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Research Paper

To model bolted parts for tolerance analysis using variational model

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Abstract

Mechanical products are usually made by assembling many parts. Among the different type of links, bolts are widely used to join the components of an assembly. In a bolting a clearance exists among the bolt and the holes of the parts to join. This clearance has to be modeled in order to define the possible movements agreed to the joined parts. The model of the clearance takes part to the global model that builds the stack-up functions by accumulating the tolerances applied to the assembly components. Then, the stack-up functions are solved to evaluate the influence of the tolerances assigned to the assembly components on the functional requirements of the assembly product.
The aim of this work is to model the joining between two parts by a planar contact surface and two bolts inside the model that builds and solves the stack-up functions of the tolerance analysis. It adopts the variational solid model. The proposed model uses the simplified hypothesis that each surface maintains its nominal shape, i.e. the effects of the form errors are neglected. The proposed model has been applied to a case study where the holes have dimensional and positional tolerances in order to demonstrate its effectiveness.

Keywords: *Clearance; Joining Processes; Tolerance Analysis; Variational Model.*

1. Introduction

Mechanical products are generally constituted by many components. The dimensional and geometric deviations from the nominal geometry of each component of the assembly have to be limited in order to assure their assembling, their interchangeability and their quality. The choice of the kind and the value of the tolerances to assign to the components or to the whole assembly is a very critical task. They influence the functional requirements of the assembly product. Moreover, the actual global competition pushes industry to produce high precision assembly by low manufacturing costs. However, precision and costs are generally in contrast: to have high precision assembly is needed to have small tolerance ranges, while to have low manufacturing costs is needed to have large tolerance ranges.
Therefore, tolerance analysis becomes an important tool to study how the tolerances assigned to the assembly components influence the functional requirements of the assembly product. It builds and resolves a set of stack-up functions due to the accumulation of both the dimensional and the geometric tolerances assigned to the assembly components and the assembly conditions between the parts. Two models are needed to carry out a tolerance analysis of an assembly: the first one schematizes each surface of each component on which a tolerance is applied to (it is called local model), while the second one builds the stack-up functions by accumulating the effects of the tolerances applied to the components (it is called global model). The global model should consider the joining conditions between the paired parts of the assembly. Some are the global models proposed by the literature where the assembly conditions are modeled [1]-[2], but they are focused on the joints with contact between the paired components. Many doubts remain in dealing with the joints with clearance. Therefore, the commercial Computer Aided Tolerancing (CAT) software, which is based on them, has the same problems and limits [3]-[4].
A typical industrial application of a joint with clearance is bolting. Despite their spread, a bolting is usually dimensioned only on the basis of the load that has to be transferred from a component to the other. The tolerances are

usually assigned on the surfaces involved in the join on the basis of the experience, while their influence on the effective assemblability of the components or on the functional requirements of the whole assembly is rarely taken into account.

The aim of this work is to model the joining between two parts by a planar contact surface and two bolts inside the global model that builds and solves the stack-up functions of the tolerance analysis. This model allows the functional design of a mating, i.e. the choice of the tolerances on the basis of their effect on the effective assemblability of the components and on the functional requirements of the whole assembly. It adopts as global model the variational solid model [5]-[7]. The proposed model uses the simplified hypothesis that each feature maintains its nominal shape, i.e. the effects of the form errors are neglected. The proposed model has been applied to a case study where the holes have dimensional and position tolerances; the results are showed and discussed.

In the following the model of the joint between two parts by a planar contact surface and two bolts inside the global model is deeply discussed (see §2). Then, the model is solved by means of worst case (see §3) and statistical (see §4) approaches. Finally, the model is applied to a case study (see §5).

2. Bolting method using variational model

The variational solid modelling is based on the idea to represent the variability due to the tolerances, imposed on the singular components, by model parameters (or variables). The bolting method aims to model the joining between two plates by a planar contact surface and a pattern of bolts. This work considers a pattern of only two holes, but the proposed model may be easily extended to more than two holes. We suppose that the two considered holes are independent and the tolerances assigned to them are shown in Figure 1.

The tolerances are assigned according to the envelope rule (or Rule # 1) of the ASME standard [8]. Being the holes and the bolts assigned dimensional and positional tolerances, the relative location of the two plates deviates from the nominal. Considering the 2D case and that the two plates have small thickness, the relative location deviation of the second plate as regards to the first plate, due to the tolerances and the clearance among the bolts and the two coupled holes, may be characterized by three contributors Δx, Δy and $\Delta\alpha$ respectively along the X-axis, along the Y-axis and around the Z-axis (see Figure 2). These relative location deviations are also named as small kinematic adjustments [9] and they are the parameters of the proposed assembly model. These are the parameters needed to build the homogeneous transformation matrix that permits to pass from the local Datum Reference Frame (DRF) of the first component, to the local DRF of the second component [9] in order to build the stack-up functions of the assembly.

