Back Iron Reduction of Single Sided Linear Induction Motor

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Abstract

Finite element analysis is introduced in this work making use of penalty functions for obtaining optimum geometry of Linear Induction Motor (LIM) such as air gap dimension, rotor plate thickness, rotor iron back iron, where optimum values are computed depending on first order penalty function. This technique illustrates the advantage of using numerical analysis where most of linear induction motor optimization previews researches are performed using analytical solution which deal with optimizing of power factor, efficiency, losses, thrust and the geometry does not take into consideration. The best computed values are defined depending on tolerance values where an iterative numerical solution is performed to achieve the excellent values of motor model geometry dimensions.

Keywords: *finite element, linear induction motor, back iron, geometry*

1. Introduction

Linear motors are preferred compared to normal rotary motors for high speed propulsion, due to the fact that rotary motor propulsion depends on friction, and is limited by the maximum achievable friction. However, in linear motor, the thrust is obtained from the traveling magnetic field, and thus, there is no theoretical limit for maximum thrust achievable. Also, linear motor, used for low speed application where gearing mechanisms, are preferably avoided [1].

LIM applications are varied starting from simple sliding doors to full control of vehicles whose weight can be many tons as there is no mechanical limitation. In addition, higher acceleration and higher speed of the moving part allow to use LIM in different industrial applications.

In recent years, the single sided LIM has been the most suitable choice for electric vehicles for both wheel and touch less type [2]. The reasons for this choice are due to its simplicity in manufacturing, capable to apply direct forces, moving and braking, independency of adhesiveness, movability over steep and turn of road, low cost of road maintenance, and high reliability .

LIM is very rapid in acceleration and deceleration. In metal industry, LIM is used as an extrusion puller using its characteristic to exert a low – speed with high- thrust pulls on a gripper head. In catapult launch system, the LIM is used for roller coasters, where cars can be accelerated to high speed in very short time, also this type of machines can produce thrust from just a few Newton's to many kilo Newton's with input from few milliamp at low voltage up to handered amperes at the supply voltage. Also sizes vary from square centimeters to many square meters.

Most of the previous papers that take LIM optimization are take into account performances optimization only such as thrust, power factor, efficiency, while the dimensions of model which have economical effects on LIM model building is not considered.

Isfahani, [3] used 2-D FEM using incomplete cholesky conjugate gradient solver , also 3-D FEM is employed to investigate the effect of overhang and edge effect. The design optimization is performed depending on analytical solution which is based on magnetic equivalent circuit used to compute power factor and efficiency. The criterion of optimum design depends on power factor and efficiency multiplying.

2. Geometry optimization design

Design optimization is a technique that seeks to optimize geometry of LIM designed model that meets most of motor requirements but with minimum expense of certain factors like weight, surface area, volume of model parts. Practically any feature of model design can be optimized, stator height, length, rotor plate thickness …etc. This optimum design is the best design in some predefined sense such as maximum peak force for magnetic motor design. In many other situations, minimization of a single function may not be the only goal, and attention must be directed to the satisfaction of predefined constrains placed on the design.

Working towards an optimum design needs to define three types of variables that characterize the design process [4]:

1- **Design variables** (DVs) are independent quantities that are varied in order to achieve the optimum design. Higher and lower limits are specified to provide constraints on the design variables. These restrictions identify the range of deviation of DVs. In the present model, the DVs are slots width, depth, teeth width, yoke depth, air gap length, rotor plate thickness, back iron depth, and any further effective factors. All the above variables are restricted to positive values.

2- **State variables** (SVs) are quantities that limit the design. They are known as dependent variables and function of design variables. Any model parameters can be used as SVs such as rotor force, current density or any other effective factors with maximum and minimum value or it may be single sided that means with only one limit.

3- **Objective functions** is the dependent variable that the optimization program attempts to minimize it. It should be a function of the DVs, that is varying the values of DVs, should result in modifying the objective function. In the optimization procedure, only one objective function is defined.

The vector of design variables is indicated by [4]:

$$X = \begin{bmatrix} X_1 & X_2 & X_3 & . & . & . & X_n \end{bmatrix} \qquad (1)$$

DVs are subjected to n constraints with upper and lower limits, that is,

$$\underline{X}_i \le X_i \le \overline{X}_i \quad (i = 1,2,3,...,n) \qquad (2)$$

Where $\underline{X}_i$ is lower limits, $\overline{X}_i$ upper limits and n is number of design variables.

