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Research paper

Optimal Age for Preventive Replacement of A System with GPP Repair Process

Yu-Hung Chien[1], Chung-Piao Chiang[2]

[1]Department of Applied Statistics, National Taichung University of Science and Technology
[2]Department of Mice & Tourism, HungKuo Delin University of Technology, Graduate Institute of Physical Education, National Taiwan Sport University

Abstract

A new and more general type of repair, called the 'GPP repair', is applied in this study, which is defined base on the Generalized Polya Process and it is more close to practical situations than the 'minimal repair'. Under the GPP repair assumption, the future reliability performance of a system becomes worse and worse as the number of system failures occurred in the past increases. Cost model is developed and the corresponding optimal replacement age is derived such that the long-run expected cost rate is minimized. Structural properties of the optimal replacement age for the GPP repairable system are obtained.

Keywords: *Age replacement, GPP repair, Cost rate, Optimization*

1. Introduction

Many works focused on the system subject to shocks which cause system failures, most of them modeled the shocks as a non-homogeneous Poisson process (NHPP). Note that the NHPP shocks only depend on time (i.e., the age of the operating system), but not on the number of failures. However, many practical systems deteriorate with time (age) as well as the number of repairable failures. To incorporate this feature, it is necessary to model the shocks by a more suitable counting process (point process). In addition, if the failure process of a system follows the NHPP, it implies that the repair type on each failure is a minimal repair. 'Minimal repair' means that the state of the system after the repair is restored to the state it had prior to the failure (i.e., as-bad-as-old condition). In other words, after a minimal repair action, the failure rate function of the product remains unchanged. Although 'minimal repair' has many advantages in the maintenance optimization and model development, it also has practical limitations. For example, when a component in a system fails, this may lead to a more hostile working environment through increased pressure, temperature, humidity, etc., causing instantaneous stress or damage to the adjacent non-failed components. It eventually results in the system degradation, hence increasing the level of the system failure rate function. As a result, a new repair type, which is 'worse-than-minimal repair', seems more close to real situations.

A new counting process suggested in Konno (2010), called the 'Generalized Polya Process (GPP)', is characterized by Cha (2014), and its definition is stated below:

Definition 1. *Generalized Polya Process (GPP)*

A counting process $\{N(t),\ t \geq 0\}$ *is called the Generalized Polya Process (GPP) with the set of parameters* $(\lambda(t),\ \theta, \beta)$, $\theta \geq 0$, $\beta \geq 0$, *if (i)* $N(0) = 0$ *and (ii)* $\lambda_t = (\theta N(t-) + \beta)\lambda(t)$.

In the Definition 1, the $\lambda(t)$ is the intensity function of the counting process, $N(t-)$ is the number of point event in $[0,\ t)$, and the λ_t is the stochastic intensity (the intensity process) which is used to mathematically describe a point process. The specific definition of λ_t is presented in (1) of Cha (2014) and Lee & Cha (2016). It is clear that the GPP with $(\lambda(t),\ \theta = 0,\ \beta = 1)$ reduces to the NHPP with the intensity function $\lambda(t)$ and, accordingly, the GPP can be understood as a generalized version of the NHPP.

Based on the GPP, Lee & Cha (2016) further defined a new type of repair which is called the 'GPP repair'. Let $\{N(t),\ t \geq 0\}$ as the failure process of a system whose failure rate $r(t)$ undergoes a new type of repair upon each failure and the duration of repair is negligible. Thus $N(t)$ can be interpreted as the total number of failures (repairs) in $[0,\ t)$.

Definition 2. *The GPP repair*

For a system with its failure rate $r(t)$, *a repair type is called the 'GPP repair' with parameter* α *if* $\{N(t),\ t \geq 0\}$ *is the GPP with the parameter set* $(\lambda(t), \theta, 1)$.

Under the GPP repair process, the corresponding stochastic intensity is specified as $\lambda_t = (\theta N(t-) + 1)r(t)$, and the parameter θ determines the 'degree of repair'. Obviously, $\theta = 0$ corresponds to the minimal repair and $\theta > 0$ implies that the repair is worse-than-minimal repair. As θ increases, the corresponding repair becomes worse and worse.

Figure 1 shows the stochastic intensities of the GPP repair process with $r(t) = 0.05t^2+0.1$ for $\theta = 0$, 0.3 and 0.5, and in which the system failures occurred at the points where "×" is marked. As shown in the figure, when $\theta = 0$ (minimal repair process) the stochastic intensity is just given by the fixed failure rate $r(t)$, which is not affected by the failure history. On the other hand, when $\theta > 0$, the stochastic intensity jumps at each failure point; thus, under the GPP repair process, after each failure, the state of the system becomes worse than it was before the failure.