Assuming the simplified hypothesis that each surface keeps its nominal shape, i.e. the effects of the form tolerances are neglected. The present work defines the mathematical assembly model to relate Δx, Δy and $\Delta\alpha$ to the assigned tolerances. The three contributors are considered independent even if the dependence is limited by the tolerance circular zone (with diameter φ_P) and, therefore, the maximum values of Δx and Δy cannot be contemporarily achieved.

Once defined the bolting model, a worst or a statistical approach may be used to model the stack-up functions of an assembly to solve by the methods of the literature [10]-[12].

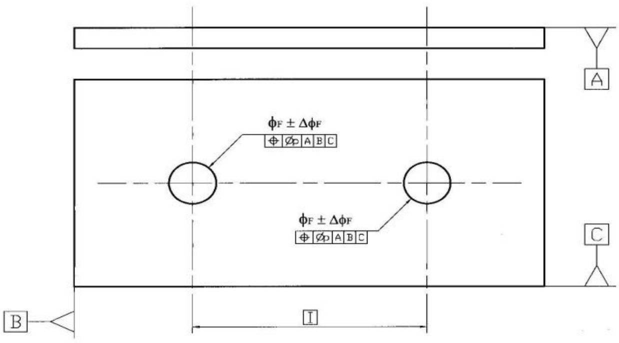

Fig. 1: Part with Two Holes.

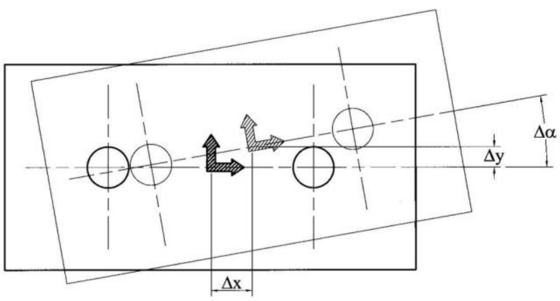

Fig. 2: Location Deviations

3. Worst case approach

The maximum deviation along the X-axis (Δx WC) between the axes of two coupled holes belonging to the two plates is obtained by considering the minimum material condition (i.e. the larger value of the hole's diameter and the smaller value of the bolt's diameter) and by considering the larger or the smaller values of the position deviation along the X-axis. Therefore, the larger value of deviation of the second plate as regards to the first plate along the X-axis is given by (see Figure 3):

$$\Delta x = \varphi_H - \varphi_B + \varphi_P + \Delta\varphi_H + \Delta\varphi_B \tag{1}$$

Where φ_H is the nominal diameter of a hole; φ_B is the nominal diameter of a bolt; φ_P is the position tolerance range of the holes; $\Delta\varphi_H$ is the half of the dimensional tolerance range of a hole and $\Delta\varphi_B$ is the half of the dimensional tolerance range of a bolt. Therefore, the whole admitted deviation of the second plate as regards to the first plate along the x-axis is given by:

$$\Delta x_{WC} = \pm\left(\varphi_H - \varphi_B + \varphi_P + \Delta\varphi_H + \Delta\varphi_B\right) \tag{2}$$

In the same way the maximum deviation along the Y-axis of the second plate as regards to the first plate is given by:

$$\Delta y_{WC} = \Delta x_{WC} \tag{3}$$

To evaluate the maximum deviation around the Z-axis (ΔαWC) between the two couples's plates, it has been considered that the deviations along the Y-axis have opposite values for the two holes as shown in Figure 4. Therefore, it may be calculated by:

$$\Delta\alpha_{WC} \cong \pm 2 \cdot \Delta y_{WC}/I = \pm 2 \cdot \Delta x_{WC}/I \tag{4}$$

Where I is the inter-axis between the holes and the considered angles have small values. It is to observe that to assembly a bolt in each couple of holes belonging to the two coupled plates it is needed that (see Figure 5):

$$I - \varphi_p + (\varphi_H - \Delta\varphi_H) - 2(\varphi_B - \Delta\varphi_B) \geq I + \varphi_p - (\varphi_H - \Delta\varphi_H) \tag{5}$$

And therefore the assembly condition of the worst case scenario is:

$$\varphi_H \geq \varphi_B + \varphi_p + \Delta\varphi_H + \Delta\varphi_B \tag{6}$$

