively.



**Fig. 1:** The Schematic Diagram of the System Design in Triple Anti-Coating Representation with the Layer Thickness  $D_1, D_2$  and  $D_3$  . the Corresponding Value for These Optical Thicknesses Will Be Determined for A Matching Materials Based on the Quarter-Quarter-Quarter (-Q-Q-Q) Wavelength Theory

The parameter  $\sigma_j$  is an effective optical thickness of the layer at a wavelength  $\lambda$  and having an expression of

$$\sigma_j = \frac{2\pi n_j d_j \cos \theta}{\lambda} \tag{2}$$

With  $\theta$  is an angle of incidence with respect to the normal axis that has to be perpendicular to the plane of the optical surface. According to Eq. (2), we can employ the effect of the incident angle in the characteristics matrix and later on the optical transmittance and reflectance. In a certain condition to refractive indices and when a light waves are out of phase which leads to the distractive interference, anti-reflection coating is possible when the geometric relations [10]:

$$n_1 = \sqrt{n_a n_2}, \quad n_2 = \sqrt{n_1 n_3} \tag{3}$$

And

$$n_3 = \sqrt{n_2 n_g} \tag{4}$$

In order to know the transmission and reflectance, first we should have to develop the optical admittance of the system based on the fundamental description in relation with the electric and magnetic fields. The optical admittance is then putting as:

$$Y = \frac{E}{B} \tag{5}$$

The reflectivity of the system is again found to be

$$|r|^2 = \left| \frac{n_a - Y}{n_a + Y} \right|^2 \tag{6}$$

At this juncture, we can find the reflectance as:

$$R = |r|^2, \tag{7}$$

To this end, employing Eqs. (6) and (7) the reflectance profile can be rewritten as:

$$R = \left( \frac{n_a - Y}{n_a + Y} \right)^* \left( \frac{n_a - Y}{n_a + Y} \right) \tag{8}$$

Based on this general expression, we are grasping in finding the matching materials to triple antireflection coatings which satisfies the relation given in Eqs. (3) and (4). In this work, we compute the spectral analysis for the optical characteristics of a glass with  $\text{MgF}_2 - \text{Al}_2\text{O}_3$  coatings with thickness equivalent to the Quarter-Quarter-Quarter (Q-Q) wavelength theory. Here, the optical spectra analysis is in a fixed wave length at  $\lambda = 650 \text{ nm}$  with angle of incidence ( $0^\circ \leq \theta \leq 90^\circ$ ). The selection of the material is retracted from the material library data base. Moreover, we generate table1 below at a fixed wavelength and make design specification for  $\text{MgF}_2 - \text{Al}_2\text{O}_3$ .

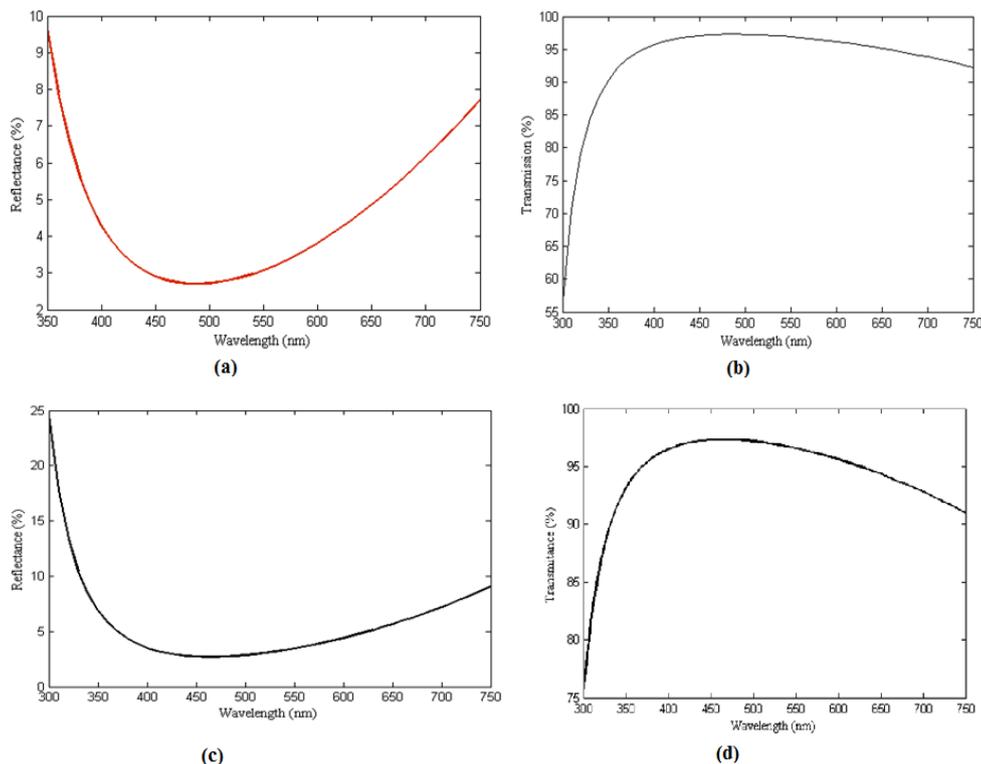
**Table 1:** The Optical Thickness and Refractive Index of the Coating Materials for Wavelength  $\lambda = 650 \text{ Nm}$  Are Obtained Based on the Quarter-Quarter Wavelength Theory with Non-Absorption Condition Applied to the Layers

