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Research Paper

About signals allowing to provide sustainability to impacts of short-term and or focused on spectrum interference

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Abstract

The growth of telecommunication technology application intensity generates the problem of mutual influence concerning the systems which generate them. This influence distorts the signals providing information interaction, which allows us to talk about the mutual interference created by them.
It is proposed to use basic functions in the work in the work to combat short-term interference in comparison with signal duration. One of the function fragments can be calculated from the others. In its turn, one should use basic functions in order to combat narrow-band interference, some fragments of the Fourier spectra of which can be calculated from the others.
It was shown that it is useful to use the eigenfunctions of symmetric operators determined non-negatively as the transmitted signals. Relations are obtained that express certain fragments of signals through others.
In order to combat narrow-band interference, it is proposed to use the intrinsic functions of subband nuclei as basic functions. They obtained the relations, which allow us to recover the distorted fragments of their spectra.
Thus, they developed the theoretical basis to counter short-term and narrow-band interference during the transmission of information.

Keywords: *The Application of Signals in the form of Eigenfunctions of Symmetric Operators Determined Non-Negatively; The Combat with Narrow-Band and / or Short-Duration Interference in Comparison with Signal Durations.*

1. Introduction

A signal is a certain function of time, the parameters of which encode the information intended for certain users (communication signals). Signals are applied widely in telecommunications, telemechanics, the remote control of objects, etc. An observed tendency of information exchange intensity increase between different users has led to the need of some signal distortion problem solution by others, which are called interference in this context. Quite often, distortions are subjected to either separate time fragments of signals or the fragments of their spectra. In the first case, the thing is about the effects of short-lived interference, while in the second one we deal with the interference centered in Fourier spectra. Short-term interference arises with the fading in their transmission channels quite often as compared to signal duration, for example, in the case of multipath propagation of radio waves or repeated reflections in terms of urban development. The appearance of narrow-band interference can also be conditioned by various causes, among which the main place is occupied by electromagnetic radiation sources. It is important that distortions occur unpredictably due to the effects of other signals, although they can be fixed during the transmission / reception of their own signals. Thus, the fight against these disturbances becomes increasingly important. One of the approaches to its solution can be the use of such signals, which allow the restoration of some fragments over the rest part of the signal or some spectrum fragments along the rest ones.
At present, such a problem has not been studied in detail by literature. Therefore, it is not possible to analyze the state of its solution. It can only be noted that one of the approaches to the development of information signals is the use of certain bases [1,2], which allow to create signal-code constructions possessing the required properties. This approach is considered in this paper, which proposes signal generation method, the fragments of which can be reconstructed (computed) by other fragments. A special basis was proposed, which also allows the reconstruction of Fourier spectrum fragments. Theoretical conclusions are illustrated by computational experiment results.
Methods and results
Then let $x(t),\ 0 \le t \le T$, is real-valued continuous time function (a signal) with a bounded Euclidean norm

$$\| x \|^2 = \int_0^T x^2(t)dt < \infty . \tag{1}$$

Let

$$x_1(t) = x(t),\ 0 \le t < T_1;\ x_2(t) = x(t+T_1),\ 0 \le t \le T_2 = T - T_1 . \tag{2}$$

The task is to develop a fairly general approach to a class of signals development that allow to calculate one of the fragments defined in (2) to be calculated in the presence of another one. Such calculations should be performed on the basis of some functionals, which are presented below in a general form

$$x_1(t) = F_1(t, x_2(\tau),\ 0 \le \tau \le T_2), 0 \le t < T_1 , \tag{3}$$

$$x_2(t) = F_2(t, x_1(\tau),\ 0 \le \tau \le T_1), 0 \le t < T_2 . \tag{4}$$

Statement 1. The signals that allow the restoration of some fragments by the rest ones can be formed on the basis of symmetric operator eigenfunctions.
Proof Without the loss of generality, let's consider the construction of such signals and functionals to restore the initial signals from their fragments (3).