The design variable constraints are often referred to as side constraints and define what is commonly called feasible design space. Now, minimize

$$f = f(X) \qquad (3)$$

Subject to

$$g_i(X) \le \overline{g}_i \quad (i = 1,2,3,...,m_1) \qquad (4)$$

$$\underline{h}_i \le h_i(X) \quad (i = 1,2,3,....m_2) \qquad (5)$$

$$\underline{w}_i \le w_i(X) \le \overline{w}_i \quad (i = 1,2,3...,m_3) \qquad (6)$$

Where f is objective function, g_i, h_i, w_i are state variables containing the design with under bar and over bar representing lower and upper bounds respectively, m_1, m_2, m_3 are number of state variables constraints with various upper and lower limit values.

2.1 Feasible Design Sets

Design arrangements that satisfy all constrains are referred to as feasible designs while design configurations with one or more violations are termed infeasible. For defining feasible design space, a tolerance is added to each state variable limit.

If X^* is a given design set defined as:

$$X^* = \begin{bmatrix} X_1^* & X_2^* & X_3^* & . & . & . & X_n^* \end{bmatrix} \qquad (7)$$

The design is defined feasible only if:

$$g_i^* = g_i(X^*) \le \overline{g}_i + \alpha_i \quad (i = 1,2,3,...,m_1) \qquad (8)$$

$$\underline{h}_i - \beta_i \le h_i^* = h_i(X^i) \quad (i = 1,2,3,....m_2) \qquad (9)$$

$$\underline{w}_i - \gamma_i \le w_i^* = w_i(X^*) \le \overline{w}_i + \gamma_i \quad (i = 1,2,3...,m_3) \qquad (10)$$

Where $\alpha_i, \beta_i, and \gamma_i$ are tolerances and:

$$\underline{X}_i \le X_i^* \le \overline{X}_i \quad (i = 1,2,3...,n) \qquad (11)$$

Since no tolerances are added to design variable constraints.

2.2 Best Design Set

The best design set is determined depending on the objective function under one of the following conditions:

1- If one or more feasible sets exist, the best design set is the feasible one with the lowest objective function value.
2- If all design sets are infeasible, the best design set is the one nearest to being feasible.

2.3 First Order Optimization Method

The methodof optimization that is used in the present work for LIM model geometry optimization is first order optimization method. Its choice of is depends on the unconstrained objective function, used for LIM optimization. This method of optimization computes and makes use of derivative information. The constrains of design parameters transformed into an unconstrained one using penalty functions. Derivatives are formed for the objective function and the state variable penalty functions, leading to a search direction in design space. A variety of steepest descent and conjugate direction searches are performed during each iteration until convergence is reached. In each iteration several analysis loops are performed in order to optimize the design variables of the analyzed model.

The unconstrained version of the problem outlined previously is formulated as shown [4]:

$$U(X,u) = \frac{f}{f_o} + \sum_{i=1}^{n} P_x(X_i) + u\left(\sum_{i=1}^{m_1} P_g(g_i) + \sum_{i=1}^{m_2} P_h(h_i) + \sum_{i=1}^{m_3} P_w(w_i) \right) \qquad (12)$$

where U is dimensionless unconstrained function, $P_x, P_g, P_h,$ and P_w are penalties applied to the constrained design and state variables. f_o is reference objective function value, selected from the current group of design sets.

Constraint fulfillment is controlled by a response surface parameter u. The penalty functions P_x are external functions and applied to the design variables while the extended interior penalty functions $P_g, P_h,$ and P_w represent the state variable constraints. The penalty function for equation 2.70 can be written as:

$$P_g(g_i) = \left(\frac{g_i}{g_i + a_i} \right)^{2\lambda} \qquad (13)$$

Where λ is integer and this function be very large when the constraint is violated and very small when it is not. The functions are used in order to maintain penalties at similar form.

As search directions are devised, a certain computational advantage can be gained if the function U is written in the form:

$$U(X,u) = U_f(X) + U_p(X,u) \qquad (14)$$

Where

$$U_f(X) = \frac{f}{f_o} \qquad (15)$$

$$U_p(X,u) =$$

$$\sum_{i=1}^{n} P_x(X_i) + u\left(\sum_{i=1}^{m_1} P_g(g_i) + \sum_{i=1}^{m_2} P_h(h_i) + \sum_{i=1}^{m_3} P_w(w_i) \right) \qquad (16)$$

First order optimization method is solved by using iterations solution. For each iteration, a search direction vector $d(i)$ is devised and at the next iteration obtained from :

$$X^{(i+1)} = X^i + s_j d^{(i)} \tag{17}$$

The range of line search parameter s_i given by:

$$0 \leq s_i \geq \frac{S_{max}}{100} s_j^* \tag{18}$$

Where s_j^* is the largest possible step size for the line search of current iteration and S_{max} is maximum line search step size (in percent).