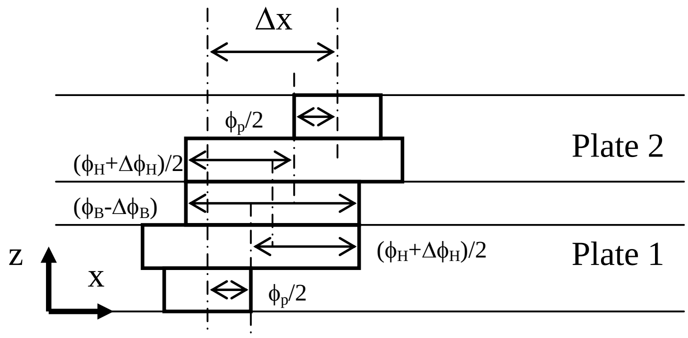

Fig. 3: Maximum Deviation between the Top Surface of the Second Plate and the Bottom Surface of the First Plate along the X-Axis.

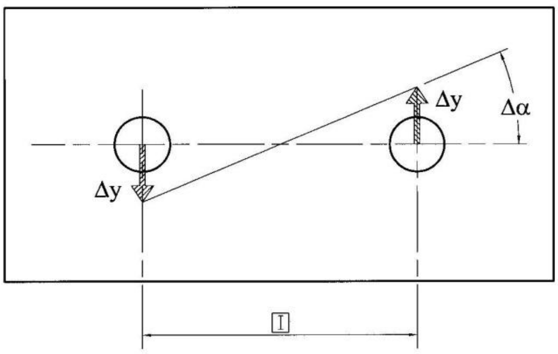

Fig. 4: Maximum Deviation between the Top Surface of the Second Plate and the Bottom Surface of the First Plate around Z-Axis.

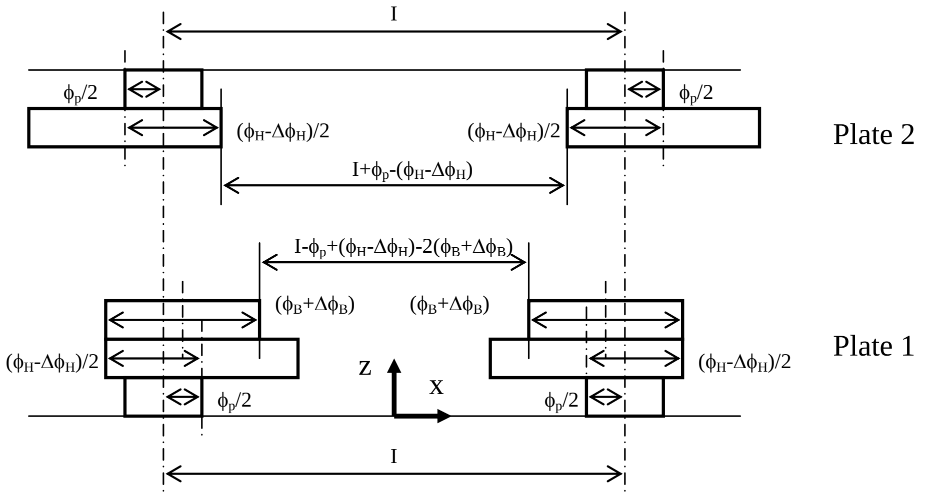

Fig. 5: Assembly Constraint in Worst Case Approach.

4. Statistical approach

The maximum deviations along the X-axis, along the Y-axis and around the Z-axis have been calculated by considering the deviations on each plate due to the applied tolerances, the dimensional tolerance of the bolts and the relative deviations due to the clearance among bolt and holes. The deviations of the two holes of the first plate have been calculated by considering that each hole is assigned dimensional and positional tolerances (see Figure 6):

$$\Delta x_{11} = R_{11}\cos(\theta_{11})$$
$$\Delta y_{11} = R_{11}\sin(\theta_{11})$$
$$\Delta x_{12} = R_{12}\cos(\theta_{12})$$
$$\Delta y_{12} = R_{12}\sin(\theta_{12}) \tag{7}$$

Where R_{11}, R_{12}, θ_{11} and θ_{12} are the model's parameters defining the deviations from the nominal due to the position tolerance of the first and the second hole of the first plate.