N <sup>o</sup> .	Design	Material	Refractive index	Extinction coefficient	Coating thickness
1	Medium	Air	1.00000	0.0000	-
	Substrate	Glass	1.52083	0.0000	-
	Medium	Air	1.00000	0.0000	-
2	Layer 1	$\text{MgF}_2$	1.33200	0.0000	121.99699699
	Substrate	Glass	1.52083	0.0000	
	Medium	Air	1.00000	0.0000	
3	Layer 1	$\text{MgF}_2$	1.33200	0.0000	121.99699699
	Layer 2	$\text{Al}_2\text{O}_3$	1.77500	0.0000	
	Substrate	Glass	1.52083	0.0000	

### 3. Result and discussions

The triple coating for the selected materials analysis could be done according to the matrix given in Eq. (2). Manipulation of the results in MatLab, we generate the figures, which describe the optical spectral in the wavelength rage of 350 nm-750 nm along with incorporating the effect of the incident angle varies from 0 to 90 degrees.

It is clearly seen from plots of Figure 2 (a) and (b) that reflectance and transmittance spectra versus the wavelength of the visible region, respectively. As one can notice clearly from these plots, the reflectance of the material is very small in the wavelength rage of 450nm-550nm in normal incidence. In this region of the visible light spectrum, the designed material has efficiency from 97.2% – 97.0% of transmission which means by



**Fig. 2:** Plots of Reflectance (A) and Transmittance (B) of A Single Layered Coating of Glass with  $\text{MgF}_2$  Designed at 650nm by Quarter Wavelength for Normal Incident ( $\theta = 0^\circ$ ). These Figures are Generated by Employing the Matrix Elements of Transformation Characteristics Matrices in the Visible Frequency Range (300nm-750nm) and Based on the Optical Thickness Obtained in Table 1. Plots of Reflectance (C) and Transmittance (D) of A Single Layered Coating of Glass with  $\text{MgF}_2$  Designed at 650nm by Quarter Wavelength for Normal Incident ( $\theta = 60^\circ$ ). These Figures are Generated by Employing the Matrix Elements of Transformation Characteristics Matrices in the Visible Frequency Range (300nm-750nm) and Based on the Optical Thickness Obtained in Table 1.

Coating a glass with a single layer anti-reflective coating material ( $\text{MgF}_2$ ), we can sufficiently minimize the reflectance for 450nm-550nm visible spectra.

To see the effect of angle of incidence, we carefully examined the plots of Figure 2 (a) and (b). Here, the plot was drawn in MatLab by considering angle of incidence to be  $\theta = 60^\circ$ . In this case, some changes were observed. We have found that in the same frequency range of an electromagnetic spectrum as shown in figure 2 (a) and (b), we have seen the effect of angle of incidence on the transmission efficiency of a coated material and how much it reflects an incident light which may related to the desparted power. We have transmission power given as shown in the table below at  $60^\circ$  angle of incidence.

**Table 2:** Transmission and Reflectance for Different Wavelengths at A Fixed Angle of Incidence ( $\theta = 60^\circ$ )

Wavelength (nm)	Transmission (%)	Reflectance (%)
460	97.32	2.68
480	97.27	2.73
510	97.07	2.93
550	94.33	5.67

Now let's analyze what natures will be possessed by double coatings of glass as given by the designs in table 1, design 3. In double coating, the glass (substrate) supposed to be coated with  $\text{MgF}_2 - \text{Al}_2\text{O}_3$ . To do this, the matrix representation given in Eq. (1) could be put as:

$$\begin{pmatrix} E \\ B \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} 1 \\ n_g \end{pmatrix}. \quad (9)$$

Where

$$M_{11} = \cos \sigma_1 \cos \sigma_2 - \frac{n_2}{n_1} \sin \sigma_1 \sin \sigma_2, \quad (10)$$

$$M_{12} = i \left( \frac{1}{n_2} \cos \sigma_1 \sin \sigma_2 + \frac{1}{n_1} \cos \sigma_2 \sin \sigma_1 \right), \quad (11)$$

$$M_{21} = i (n_1 \cos \sigma_2 \sin \sigma_1 + n_2 \cos \sigma_1 \sin \sigma_2), \quad (12)$$