At the first iteration, the search direction is assumed to be negative of the gradient of the unconstrained objective function:

$$d^{(0)} = -\nabla U\left(X^{(0)}, u\right) = d_f^{(0)} + d_p^{(0)} \tag{19}$$

in which $u = 1$ and

$$d_f^{(0)} = -\nabla U_f\left(X^{(0)}\right) \tag{20}$$

$$d_p^{(0)} = -\nabla U_P\left(X^{(0)}\right) \tag{21}$$

For subsequent iterations, conjugate directions are formed according to recursion formula:

$$d^{(i)} = -\nabla U\left(X^{(i)}, u_k\right) + r_{j-1} d^{(i-1)} \tag{22}$$

$$r_{i-1} = \frac{\left(\nabla U\left(X^{(i)}, u\right) - \nabla U\left(X^{(i-1)}, u\right)\right)^T \nabla U\left(X^i, u\right)}{\left|\nabla U\left(X^{(i-1)}, u\right)\right|^2} \tag{23}$$

When all design variable constrains are satisfied $P_x\left(X_i\right) = 0$, that means u can be factored out of U_p and can be written as:

$$U_p\left(X^{(i)}, u\right) = u U_p\left(X^{(i)}\right)$$
$$\text{if } \overline{X}_i \leq X_i \leq \underline{X}_i \quad (i = 1, 2, 3, \ldots, n) \tag{24}$$

For suitable correction, u can be changed from iteration to iteration without destroying the conjugate nature of equation 18. By adjusting u, the constraints will be pushed to their limit value and convergence is achieved.

The algorithm is infrequently restarted by setting $r_{i-1} = 0$, forcing a steepest decent iteration. Restarting is employed whenever ill- condition is detected, convergence is nearly achieved. So far, it has been assumed that the gradient vector is available. The gradient vector is computed using an approximation as follows:

$$\frac{\partial U\left(X^{(i)}\right)}{\partial X_i} \approx \frac{U\left(X^{(i)} + \Delta X_i e\right) - U\left(X^{(i)}\right)}{\Delta X_i} \tag{25}$$

Where e is vector with 1 in its ith component and 0 for all other components.

$$\Delta X_i = \frac{\Delta D}{100}\left(X_i - \underline{X}_i\right)$$

And ΔD is forward difference step size (in percent).

first order iterations continue until convergence of solution is achieved or termination occurred (infeasible design set). The criteria of solution convergence is depend on objective function tolerance.

3. Geometry Optimization

3.1. Implementation

The optimization program that has been used for geometry optimization of LIM analyzed model is provided in ANSYS software. This program uses a parametric design language. Different methods of optimization are provided in this program to perform model geometry optimization. In the present work, the first order method is used because its perfect for unconstrained objective function. In order to make use of this program, the following must be executed to perform the geometry optimization:

1- Create analysis file to be used during looping through analysis cycle. This file represents a complete analysis sequence including building the model parametrically in terms of initial values of DVs representing starting of design and obtains the solution. Recover results data and assign them to parameters that represent DVs and SVs and objective function.

2- Establish model parameters in the program database as first design set.

3- Define optimization variables DVs, SVs, and objective function. Minimum and maximum constraints are specified for DVs and SVs, while no constrains are needed for objective function. Every variable has a tolerance value related to it.

4- Choose optimization method where each one of the analyzing methods has certain looping controls associated with it, such as number of iterations.

6- Initiate optimization analysis after all appropriate controls have been specified. At the end of each iteration, a check for convergence is made. The problem is said to converge if the current, previous, or best design is feasible.

7- Review design sets after optimization computation is completed. The flow chart for optimization program is shown in Appendix [A] .

4. Results

The geometry optimization program starts first defining the design variables, which represent the model geometry and each design variable value is defined within lower and upper limits with tolerance value which is defined by [0.01* (maximum value-minimum value)]. The design variables used for geometry optimization are:

1- Rotor plate thickness.
2- Rotor back iron depth.
3- Stator yoke.
4- Air gap length.

Rotor force is chosen as state variable with lower and upper value. The tolerance is defined in similar way as that used for design variable. At the end of each iteration, the force is computed and compared with the set value. If the tolerance is within the appropriate limit, the iteration solution is stopped.

The objective function of the optimization program is total model volume with tolerance value [0.01*present value]. Both state variables and objective function tolerance define the convergence to a minimum objective function through varying design variable, while the rotor force remains within a specified value higher than the lower value.