Therefore, the inter-axis between the holes moves to:

$$I_1 = \sqrt{\left(I + \Delta x_{12} - \Delta x_{11}\right)^2 + \left(\Delta y_{12} - \Delta y_{11}\right)^2} \tag{8}$$

And the datum reference frame of the first plate, those are centred in the barycentre of the plate and have the axes parallel to the plate's boundary planes, moves to:

$$\Delta x_1 = \frac{\Delta x_{11} + \Delta x_{12}}{2}$$
$$\Delta y_1 = \frac{\Delta y_{11} + \Delta y_{12}}{2} \tag{9}$$
$$\Delta \alpha_1 = \frac{\Delta y_{12} - \Delta y_{11}}{I + \Delta x_{12} - \Delta x_{11}}$$

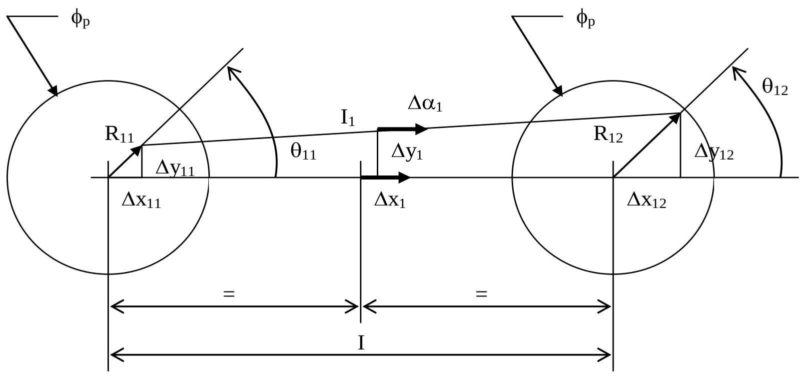

Fig. 6: Deviations of Plate 1 through the Statistical Approach.

To completely define the problem it is needed to assign the probability density functions to the model's parameters R_{11}, R_{12}, θ_{11} and θ_{12}. These probability density functions depend by the drilling process. Therefore, the radii R_{11} and R_{12} have been considered distributed according to a Gaussian probability density function with mean μ equal to zero and standard deviation σ equal to one sixth of the position tolerance range, while the angles θ_{11} and θ_{12} have been considered distributed according to a uniform probability density function inside the range 0°-360°:

$$
\begin{aligned}
&R_{11} = N(0, \varphi_p/6)\\
&R_{12} = N(0, \varphi_p/6)\\
&\theta_{11} = U[0{,}360°]\\
&\theta_{12} = U[0{,}360°]
\end{aligned} \tag{10}
$$

The diameters of the two holes have been considered distributed according to a Gaussian probability density function with mean equal to the nominal value of the diameter of the holes and standard deviation equal to one sixth of the dimensional tolerance range (2 $\Delta\varphi_H$):

$$
\begin{aligned}
&\varphi_{H_{11}} = N(\varphi_H, \Delta\varphi_H/3)\\
&\varphi_{H_{12}} = N(\varphi_H, \Delta\varphi_H/3)
\end{aligned} \tag{11}
$$

The deviations of the two holes of the second plate have been calculated in the same way as the holes of the first plate:

$$
\begin{aligned}
&\Delta x_{21} = R_{21}\cos(\theta_{21})\\
&\Delta y_{21} = R_{21}\sin(\theta_{21})\\
&\Delta x_{22} = R_{22}\cos(\theta_{22})\\
&\Delta y_{22} = R_{22}\sin(\theta_{22})\\
&I_2 = \sqrt{(I + \Delta x_{22} - \Delta x_{21})^2 + (\Delta y_{22} - \Delta y_{21})^2}\\
&\Delta x_2 = \frac{\Delta x_{21} + \Delta x_{22}}{2}\\
&\Delta y_1 = \frac{\Delta y_{21} + \Delta y_{22}}{2}\\
&\Delta\alpha_2 = \frac{\Delta y_{22} - \Delta y_{21}}{I + \Delta x_{22} - \Delta x_{21}}
\end{aligned} \tag{12}
$$

With:

$$
\begin{aligned}
&R_{21} = N(0, \varphi_p/6)\\
&R_{22} = N(0, \varphi_p/6)\\
&\theta_{21} = U[0{,}360°]\\
&\theta_{22} = U[0{,}360°]\\
&\varphi_{H_{21}} = N(\varphi_H, \Delta\varphi_H/3)\\
&\varphi_{H_{22}} = N(\varphi_H, \Delta\varphi_H/3)
\end{aligned} \tag{13}
$$

It is to observe that these contributors are obtained considering the displacements of the real DRF of Part 2 from its nominal position, and not to its nominal position; then, when will be make the algebraic sum of the contributors, these terms have had took with as negative.

The diameter of the bolts (φ Bolt) have been considered distributed according to a Gaussian probability density function with mean equal to the nominal value of the diameter (φ_B) and standard deviation equal to one sixth of the dimensional tolerance range (2 $\Delta\varphi_B$):

$$
\begin{aligned}
&\varphi_{B_1} = N(\varphi_B, \Delta\varphi_B/3)\\
&\varphi_{B_2} = N(\varphi_B, \Delta\varphi_B/3).
\end{aligned} \tag{14}
$$

The existence of a clearance among each couple of holes of the two plates and the corresponding bolt causes a deviation of the second plate as regards to the first plate along the X and Y axes and around the Z-axis. The maximum and minimum values of these deviations have been calculated and accumulated in a stack-up function. The effect of gravity has not been considered, it may involve preferential assembly directions.