Let $A(s,u),\ 0 \le s,u \le T$ is a symmetric kernel

$$A(s,u) = A(u,s),\ 0 \le s,u \le T \tag{5}$$

Of some integral operator that satisfies the conditions of uniform expansion in a series [3]

$$A(s,u) = \sum_{k=1}^{\infty} \lambda_k g_k(s) g_k(u) \tag{6}$$

According to orthonormal eigenfunctions

$$\lambda_k g_k(s) = \int_0^T A(s,u) g_k(u)du,\ 0 \le s \le T;\ k = 1,2,...; \tag{7}$$

$$(g_k, g_i) = \int_0^T g_k(t) g_i(t)dt = 0, i \ne k\ ;. \tag{8}$$

$$\| g_k \|^2 = (g_k, g_k) = 1\ . \tag{9}$$

It is essential for further discussion that the eigenfunctions of the symmetric kernel develop a complete orthonormal basis of the space L_2 [3].
Let's make the following for definiteness

$$x(t) \equiv c g_1(t),\ 0 \le t \le T\ . \tag{10}$$

Here c is some constant.
Then, in accordance with the definition (7), it is not difficult to obtain the following relation

$$\lambda_1 x_1(t) - \int_0^{T_1} A_{11}(t,u) x_1(u)du = y_1(t)\ , \tag{11}$$

Where

$$y_1(t) = \int_0^{T_2} A_{12}(t,u) x_2(u)du\ ; \tag{12}$$

$$A_{11}(s,u) = A(s,u),\ \ 0 \le s,u \le T_1\ ; \tag{13}$$

$$A_{12}(s,u) = A(s,u+T_1),\ \ 0 \le s \le T_1;\ \ 0 \le u \le T_2\ . \tag{14}$$

It is clear that the relation (11) is the Fredholm integral equation of the second kind [4] with respect to the fragment (3). The possibility of its solution calculation is determined by integral kernel properties in (11) and by the value λ_1. In order to complete the proof of the statement, let's develop the representation for a desired signal fragment, which, under certain conditions, permits to perform the corresponding calculations.
It is obvious that the kernel in (11) is also a symmetric one. This allows us to use the representation of the form (6)

$$A_{11}(s,u) = \sum_{k=1}^{\infty} \lambda_{1k} g_{1k}(s) g_{1k}(u), 0 \le s,u \le T_1\ ; \tag{15}$$

$$\lambda_{1k} g_{1k}(s) = \int_0^{T_1} A_{11}(s,u) g_{1k}(u)du,\ 0 \le s \le T_1;\ k = 1,2,...; \tag{16}$$

$$(g_{1k}, g_{1i}) = \int_0^{T_1} g_{1k}(t) g_{1i}(t)dt = 0, i \ne k\ ; \tag{17}$$

$$\| g_{1k} \|^2 = 1\ . \tag{18}$$

It is clear that the signals under consideration in the framework of the theory of spaces L_2 are the elements of lineals [5]. Therefore, due to the completeness of the kernel eigenfunction basis (2.13), the following representations are true:

$$x_1(t) = \sum_{k=1}^{\infty} \alpha_{1k} g_{1k}(t)\ , \tag{19}$$

$$y_1(t) = \sum_{k=1}^{\infty} \beta_{1k} g_{1k}(t)\ , \tag{20}$$

Whose coefficients are the scalar products of the following form

$$\alpha_{1k} = (x_1, g_{1k})\ ;\ \beta_{1k} = (y_1, g_{1k})\ . \tag{21}$$

The substitution of the representations (15), (19) and (20) into the equation (2.11) gives the following identity

$$\sum_{k=1}^{\infty} [\alpha_{1k}(\lambda_1 - \lambda_{1k}) - \beta_{1k}) g_{1k}(t) \equiv 0,\ 0 \le t \le T_1\ , \tag{22}$$

A necessary and a sufficient condition for the achievement of this is the fulfillment of the following equalities

$$\alpha_{1k}(\lambda_1 - \lambda_{1k}) = \beta_{1k},\ k = 1,2,...\ . \tag{23}$$

Suppose that none of kernel eigenvalues (13) coincides with λ_1

$$\lambda_{1k} \ne \lambda_1,\ \forall k = 1,2...\ . \tag{24}$$

Then, taking into account the equalities (2.23), the representation (2.19) takes the following form for the required signal fragment

$$x_1(t) = \sum_{k=1}^{\infty} \beta_{1k} g_{1k}(t) / (\lambda_1 - \lambda_{1k})\ . \tag{25}$$

