



# Fuzzy Critical Path Method Using Triangular Bipolar Fuzzy Numbers

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## Abstract

The basic issue is a traditional and vital system improvement showing up in numerous applications, particularly in arranging also and controlling the complex projects. Anyway after the effect of traditional basic way strategy calculation can't appropriately coordinate this present reality issue. In this paper, two unique calculations are exhibited to acquire the basic way in a fuzzy system, where the edge weights are taken as triangular bipolar fuzzy numbers and triangular fuzzy numbers separately.

**Keywords:** Critical path method, Fuzzy project network, Triangular bipolar fuzzy numbers, Triangular fuzzy numbers.

## 1. Introduction

System advancement is an extremely prevalent and frequently connected field among the well-studied areas of operations research. Numerous handy issues emerging in the genuine circumstances can be figured as system models. Each expansive project comprises of numerous activities. A critical part of project management is planning the action time accurately. This the basic part of project planning as this will be the due date for the fruition of a task. As indicated by the basic way strategy, the executive can control the time and cost of the task and enhance the productivity of asset designation to guarantee the undertaking quality.

## 2. Preliminaries

Definition. 2.1: [2]. A fuzzy graph with  $V$  as the underlying set is a pair of functions  $G = (\sigma, \mu)$  where  $\sigma: V \rightarrow [0, 1]$  is a fuzzy subset and  $\mu: V \times V \rightarrow [0, 1]$  is a symmetric fuzzy relation on the fuzzy subset  $\sigma$  for all  $u, v \in V$  such that  $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ . The underlying crisp graph  $\tilde{G} = (\sigma, \mu)$  of is denoted by  $G = (V, E)$  where  $E \subseteq V \times V$ . A fuzzy relation can also be expressed by a matrix called fuzzy relation matrix  $M = [a_{ij}]$  where  $a_{ij} = \mu(u_i, u_j)$

Definition. 2.2: [3]. A bipolar fuzzy graph  $G = (V, A, B)$  is a nonempty set  $V$  together with a pair of functions  $A = (\mu_A^P, \mu_A^N): V \rightarrow [0, 1] \times [-1, 0]$  and  $B = (\mu_B^P, \mu_B^N): V \times V \rightarrow [0, 1] \times [-1, 0]$  such that for all  $x, y \in V$

$$\mu_B^P(x, y) \leq \min(\mu_A^P(x), \mu_A^P(y))$$

$$\mu_B^N(x, y) \geq \max(\mu_A^N(x), \mu_A^N(y))$$

Definition. 2.3: [1]. A bipolar fuzzy graph  $G = (V, E, \mu, \rho)$  is said to be strong if  $\rho^P(xy) = \min(\mu^P(x), \mu^P(y))$  and  $\rho^N(xy) = \max(\mu^N(x), \mu^N(y))$  for all  $xy \in E$ .

Definition. 2.4: [1]. A bipolar fuzzy graph  $G = (V, E, \mu, \rho)$  is said to be complete if  $\rho^P(xy) = \min(\mu^P(x), \mu^P(y))$  and  $\rho^N(xy) = \max(\mu^N(x), \mu^N(y))$  for all  $x, y \in V$ .

Definition. 2.5: A triangular fuzzy number  $A$  can be defined by a triplet  $(t_1, t_2, t_3)$ ,  $t_1, t_2, t_3$  where  $t_1, t_2, t_3 \in \mathfrak{R}$ . The membership function

$$\mu_A(x) = \begin{cases} \frac{x - t_1}{t_2 - t_1} & ; \text{if } t_1 \leq x \leq t_2 \\ 1 & ; \text{if } x = t_2 \\ \frac{t_3 - x}{t_3 - t_2} & ; \text{if } t_2 \leq x \leq t_3 \\ 0 & ; \text{otherwise} \end{cases}$$