The maximum displacement along the positive X-axis of the second plate as regards to the first plate may be calculated as the minimum of the two gaps between each bolt coupled with each hole of the first plate and the corresponding hole of the second plate, as shown in Figure 7:

$$
\Delta x^+ = \min(\Delta x_l^+; \Delta x_r^+) \tag{15}
$$

Where Δx_l^+ and Δx_r^+ are given by:

$$\Delta x_l^+ = \frac{I_2 + \varphi_{H21}}{2} - \frac{I_1 + \varphi_{H11}}{2} - \varphi_{B_1} \tag{16}$$

$$\Delta x_r^+ = \frac{I_1 + \varphi_{H12}}{2} - \frac{I_2 + \varphi_{H22}}{2} - \varphi_{B_2} \,.$$

The hole have been considered independent, while the sign + near to the calculated deviations is due to the fact that the second plate may moves as regards the first one, due to the clearance among the holes, in the same versus of the X-axis. Equation (15) is due to the fact that when one bolt matches the cylindrical surface of a hole, even the other bolt has to stop its moving.

The maximum deviation along the negative x-axis of the second plate as regards to the first plate may be calculated by means of the minimum of the two gaps between each bolt coupled with each hole of the first plate and the corresponding hole of the second plate, as shown in Figure 8:

$$\Delta x^- = -\min(\Delta x_l^-; \Delta x_r^-) \tag{17}$$

Where Δx_l^- and Δx_r^- are given by:

$$\Delta x_l^- = \frac{I_1 + \varphi_{H11}}{2} - \frac{I_2 + \varphi_{H21}}{2} - \varphi_{B_1} \tag{18}$$

$$\Delta x_r^- = \frac{I_2 + \varphi_{H22}}{2} - \frac{I_1 - \varphi_{H12}}{2} - \varphi_{B_2} \,.$$

The sign – in equation (17) is due to the fact the displacement of the second plate as regard to the first place is opposite to the positive versus of the X-axis.

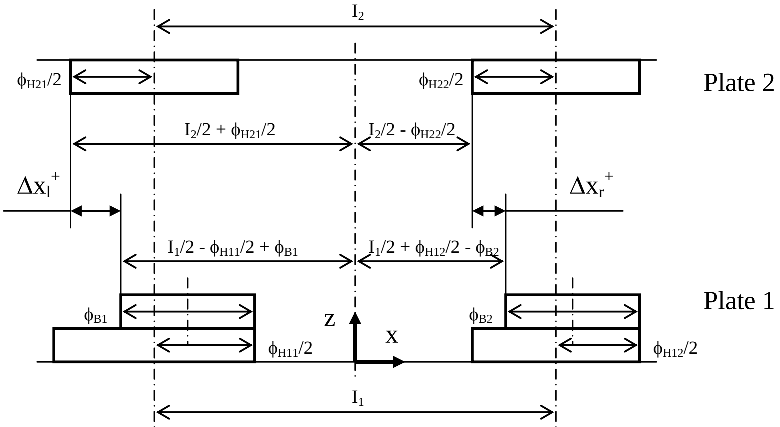

Fig. 7: Displacement of the Second Plate as Regards to the First Plate along the Positive X-Axis.

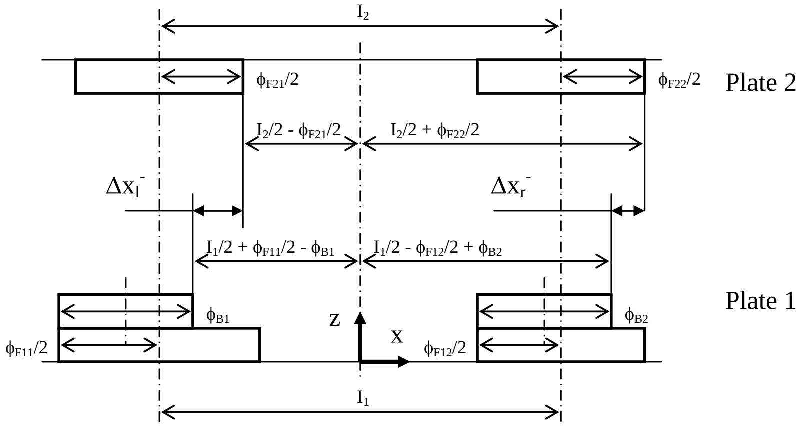

Fig. 8: Displacement of the Second Plate as Regards to the First Plate along the Negative X-Axis.