Thus, the resulting relation determines one of the signal fragments of the form (10) through the other. The proof is complete.
The following is true.
Statement 2 Let the kernel (6) is a positive definite [4] and its eigenvalues are ordered in a descending order

$$\lambda_1 \ge \lambda_2 \ge .... > 0\ , \tag{26}$$

At that the maximum of which λ_1 is a unique one. Then the following inequalities are performed for the eigenvalues of the kernel (15)

$$\lambda_{1k} > 0\ ;\ \lambda_1 \ge \lambda_{1k}, k = 1,....\ , \tag{27}$$

At that the equality sign in the last inequality corresponds to the coincidence of kernel eigenfunctions (6) and (15).
Proof it is easy to show that at any $T_1 < T$ the kernel (13) is determined positively. Indeed, let

$$z(t) = g_{1k}(t),\ 0 \le t \le T_1; z(t) \equiv 0, T_1 < t \le T\ . \tag{28}$$

Then, due to a positive definiteness of an original kernel, it is not difficult to obtain the following inequality

$$d(T_1) = \int_0^T \int_0^T A(t_1,t_2) z(t_1) z(t_2) dt_1 dt_2 = \int_0^{T_1} \int_0^{T_2} A_{11}(t_1,t_2) g_{1k}(t_1) g_{1k}(t_2) dt_1 dt_2 > 0\ .$$

In accordance with the relations (16) and (18), we obtain the following inequalities

$$d(T_1)=\lambda_{1k}>0, \forall k\geq 1, \quad (29)$$

Which is a necessary and a sufficient condition for a kernel positive definiteness (13). On the other hand, the variational inequality is performed for any normalized signal $y(t)$, $0\leq t\leq T$; $\|y\|=1$

$$\lambda_1=\int_0^T\int_0^T A(t_1,t_2)g_1(t_1)g_1(t_2)dt_1dt_2=\max\int_0^T\int_0^T A(t_1,t_2)y(t_1)y(t_2)dt_1dt_2. \quad (30)$$

The comparison of (30) and (29) shows that the inequalities (27) are satisfied. This completes the last statement proof.

Consequence. If the eigenfunctions of the kernels (6) and (15), advising the maximum eigenvalues, do not coincide, then the relation (25) is a correct mathematical representation of a signal fragment of the form (10) with any duration satisfying the inequality $T_1<T$.

Thus, the representation (25) can serve as a theoretical basis to calculate the fragments of signals of the form (10) concerning the remaining fragment. It is clear that the implementation of such calculations requires the discretization of kernel definition domain. In accordance with this, the kernels are approximated by the means of the following matrices

$$A=\{a_{ik}\}=(\Delta t)^2\{A(i\Delta t,k\Delta t)\},\ i,k=1,..,N\ ; \quad (31)$$

$$A_{11}=\{a_{ik}\}, i,k=1,...,M<\mathrm{N} \quad (32)$$

$$A_{12}=(\Delta t)^2\{A(i\Delta t,(M+n)\Delta t)\},\ i=1,...,M;\ n=1,...,N-M\ . \quad (33)$$

In accordance with this, it is advisable to consider the sampled signals

$$x_k=x(k\Delta t),\ k=1,..,N; N\Delta t=T\ ; \quad (34)$$

$$\vec{x}=(x_1,..,x_M,x_{M+1},..,x_N)'=(\vec{x}_1',\vec{x}_2')';\ M\Delta t=T_1\ , \quad (35)$$

Where the prime denotes transposition.

The choice of sampling step is carried out on the basis of specific kernel analysis, ensuring, for example, the properties of positive definiteness for the resulting matrices. It is important that the matrices (31) are symmetric ones, whose eigenvector sets form complete orthonormal bases in the sense of N-dimensional real vector space [6], [7].