Definition. 2.6: The bipolar fuzzy numbers for triangle is defined by  $A = \{x, \mu_A^P(x), \mu_A^N(x)\}$  where  $\mu_A^P(x)$  and  $\mu_A^N(x)$  are positive membership and negative membership functions respectively. The membership function is given as

$$\mu_A^P(x) = \begin{cases} \frac{x-t_L}{t_P-t_L}; & \text{if } t_L \leq x < t_P \\ \frac{x-t_R}{t_P-t_R}; & \text{if } t_P < x \leq t_R \\ 0; & \text{otherwise} \end{cases}$$

$$\mu_A^N(x) = \begin{cases} -\frac{(x-t_L)}{t_N-t_L}; & \text{if } t_L \leq x < t_N \\ -\frac{(x-t_R)}{t_N-t_R}; & \text{if } t_N < x \leq t_R \\ 0; & \text{otherwise} \end{cases}$$

Definition 2.7: The set of two bipolar fuzzy numbers  $a = (a_L, a_P, a_N, a_R)$  and  $b = (b_L, b_P, b_N, b_R)$  to denote the addition operation on triangular bipolar fuzzy numbers

$$A \oplus B = (a_L + b_L, a_P + b_P, a_N + b_N, a_R + b_R).$$

Definition 2.8: The set of two bipolar fuzzy numbers

$$a = (a_L, a_P, a_N, a_R) \text{ and } b = (b_L, b_P, b_N, b_R)$$

to denote the L operation on triangular bipolar fuzzy numbers  $L = ((L_{MIN}, L_{MAX}))$  where as  $L_{MIN}$  denotes the positive numbers of triangular bipolar fuzzy and  $L_{MAX}$  denotes the negative numbers of triangular bipolar fuzzy.

Definition 2.9: The set of triangular bipolar fuzzy number  $T = (T_L, T_P, T_N, T_R)$  the  $\alpha$ -cut ranking technique of T is denoted by

$$R(T) = \left( \left( \frac{T_L + 4T_P + T_R}{6} \right), - \left( \frac{T_L + T_N + T_R}{3} \right) \right)$$

Definition 2.10: The path of length of triangular bipolar fuzzy is denoted by  $L_j = ((T_{L_j}, Q_{P_j}, S_{R_j}), (T_{L_j}, Q_{N_j}, S_{R_j}))$  and  $L = ((T, Q, S), (T, Q', S))$  then the Euclidean ranking is denoted by

$$E(L_j) = (\sqrt{(T-T_L)^2 + (Q-Q_P)^2 + (S-S_{R_j})^2}, \sqrt{(T-T_L)^2 + (Q'-Q_{N_j})^2 + (S-S_{R_j})^2})$$

Definition 2.11: The path of length of triangular bipolar fuzzy is denoted by  $L_j = ((T_{L_j}, Q_{P_j}, S_{R_j}), (T_{L_j}, Q_{N_j}, S_{R_j}))$  and  $L = ((T, Q, S), (T, Q', S))$  be the bipolar fuzzy longest length then the similarity ranking of bipolar fuzzy number is denoted by

$$S(T_j, L) = \begin{cases} \phi; & T_j \cap L = \phi \\ \left( \frac{(S_{R_j} - T)^2}{2((Q-T) + (S_{R_j} - Q_P))}, \frac{(S_{R_j} - T)(Q - Q_{N_j})}{2((Q'-T) + (S_{R_j} - Q_{N_j}))} \right); & T_j \cap L \neq \phi \end{cases}$$

### 3. Proposed Methods

#### 3.1. Algorithm for triangular bipolar fuzzy numbers

**Step: 1** construct a network with vertex and edges. The triangular bipolar fuzzy numbers associated with edges has a length transfer from each node to node.