Once calculated the minimum and the maximum values of the possible deviations of the second plate as regards to the first plate along the X-axis, the actual deviation has been considered distributed according to a uniform probability density function U(Δx^- , Δx^+), as shown in Figure 9. Therefore, the deviation of the second plate as regards to the first plate along the x-axis due to the clearance among the holes and the bolts has been calculated as:

$$\Delta x_3 = \Delta x^- + \left(\Delta x^+ - \Delta x^-\right) \cdot u \tag{19}$$

Where $u \approx U(0,1)$. The maximum deviation along the positive Y-axis of the second plate as regards to the first plate may be calculated through the two gaps between each bolt coupled with each hole of the first plate and the corresponding hole of the second plate, as shown in Figure 10:

$$\Delta y_{B_1}^+ = \frac{\varphi_{H_{11}}}{2} - \varphi_{B_1} + \frac{\varphi_{H_{21}}}{2} = \frac{\varphi_{H_{11}} + \varphi_{H_{21}} - 2\varphi_{B_1}}{2} \tag{20}$$

$$\Delta y_{B_2}^+ = \frac{\varphi_{H_{12}}}{2} - \varphi_{B_2} + \frac{\varphi_{H_{22}}}{2} = \frac{\varphi_{H_{12}} + \varphi_{H_{22}} - 2\varphi_{B_2}}{2} .$$

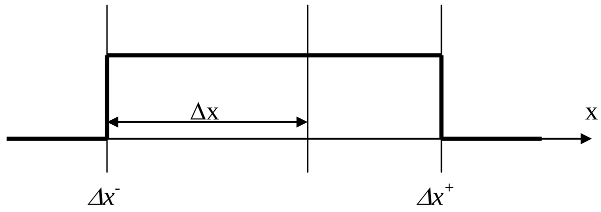

Fig. 9: Probability Density Functions of the Displacement of the Second Plate as Regards to the First Plate along the X-Axis.

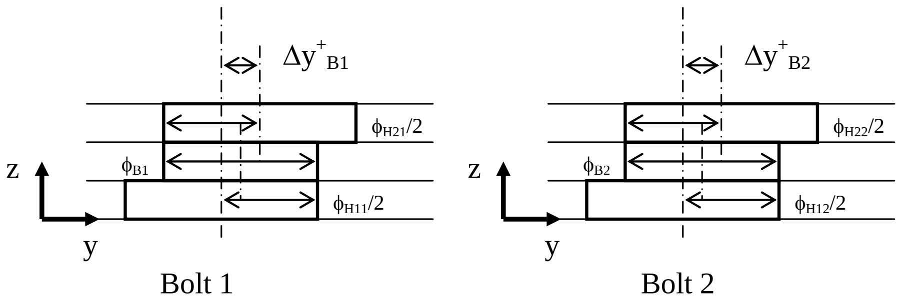

Fig. 10: Displacement of the Holes of the Second Plate as Regards to the Bolts Coupled with the Holes of the First Plate.

The deviation of the two holes of the second plate as regards to the corresponding bolts coupled with the holes of the first plate has been considered distributed according with a uniform probability density function:

$$\Delta y_{B_1} = \Delta y_{B_1}{}^+ \cdot u_{B_1} \tag{21}$$

$$\Delta y_{B_2} = \Delta y_{B_2}{}^+ \cdot u_{B_2}$$

Where $u_{B_1,B_2} \approx U(-1,1)$ for the symmetry as regards the x-axis of the considered datum reference frame. Therefore, the deviation of the second plate as regards to the first plate along the Y-axis due to the clearance among the holes and the bolts has been calculated as:

$$\Delta y_3 = \left(\Delta y_{B_1} + \Delta y_{B_2}\right)/2 \tag{22}$$

As shown in Figure 11. The deviation around the Z-axis ($\Delta\alpha_3$) of the second plate as regards to the first plate may be calculated by means of the following equation that takes into consideration the small values of the rotational deviation due to the small values of the applied tolerances:

$$\Delta\alpha_3 \cong 2\left(\Delta y_{B_2} - \Delta y_{B_1}\right)/\left(I_1 + I_2\right) \tag{23}$$