The eigenvalues of the matrices (31) and (32) below are indicated by the same symbols as for the kernels they approximate for simplicity, if this does not cause difficulties in their interpretation.

If the signal samples are selected on the basis of the matrix (31) eigenvector corresponding to the maximum eigenvalue

$$\vec{x}=c\vec{q}_1, \quad (36)$$

Then it is not difficult to show that equation (2.11) turns into the matrix relation

$$(\lambda_1 I-A_{11})\vec{x}_1=A_{12}\vec{x}_2. \quad (37)$$

If the eigenvectors of the matrices (31) and (32) corresponding to the maximal eigenvalues do not coincide, then the representation of the equation (37) solution has the following form

$$\vec{x}_1=(\lambda_1 I-A_{11})^{-1}A_{12}\vec{x}_2. \quad (38)$$

The obtained relations can be generalized when multiple ones are present among the eigenvalues of the matrix (31). Without the loss of generality, let's assume that the following equalities are performed

$$\lambda_1=\lambda_2=...=\lambda_M=\lambda. \quad (39)$$

As is well known [6], each of these eigenvalues will correspond to orthogonal eigenvectors. Therefore, any linear combination of the following form

$$\vec{x}=\sum_{m=1}^{M}c_m\vec{q}_m. \quad (40)$$

Will also be the eigenvector of the matrix (31) corresponding to the eigenvalue (39), so that the relation (37) takes the following form

$$(\lambda I-A_{11})\vec{x}_1=A_{12}\vec{x}_2. \quad (41)$$

The variables included here are defined above. In its turn, if the eigenvectors of the matrices (31) and (32) corresponding to the maximal eigenvalues do not coincide, then the representation of the equation (41) solution has the following form

$$\vec{x}_1=(\lambda I-A_{11})^{-1}A_{12}\vec{x}_2. \quad (42)$$

Let's note that it is not difficult to obtain the relations for the reconstruction of any of the fragments occupying a continuous subdomain inside the domain of an original signal determination by analogy. In this case, the definitions of the kernel (13) and the function (12) and their discretized analogs must be changed in an obvious way.

It should only be kept in mind that it is difficult to determine the conditions for the fulfillment of type (24) inequalities by the analogy with Statement 2 in the presence of several identical eigenvalues.

In order to illustrate the proposed approach to the development of signals with recoverable fragments, let us use the computational experiments. As the elements of symmetric matrices for the discrete approximation of the considered reconstruction problem, we use the following ones:

$$a_{ik}=\exp(-4ik/N^2); \quad (43)$$

$$a_{ik}=\cos(4\pi(i-k)/N); \quad (44)$$

$$a_{ik}=\sin(4\pi(i-k)/N)/(\pi(i-k)); \quad (45)$$

$$a_{ik}=\sin(32\pi(i-k)/N)/(\pi(i-k)). \quad (46)$$

Table 1 below shows the results of their eigenvalue calculations at $N=256$, Table 2 - at $N=255$, and Table 3 at $N=252$. The data of the tables indicate that it is possible to reconstruct the fragments of their eigenvectors shorter than 252 samples for the matrix (46), whereas the segments of the eigenvectors of the matrices (43) and (44) can be reconstructed from one sample.

Thus, in order to determine the lengths of the signal fragments available for reconstruction, first of all it is necessary to establish the dependence of the eigenvalues on the durations of the original eigenfunctions (the sizes of the nuclear determination domains) for each of the nuclei.