The effect of each model geometry parameter is analyzed separately through optimization program to explain the major effect of this parameter on the magnetic vector potential value and distribution in the model. Also flux density, current density and any other field quantity can be analyzed.

Starting from rotor plate thickness, the variation of plate thickness with force is depicted in figure 1, where the limit of variation is taken from 1.5 to 4 mm , within a tolerance of 0.0025 . The force increases from 807 to 937 N while the thickness reduces to the optimized value of 2.98 mm. The increase of A_z is shown in figure 2.
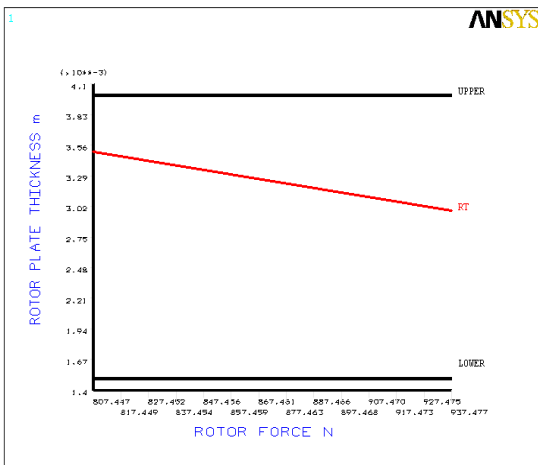
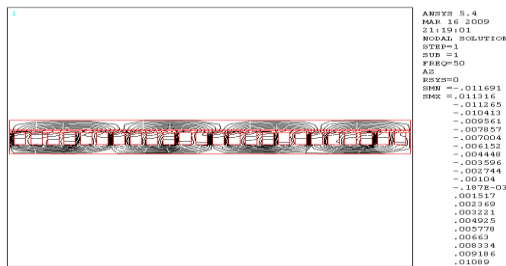**Fig 1**. Variation of force with rotor plate thickness

Fig 2: Variation of magnetic vector potential With rotor plate thickness

As a result of A_z increasing, the flux density is increased in the motor domain where figure 3 exhibits this increase.
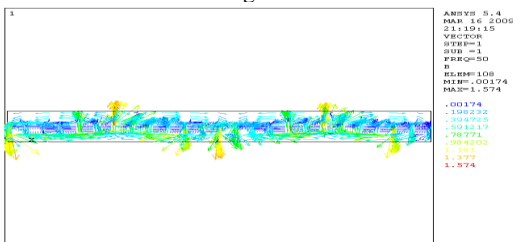
Fig 3: Variation of flux density with rotor plate thickness

The rotor back iron cooperate considerable effect for completing the stator magnetic circuit. For long rotor LIM, the depth of back iron is effective for this type of application due to the quantity of iron that is used for model construction. The computation of back iron depth in the analytical computation depends on the assumed flux density in rotor back iron and flux density shape function, which depends on the magnetic saturation coefficient of teeth and flux density in the air gap, where all of these factor are assumed values and do not give the appropriate depth value. By using finite element modeling, the back iron depth value can be optimized to the optimum value, which can be used for LIM application. The variation of back iron is taken from 14.1 to 6 mm within tolerance value of 0.008 . The optimized value is 6 mm and the reduction in depth is 42.5% of the original value. Figure 4 depicts the variation of back iron depth with the total motor volume. Also this variation has no effect on rotor force where this force value remains approximately constant.
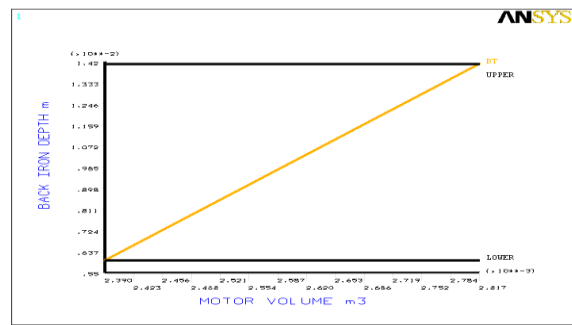
Fig 4: Effect of back iron reduction on motor volume

The flux density distribution in the model after back iron reduction is shown in figure 5, where the concentration in back iron is clearly shown and the maximum value of flux density does not change. Also the effect of using solid back iron in state of lamination is also studied. In this case, the optimized back iron value is 5.31 mm. As mentioned earlier, the rotor force is reduced when using solid back iron. This reduction is due to the eddy current induced in the back iron which causes also further consumption of power due to power losses in back iron.
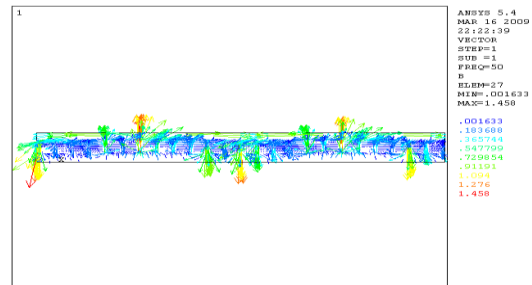
Fig 5. Flux density distribution for optimized back iron