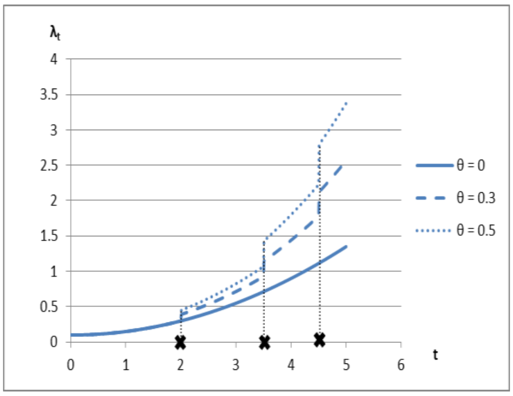

Figure 1: The stochastic intensities of the GPP repair processes.

Based on the above descriptions of the practicality of GPP repair and the demonstration of the mathematical characteristics of the model, it can be realized that the GPP repair process has a failure rate which depends on both the system's age and the system's failure history, and this study believes that it is worth applying the GPP repair model to the traditional age-replacement policy for a system subject to shocks. Thus, a maintenance model with the GPP shocks is a generalization of the existing models and can be applied in production, insurance, epidemiology, and load-sharing systems.

2. General Model Formulation

Suppose a system is subject to shocks that arrive according to a counting process $\{N(t),\ t \geq 0\}$ with intensity function $\lambda(t)$ and the mean value function $\Lambda(t)$ where t is the age of the system. As a shock occurs, it results the system in one of the two types of failures: type I failure is a minor failure, which occurs with probability q and can be corrected by a GPP repair; type II failure is a major failure, which occurs with probability $p=1-q$ and the system has to be replaced. The replacement policy is that the system is replaced at first type II failure or at age (time) T, whichever occurs first. Assume the distribution of the lifetime of the system is proper, i.e., $\Lambda(\infty) = \infty$, and without loss of generality that $\lim_{t\to\infty} \lambda(t) > 0$. After a replacement, the shock process resets to 0. The repair and replacement process is repeated again and again.

Let c_2 denote the cost of replacement at time T, and c_3 denote the cost of replacement at the first type II failure. It is reasonable to assume that $c_2 < c_3$ because the cost for a corrective replacement is usually more expensive than the cost for a preventive replacement. The cost of performing a GPP repair is c_1; all failures are instantly detected and repaired.

Let $^*Y_i$ denote the length of the i-th successive replacement cycle for $i = 1, 2, \ldots$, $^*R_i$ denote the operational cost over the cycle $^*Y_i$, thus $\{(^*Y_i,\ ^*R_i)\}$ constitutes a renewal reward process. If $D(t)$ denote the expected cost of operating the system over time interval $[0,\ t]$, then it is well-known that $\lim_{t\to\infty} D(t)/t = \mathrm{E}(^*R_i)/\mathrm{E}(^*Y_i)$ and we denote the right-hand side by $C(T;\ p)$.

In the example 1 of Cha (2014), it stated that if an event from the GPP occurs at time t, then independently of all else, it is classified as being a type I event with probability $q(t)$ and a type II event with probability $p(t)=1-q(t)$. Let $N_i(t)$, $i = 1, 2$, represent the number of type i events that occur by time t, thus by using the formulation results that derived in Cha (2014), we can get the following results

$$P(N_1(t)=n) = \frac{\Gamma(1/\theta+n)}{\Gamma(1/\theta)n!}\left[\frac{\theta\int_0^t q\lambda(x)e^{\theta\Lambda(x)}dx}{1+\theta\int_0^t q\lambda(x)e^{\theta\Lambda(x)}dx}\right]^n\left[\frac{1}{1+\theta\int_0^t q\lambda(x)e^{\theta\Lambda(x)}dx}\right]^{1/\theta},$$

and

$$P(N_2(t)=n) = \frac{\Gamma(1/\theta+n)}{\Gamma(1/\theta)n!}\left[\frac{\theta\int_0^t p\lambda(x)e^{\theta\Lambda(x)}dx}{1+\theta\int_0^t p\lambda(x)e^{\theta\Lambda(x)}dx}\right]^n\left[\frac{1}{1+\theta\int_0^t p\lambda(x)e^{\theta\Lambda(x)}dx}\right]^{1/\theta}.$$