Table 1: $N=256$

k	1	2	3	4	5	6
(2.43)	9,9421	2,2840	0,4544	0,0819	0,0134	0,0020
(2.44)	128,0000	128,0000	0,0000	0,0000	0,0000	0,0000
(2.45)	0,9999	0,9976	0,9594	0,7216	0,2747	0,0430
(2.46)	1,0000	1.0000	1.0000	0,9994	0,9925	0,9367

Table 2: $N = 255$

k	1	2	3	4	5	6
(2.43)	8,7604	2,0763	0,4498	0,0742	0,0133	0,0018
(2.44)	127,5000	0,1571	0,0000	0,0000	0,0000	0,0000
(2.45)	0,9999	0,9976	0,8298	0,7218	0,0430	0,0097
(2.46)	1,0000	1,0000	1,0000	0,9995	0,9930	0,9403

Table 3: $N = 252$

k	1	2	3	4	5	6
(2.43)	0,0798	0,0131	0,0118	0,0019	0,0004	0,0002
(2.44)	0,0004	0,0001	0,0000	0,0000	0,0000	0,0000
(2.45)	0,3582	0,1708	0,0211	0,0082	0,0016	0,0005
(2.46)	0,9996	0,9944	0,9511	0,7424	0,3417	0,0818

Development of signals based on the eigenfunctions of sub-band nuclei

As an important concretization of the proposed approach to the development of signals that allow one to recover the fragments from the rest ones, let us consider the use of subband nuclei introduced in [8]. The expediency of their use is conditioned in particular to the fact that they are based on the optimization of the procedures for subband analysis and signal synthesis and, as will be shown below, they allow to develop the reconstruction procedures of Fourier spectra certain fragments by the rest ones.

In order to preserve the integrity of the presentation, it seems necessary to give some information here from the theory of subband nuclei.

According to [8], the subband nucleus is determined for the frequency interval (a band)

$$\Omega=[-\Omega_2,-\Omega_1)\cup[\Omega_1,\Omega_2),\ \Omega_2>\Omega_1\geq 0 \tag{47}$$

Based on Fourier transformation

$$A_\Omega(t)=\int_{z\in\Omega}\exp(-jzt)dz\ /\ 2\pi\ , \tag{48}$$

Which results in

$$A_\Omega(t-\tau)=2A_0(t-\tau)\cos(\omega_s(t-\tau),\ A_\Omega(0)=A_0(0)=2D\ /\ \pi\ , \tag{49}$$

Where

$$A_0(t-\tau)=\sin(D(t-\tau))\ /\ \pi(t-\tau)\ ; \tag{50}$$

$$D=(\Omega_2-\Omega_1)\ /\ 2\ ;\ \omega_s=(\Omega_2+\Omega_1)\ /\ 2\ . \tag{51}$$

Let's note that the kernels of the form (50) were considered earlier due to the study of a class of functions with double orthogonality (see, for example, [9], [10]).

It follows from the definition (48) that the subband core (49) is a symmetric and a positive definite one. Therefore, all the conclusions stated in the previous section are valid for it.

Now, let us consider the reconstruction problem of the signal spectra of the form (10) affected by narrowband interference.

On the basis of definitions (7) and (48), it is not difficult to obtain the following representation for the eigenfunctions of a subband nucleus.

$$\lambda_k g_k(t)=\int_{z\in\Omega}G_k(z)\exp(jzt)dz\ /\ 2\pi \tag{52}$$

Through the segment of their Fourier transformations (a transformant, a spectrum)

$$G_k(z)=\int_0^T g_k(t)\exp(-jzt)dt\ . \tag{53}$$

The following relations are true [8]

$$\lambda_k=\int_{z\in\Omega}|G(z)|^2\,dz\ /\ 2\pi\leq 1,\ k=1,... \tag{54}$$

And the following equations are fulfilled with a high accuracy

$$\lambda_k\approx 0.\ k>J\ , \tag{55}$$

Where

$$J=2[TD\ /\ \pi]+4\ . \tag{56}$$

The following is true.

Statement 3 The fragments of kernel eigenfunction spectrum (49) corresponding to nonzero eigenvalues outside the frequency interval (47) can be calculated by the fragments inside it on the basis of the following relation

$$\lambda_k G_k(y)=\int_{z\in\Omega}G_k(z)(1-\exp(-jT(y-z)))\ /\ j2\pi(y-z)dz\ . \tag{57}$$

Proof The relation (57) is obtained by multiplying the left and right part of the representations (52) by $\exp(-jyt)$ and the integration in accordance with the definition (53) with respect to the variable t within $0\leq t\leq T$. It is obvious that the relation (57) determines the method of analytic continuation of eigenfunction spectra from the chosen interval (47) to the entire frequency axis. In other words, it is possible to calculate the fragments of eigenvector spectra outside a selected frequency interval from by a fragment inside it, which completes the proof of the Statement 3.