Stator yoke depth is taken into optimization computation where the limit of its value is from 5 to 14 mm within a tolerance value 0.09 . The optimized value is 5.82 mm with constant rotor force. The variation of stator yoke depth and model volume is shown in figure 6, also, distribution and value of magnetic vector potential in the motor domain are depicted in figure 7, where its concentration in stator yoke after optimization computation is performed with maximum value of 0.009818 in the positive direction and -0.010569 in the negative direction. These values are slightly less than its value before optimization computation. The reduction of stator yoke has no significant effect on rotor force, where the reduction in force value is about 0.2% of force value before optimization.
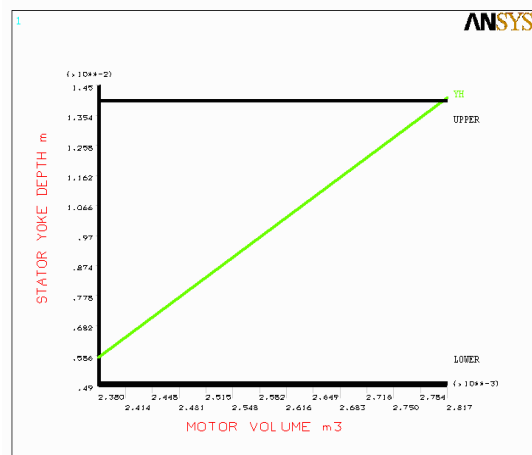
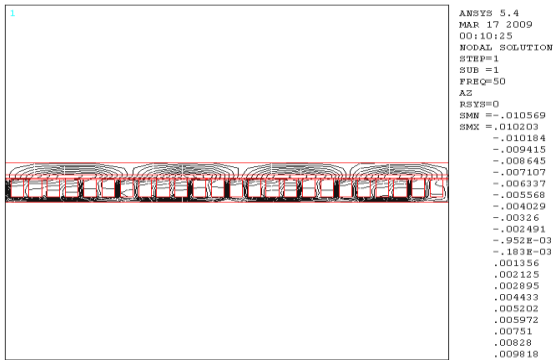**Fig 6**. Motor volume reduction with stator yoke depth

Fig 7: Variation of magnetic vector potential with stator yoke

Air gap length of LIM is in general case more than 2 mm due to the topology of this type of motor [3,5]. In the proposed model, the air gap is taken to be 2 mm and the limit of air gap for optimization purpose is taken from 0.5 to 3 mm with a tolerance of 0.025. The optimum air gap is 1.84 mm with reduction in rotor force about 5% from the designed force. The variation of air gap with rotor force is shown in figure 8, where the force increases slightly with increasing air gap.
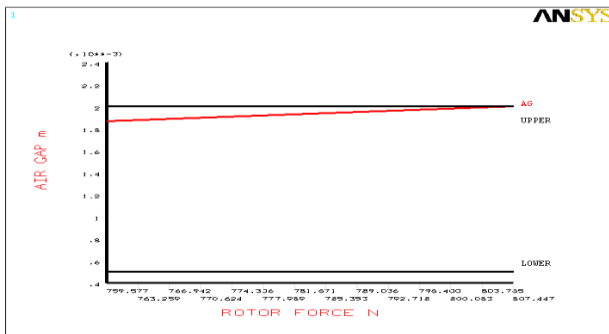

Fig 8: Air gap variation with rotor force

Table B shows the results of the proposed model optimization using first order optimization method, Appendix [B].

5. Conclusions

By making use of finite element analysis, the geometry dimension of main parts of LIM model are optimized using first order optimization method. The geometries restricted for only rotor plate thickness, back iron depth, stator yoke height, and air gap dimension. The optimized values is defined by making use of feasible set is found to be less than the design value with increasing motor force. Both of back iron and stator yoke depth are optimized with no effect on motor force. The air gap dimension is optimized to a certain value with small reduction in motor force. The reduction of rotor plate thickness is so effective on rotor construction where this reduction is an important point in high-speed LIM application, because it has great effects in the costs of long rotor rail applications. This type of optimization is performed by using numerical analysis where finite element analysis is on of the best methods that give the optimum model dimensions, in addition to the magnetic field distribution after geometry optimization.

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