And

$$M_{22} = \cos \sigma_1 \cos \sigma_2 - \frac{n_1}{n_2} \sin \sigma_1 \sin \sigma_2. \quad (13)$$

In which,  $\sigma_j$  ( $j=1,2$ ) is given in Eq. (2). Then the matrix elements in Eq. (9), can be given by

$$E = M_{11} + n_g M_{12}, \quad (14)$$

In which,  $\sigma_j$  ( $j=1,2$ ) is given in Eq. (2). Then the matrix elements in Eq. (9), can be given by

$$E = M_{11} + n_g M_{12}, \quad (14)$$

$$B = M_{21} + n_g M_{22}. \quad (15)$$

Then employing Eqs. (14) and (15) into Eq. (5), one gets the optical admittance of the system

$$Y = \frac{M_{11} + n_g M_{12}}{M_{21} + n_g M_{22}}. \quad (16)$$

Now, inserting Eq.16 into Eq. (8), we the reflectance expression to the system is given by the following powerful equation that governs for the double coated material:

$$R = \left( \frac{n_a - \frac{M_{11} + n_g M_{12}}{M_{21} + n_g M_{22}}}{n_a + \frac{M_{11} + n_g M_{12}}{M_{21} + n_g M_{22}}} \right) \left( \frac{n_a - \frac{M_{11} + n_g M_{12}}{M_{21} + n_g M_{22}}}{n_a + \frac{M_{11} + n_g M_{12}}{M_{21} + n_g M_{22}}} \right)^* \quad (17)$$

This Equation can be put in the form of

$$R = A^2 + B^2 \quad (18)$$

In which the symbols A and B are given by:

$$A = \frac{1 - \frac{1}{(d^2 + n_g^2 c^2)^2} [(ad + n_g^2 bc)^2 + (n_g bd - n_g ca)^2]}{\left( 1 + \frac{ad + n_g^2 bc}{d^2 + n_g^2 c^2} \right)^2 + \left( \frac{n_g bd - n_g ca}{d^2 + n_g^2 c^2} \right)^2} \quad (19)$$

And

$$B = \frac{\frac{2n_g}{(d^2 + n_g^2 c^2)^2} (bd - ca)}{\left(1 + \frac{ad + n_g^2 bc}{d^2 + n_g^2 c^2}\right) + \left(\frac{n_g bd - n_g ca}{d^2 + n_g^2 c^2}\right)^2}. \quad (20)$$

Where

$$a = \cos \sigma_1 \cos \sigma_2 - \frac{n_2}{n_1} \sin \sigma_1 \sin \sigma_2, \quad (21)$$

$$b = \frac{1}{n_2} \cos \sigma_1 \sin \sigma_2 + \frac{1}{n_1} \cos \sigma_2 \sin \sigma_1, \quad (22)$$

$$c = n_1 \cos \sigma_2 \sin \sigma_1 + n_2 \cos \sigma_1 \sin \sigma_2, \quad (23)$$