The problem of spectrum fragments reconstructuring within the selected frequency interval (47) is solved with more difficulties.

Let

$$\hat{G}_k(y)=\exp(jTy\ /\ 2)G_k(y)\ , \tag{58}$$

$$Ty\ /\ 2=u-T\Omega_1\ /\ 2=u-\bar{\Omega}_1\ , \tag{59}$$

$$Tz\ /\ 2=v-T\Omega_1\ /\ 2=v-\bar{\Omega}_1\ , \tag{60}$$

$$F_k(u)=\hat{G}_k(2u/T+\Omega_1)\ ,\ \bar{D}=TD\ . \tag{61}$$

Let's note that the representation (58) in (53) corresponds to the replacement (a simple shift by $-T\ /\ 2$) of integration variable. At that we obtain the following relation from (57)

$$\lambda_k F_k(u)=\int_{-\bar{D}}^{\bar{D}}F_k(v)\sin(u-v)\ /\ (\pi(u-v))dv\ , \tag{62}$$

Which shows that the functions (61) are the eigenfunctions of its integrand kernel.

As was shown in the previous section, the possibility to restore certain fragments of the functions (61) by other fragments is determined by the properties of the kernel eigenvalues in equation (62).

The discretization of definition domain by both variables with the following increment

$$\Delta\bar{D}=\bar{D}\ /\ N\ , \tag{63}$$

Allows to obtain a matrix approximating a core

$$S=\{s_{ik}\},\ i,k=1,...,2N\ , \tag{64}$$

With the following elements

$$s_{ik}=\sin(\Delta\bar{D}(i-k))\ /\ \pi(i-k)\ . \tag{65}$$

Taking into account (63), it is easy to see that the representations (65), (45), (46) are identical. Therefore, the results of the Tables 1, 2 and 3 can also be interpreted within the framework of function fragment recovering task (61). Let's note that the possibilities of this are determined by the magnitude of the product of the following form $\bar{D} = TD$. In particular, the comparison of the data from the Tables 2 and 3 shows that its increase can lead to the dimension decrease of eigenvector recovered segment of the matrix (64).

2. Discussion

The problem of signal generation is considered in the paper. Some of signal fragments can be reconstructed by the others. It is shown that the class of such signals is not empty and can be realized on the basis of the eigenfunctions of symmetric integral operators. Fredholm integral equations of the second kind are obtained, which are satisfied by recoverable fragments and the conditions are set for the existence of their solutions. The computational aspects of recovery are considered. They presented the results of computational experiments illustrating the effect of original nucleus properties (approximating matrices) on the fragment durations available for recovery.

The class of integral operators based on subband nuclei was considered as an important concretization of the proposed approach [8]. These integral operators are the generalization of the operators considered in [9]. The eigenfunctions of these kernels make it possible to generate the signals not only with restorable fragments from other fragments, but also to recover the fragments of their Fourier spectra. The relations are obtained that make it possible to implement such reconstructions.

The relevance of the obtained results is conditioned by the fact that the number of electromagnetic radiation sources is increased significantly at present, which make distorting effect on each other. In particular, control and communication signals, generated by extraneous sources, are subject to the effects of electromagnetic radiation. The approach developed in the article to the development of signals is the contribution to electromagnetic compatibility problem solution during the realization of interaction at a distance.

3. Conclusions

In order to increase the vitality of information transmission systems under the influence of short-term interference, it is proposed to use the signals, some fragments of which can be recovered from the remaining fragments. It is shown that the eigenfunctions of symmetric positive definite operators can be used as such signals. The relations are obtained that allow to perform restoration. They determined the factors that influence the duration of recoverable fragments.

It was shown that the vitality of information transmission systems under the influence of narrow-band noise is enhanced by the use of subband nucleus eigenfunctions as the signals and the relations for the reconstruction of their spectrum fragments are obtained.

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