The whole deviations of the second plate as regards to the first plate along the X-axis, the Y-axis and around the Z-axis have been evaluated by means of the following equations:

$$\Delta x_{SC} = \Delta x_1 - \Delta x_2 + \Delta x_3$$

$$\Delta y_{SC} = \Delta y_1 - \Delta y_2 + \Delta y_3 \tag{24}$$

$$\Delta\alpha_{SC} = \Delta\alpha_1 - \Delta\alpha_2 + \Delta\alpha_3$$

Those are the stack-up functions due to the accumulation of the contributions due to the tolerances applied to the holes, to the tolerances applied to the bolts and to the clearance. Also in this case it is to note that to assure the coupling between the two plates the clearance has not to be negative, and then the assembly condition of the statistical scenario is:

$$\Delta x^+ \geq \Delta x^- \tag{25}$$

Where Δx^{+} and Δx^{-} are given by equations (15) and (17). While the assembly condition of the worst case scenario (6) can be used only to predict if the bolts can be effectively assembled or not, in the statistical scenario the assembly condition (25) can be used to predict the percentages of not conformity.

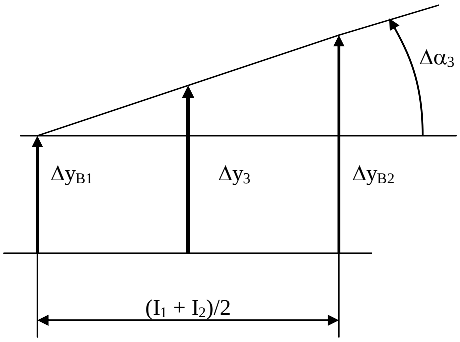

Fig. 11: To Calculate Δy_3 And $\Delta\alpha_3$.

5. Material condition modifiers

The proposed model allows modelling the material condition modifiers too. When a material condition modifier is assigned to a position tolerance of a hole, the assigned location tolerance is represented as shown in Figure 12 for the case of maximum material condition (MMC).

When a material condition modifier is applied to a location tolerance of a hole, the range of the location tolerance may increase of the difference between the actual value of the whole diameter and the value of the diameter at MMC, i.e. the tolerance bonus:

$$b_{ij} = \varphi_{H_{ij}} - (\varphi_H - \Delta\varphi_H) \tag{26}$$

Where b_{ij} is the bonus of the j-hole of the i-plate $\varphi_{H_{ij}}$ is the actual diameter of the j-hole of the i-plate and $(\varphi_H - \Delta\varphi_H)$ gives the value of the whole diameter at MMC. The position tolerance range increase of the bonus b_{ij} and the parameters of the proposed model become:

$$\begin{aligned} R_{11} &= N(0, \varphi_p + b_{11}/6) \\ R_{12} &= N(0, \varphi_p + b_{12}/6) \\ R_{21} &= N(0, \varphi_p + b_{21}/6) \\ R_{22} &= N(0, \varphi_p + b_{22}/6) \end{aligned} \tag{27}$$

That substitute equations (10) and (13), while the other equations of the proposed model keep valid.

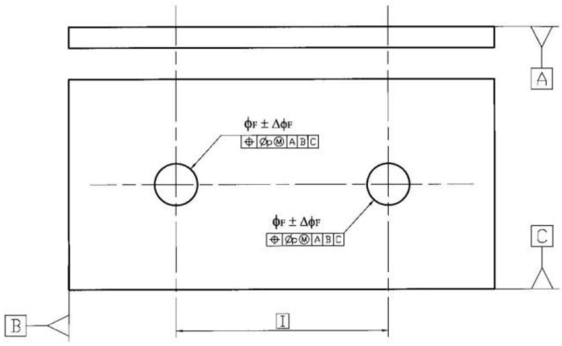

Fig. 12: To Calculate Δy_3 And $\Delta\alpha_3$.

6. Application example

Drawing in Figure 13 shows an assembly constituted by five components: a top plate, a right and a left axle support, an axle and a wheel. It has been developed by Jensen, Helsel and Voisinet [13] and has been widely used to test the performances of different design methods. Its structure looks like a simplified belt drive. Therefore, dimensional, position, orientation and form tolerances are assigned to each component by taking into consideration the assembly functional requirements.

The aim is to apply the proposed model to evaluate the relative deviation from the nominal between one support and the top plate to which are assigned the tolerances shown in Figure 14. Once calculated the relative deviations (Δx, Δy, $\Delta\alpha$) of the two support as regards the top plate, both in worst and statistical cases, these can be used to model the stack-up functions of the whole assembly.