It can be seen that the distribution of $N_i(t)$ follows a negative binomial distribution and thus

$$E[N_1(t)] = \int_0^t q\lambda(x)e^{\theta\Lambda(x)}dx$$

and

$$E[N_2(t)] = \int_0^t p\lambda(x)e^{\theta\Lambda(x)}dx.$$

Now let Y_1 denote the waiting time until the first type II failure, then base on the results presented above, the survival function of Y_1 is

$$\bar{F}_p(t) = P(Y_1 > t) = P(N_2(t)=0) = \left[1+\theta\int_0^t p\lambda(x)e^{\theta\Lambda(x)}dx\right]^{-1/\theta}, \quad (1)$$

and for our model we have

$$^*Y_1 = \begin{cases} Y_1, \text{if } Y_1 \leq T, \\ T, \text{if } Y_1 > T, \end{cases}$$

thus the expected length of a replacement cycle is given by

$$E(^*Y_1) = \int_0^T t\,dF_p(t) + T\cdot\bar{F}_p(T) = \int_0^T\left[1+\theta\int_0^t p\lambda(x)e^{\theta\Lambda(x)}dx\right]^{-1/\theta}dt \quad .(2)$$

Next, the operational cost over the replacement cycle $^*Y_1$ can be expressed as

$$^*R_1 = \sum_{i=1}^{\infty} c_1 I_{(S_1,\infty)}(Y_1)I_{[0,T]}(S_1) + c_3 I_{[0,T]}(Y_1) + c_2 I_{(T,\infty)}(Y_1), \quad (3)$$

where S_i is the arrive time of i-th type I failure and $I_A(x)$ is the indicator function of the set A such that

$$I_A(x) = \begin{cases} 1, & \text{if } x \in A, \\ 0, & \text{otherwise.} \end{cases}$$

Thus by (3), the expected operational cost over the cycle $^*R_1$ is

$$E(^*R_1) = c_3F_p(T) + c_2\bar{F}_p(T) + c_1\frac{(1-p)}{p(1-\alpha)}\left\{1-\left[1+p\left(e^{\alpha\Lambda(T)}-1\right)\right]\bar{F}_p(T)\right\}. \quad (4)$$

Therefore, for the infinite-horizon case, we want to obtain the optimal T which minimizes

$$C(T;p) = \frac{E(\ ^*R_1)}{E(\ ^*Y_1)}, \quad (5)$$

the total expected long-run cost per unit time

3. Optimization

Now we want to derive the optimal age-replacement policy for a system that subject to a GPP repair shocks. Let T^* be the optimal time which would minimizes $C(T;\ p)$. Take the first derivative of (5) with respect to T^* and set it equal to 0, we have

$$\int_0^T [\psi(T) - \psi(t)]\bar{F}_p(t)dt = c_2, \quad (6)$$

where $\psi(t) = \left\{c_1(1-p) + \frac{(c_3 - c_2)p}{1 + p[e^{\theta\Lambda(t)} - 1]}\right\}\lambda(t)e^{\theta\Lambda(t)}$. Thus if $\psi(t)$ is continuous and increases to ∞, then there exists at least one finite positive solution T^* satisfying $\int_0^{T^*} [\psi(T^*) - \psi(t)]\bar{F}_p(t)dt = c_2$. And if $\psi(t)$ is strictly increasing and continuous, then T^* is unique and $C(T^*;\ p) = \psi(T^*)$. Differentiating $\psi(t)$ we obtain $\psi'(t) = \tau(t)\ e^{\theta\Lambda(t)}$ where

$$\tau(t) = \left\{c_1 + (c_3 - c_2)\frac{p}{[1 + p(e^{\alpha\Lambda(t)} - 1)]^2}\right\}(1-p)\alpha[\lambda(t)]^2 + \left\{c_1(1-p) + \frac{(c_3 - c_2)p}{1 + p[e^{\theta\Lambda(t)} - 1]}\right\}\left[\frac{d}{dt}\lambda(t)\right] \quad (7)$$

It is obviously that if $\tau(t) > 0$ for all $t > 0$, then there exists a unique and finite optimal T^*. It is worth to note that if $\lambda(t)$ is increasing (i.e., IFR), then it must $\tau(t) > 0$ and the unique and finite optimal T^* exists; however, $\tau(t) > 0$ does not necessary require $\lambda(t)$ should be IFR. Even decreasing $\lambda(t)$ can satisfy $\tau(t) > 0$.

4. Conclusion

This paper discusses the optimality of age replacement policy for a GPP repairable system. GPP repair is a new way to model the effect of repair on a system's failure rate. It can be used to study the maintenance problem where a multi-component product becomes less reliable after a failed component repair. Thus it is a "worse than minimal repair" type. We show that under reasonable assumptions, there exists a unique optimal replacement age that can minimize the long run expected cost per unit time. Our results offer an important analytical tool for reliability engineer to design more competitive maintenance policy.

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