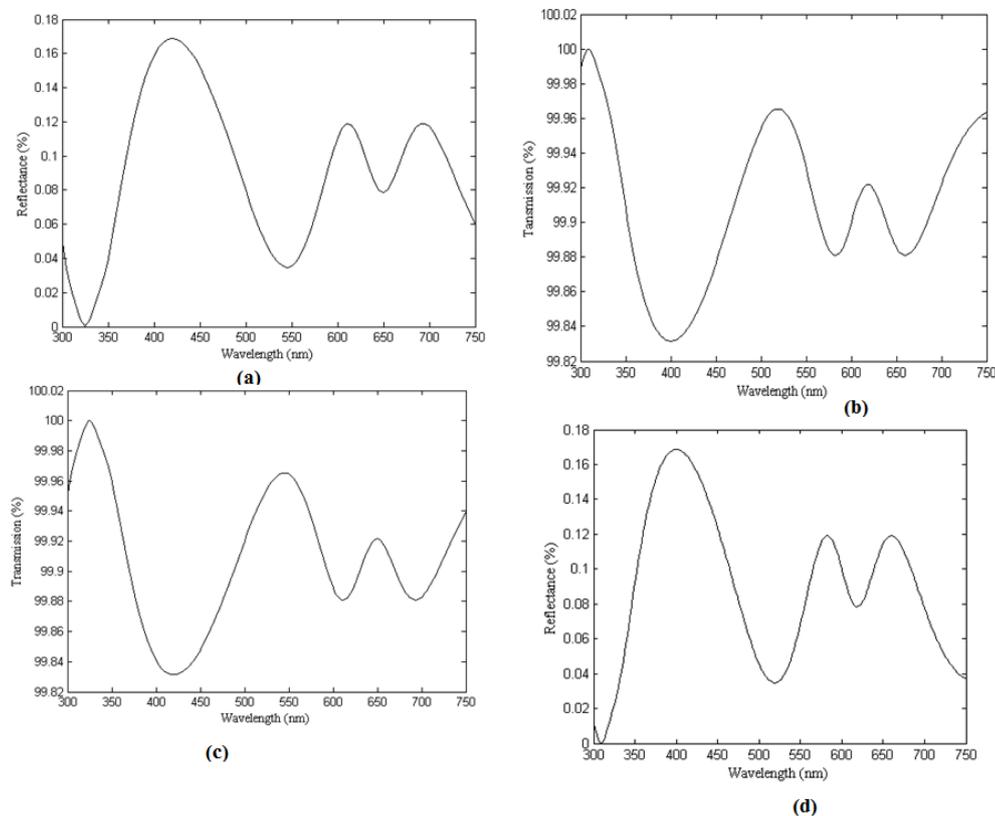
And

$$d = \cos \sigma_1 \cos \sigma_2 - \frac{n_1}{n_2} \sin \sigma_1 \sin \sigma_2. \quad (24)$$

From the plots in figure 4, we noticed that the transmittance and reflectance spectra versus the wavelength of the visible regime. In plot (b), the minimum transmission and corresponding maximum reflection spectrum values are 99.83% and 0.17%, respectively. These values occur at 418 nm for normal incidence ( $\theta = 0^\circ$ ). In the case of the incident wavelength at an angle of  $\theta = 60^\circ$ , this minimum Transmission is occurred at a wavelength of 400nm.

When we see the plots in figure 2 and figure 4, the reflectance is viewed less in figure 4 than its appearance in figure 2. This is clearly showing that the amount of transmission spectrum becomes greater for double coating system (coating of glass surface with  $\text{MgF}_2 - \text{Al}_2\text{O}_3$ ). On the other hand, the reflectance for a single layer coating system (coating of glass surface with  $\text{MgF}_2$ ) greater when it compared with the reflectance tendency observed for double layer coating system as expected. Thus it is advisable to coat a material with double layer coating system rather than single one to enhance the material's ability grasping and harvesting the incident energy to the system which may be crucial for different electrical and optical purpose.

Now, in this investigation of anti-reflection coating for an optical purposes is potentially gets into action as our mathematical calculation and MatLab program to generate the figure in this work. Physically, it is used to for the development of solar science technology. So we have seen that the design is based on the Quarter-Quarter (Q-Q) wavelength theory to have the compatible result in the design.



**Fig. 4:** Plots of Reflectance (A) and Transmittance (B) of A Double Layered Coating of Glass with  $\text{MgF}_2 - \text{Al}_2\text{O}_3$  Designed at 650 Nm by Quarter Wavelength for Normal Incident ( $\theta = 0^\circ$ ). These Figures Are Generated by Employing the Matrix Elements of Transformation Characteristics Matrices in The Visible Frequency Range (300nm-750nm) and Based on the Optical Thickness Obtained in Table 1. Plots of Reflectance (C) and Transmittance (D) of A Single Layered Coating of Glass with  $\text{MgF}_2 - \text{Al}_2\text{O}_3$  Designed at 650nm by Quarter Wavelength for Normal Incident ( $\theta = 60^\circ$ ). These Figures sre Generated by Employing the Matrix Elements of Transformation Characteristics Matrices in the Visible Frequency Range (300nm-750nm) and Based on the Optical Thickness Obtained in Table 1.

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## Conclusion

Antireflection coating design for optical surface has been studied cordially in the visible frequency regime of electromagnetic wave. In this study, we have developed an original equation (Eq. (17)) which enables us to simulate the optical characteristics properties of the material. The paper addresses two issues: The first is single layer antireflection coating of glass with  $\text{MgF}_2$ . In this case, the designed material possesses about 97.2% transmission of the incident light for normal incident. This numerical value is recorded as a maximum value in our study which was observed at a wavelength of 450 nm.

In the second part of the paper, we analyzed the optical properties statistics for doubled layer coated glass with  $\text{MgF}_2 - \text{Al}_2\text{O}_3$  antireflection coating material. In this case, we employed the matrix multiplication of the characteristics matrix to get an equation which enables us to set all the algebraic expressions. Our findings show that about 100% transmission can be occurred at wavelength of 324 nm with normal incident for the second case. In general, our investigation leads us to conclude double layer antireflection coating is more optimal and better for an optical design than that of single layer one.

Finally, we wanted address the case of our initial point. We considered Quarter-Quarter (Q-Q) wavelength theory and non-absorbing optical surfaces for both single and double layer antireflection coatings.

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