The bolts used to assembly the support with the top plate have a nominal diameter (ϕB) of 10 mm and a dimensional tolerance range ($\pm\Delta\varphi_B$) of ± 0.58 mm. Considering the assigned tolerances on the hole, i.e. a position tolerance (φ_P) of 0.20 mm and a dimensional tolerance ($\pm\Delta\varphi_H$) of ±0.10 mm, equation (6) gives the minimum value of the hole diameter that assures the coupling with the bolt:

$$\varphi_H \geq 10.88mm \tag{28}$$

Then the nominal value of the whole's diameter is assumed as 10.90 mm higher that the calculated minimum admitted value to ensure the clearance. The worst case and the statistical approaches have been implemented by means of Matlab® macro. The results are shown in Table 1. The statistical approach has been implemented by means of Monte Carlo simulations with 105 runs; the ranges of Δx, Δy and $\Delta\alpha$ deviations have been calculated as three times the standard deviation of the obtained probability density function.

As expected, the deviations calculated with the statistical approach are more contained than those calculated with worst case approach. Also the results obtained by the statistical approach show how the deviation along the X-axis is larger than the deviation along the Y-axis. This is due to the effect of the interaction between the two bolts that is more influent along the direction of the transversal direction (Y-axis), than the other one (X-axis).

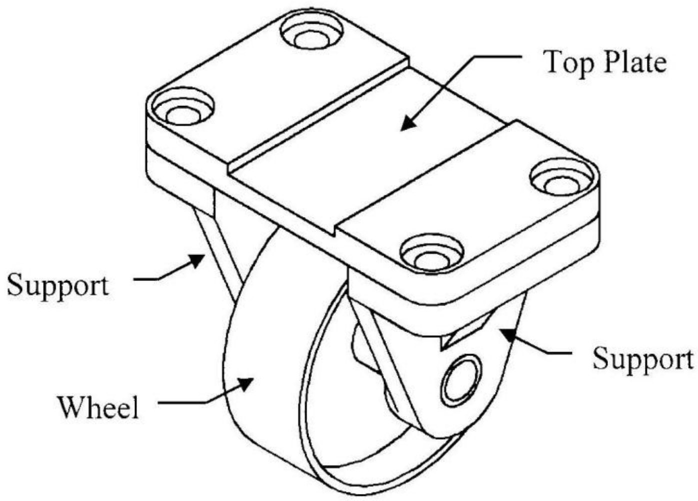

Fig. 13: Scheme of a Drive Belt.

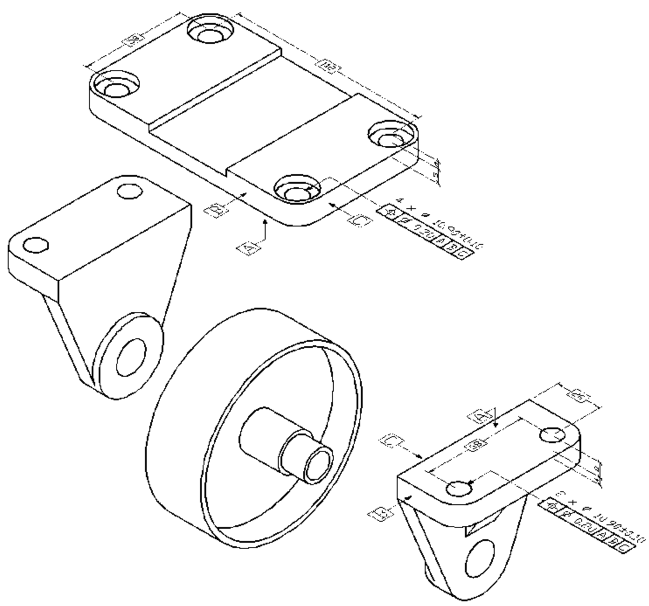

Fig. 14: Components and Tolerances of A Drive Belt.

Table 1: Application Example's Results.

Error	Worst Case	Statistic (105 run)
Δx [mm]	± 1.78	± 1.40
Δy [mm]	± 1.78	± 1.13
$\Delta\alpha$ [Rad]	± 0.061	± 0.039

7. Conclusion

The present work shows a model of the clearance among a pattern of holes of two coupled plates and the corresponding bolts. It permits to evaluate the relative position deviations between the two joined plates due to the tolerances applied on the components, and then to build the transformation matrix needed to model and to solve the tolerance analysis of the assembly. It can be used both in worst case and statistical scenario. In the worst case scenario the developed assembly condition permits to predict if the two parts can be effectively assembled together, while in statistical scenario the developed assembly condition permits to predict the percentage of not conformity (i.e. the plates that do not assembly). The model may be used for the function design of parts joined by bolts, since it allows to evaluate, once assigned a set of tolerances to the holes and the corresponding bolts, the relative position deviations between the two joined parts and, therefore, the possibility to join the parts by bolts.
Current researchers are focused on a pattern of more than two holes and the composite tolerances assigned to the holes.

Acknowledgements

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