**Step: 2** Calculate all the possible path  $H_j$  and  $L = ((L_{MIN}, L_{MAX}))$

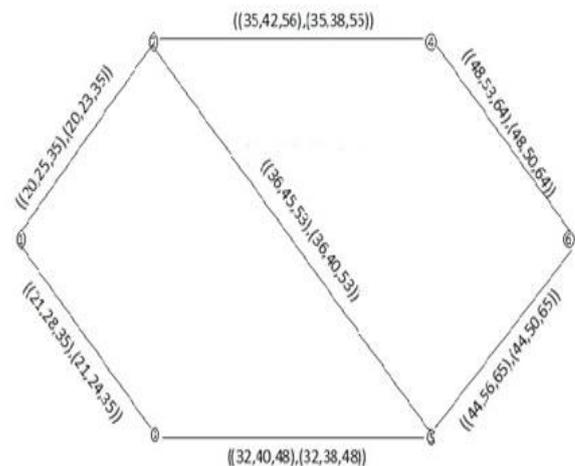
**Case (i):** The way having the upmost  $\alpha$ -cut positioning for positive membership function and the lowermost  $\alpha$ -cut positioning for negative membership function.

**Case (ii):** The way having the lowermost Euclidean ranking for both positive and negative membership function.

**Case (iii):** The way having the upmost resemblance ranking for positive membership function and the lowermost resemblance ranking for negative membership function.

#### Illustrative Example 1:

**Step: 1**



**Step: 2**

SL.No.	Path	Path Length
1	1-2-4-6	$((103,120,155), (103,111,155))$
2	1-2-5-6	$((100,126,153), (100,113,153))$
3	1-3-5-6	$((97,124,148), (97,112,148))$

**Step: 3**

$$L_{MIN} = (97,120,148) \text{ and } L_{MAX} = (103,113,155)$$

SI.No.	Path	$R(T)$	$E(L_j)$	$S(T_j, L)$	Ranking
1	1-2-4-6	(123,-123)	(9.22,2)	(29,0.96)	2
2	1-2-5-6	(126.16,-122)	(8.36,3.61)	(31.36,0)	1
3	1-3-5-6	(123.5,-119)	(4,9.27)	(27.67,0.48)	3

Here Path 1-2-5-6 is identified as fuzzy critical path.

### 3.2. Algorithm for triangular fuzzy numbers

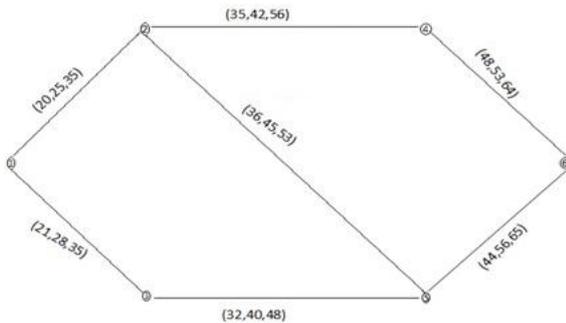
Step: 1 construct a network with vertex and edges. The triangular fuzzy numbers associated with edges has a length transfer from each node to node.

#### Step: 2 Calculate

$$g^*(j) = \max \left\{ \frac{a_{jz}^1 + 7a_{jz}^2 + a_{jz}^3}{12} + g^*(k) / j, k \in E \right\}$$

#### Illustrative Example 2:

##### Step: 1



##### Step: 2

$$a_{12}^* = 19.16; a_{24}^* = 32.1; a_{46}^* = 40.25; a_{13}^* = 21; a_{35}^* = 30; a_{56}^* = 41.75; a_{25}^* = 33.66$$

$$g^*(6) = 0; g^*(5) = 41.75; g^*(4) = 40.25; g^*(3) = 71.75; g^*(2) = 75.41; g^*(1) = 94.57$$

Then

$$g^*(1) = a_{12}^* + g^*(2) = a_{12}^* + a_{24}^* + g^*(4) = a_{12}^* + a_{24}^* + a_{46}^* + g^*(6) = 94.57$$

Path 1-2-5-6 is identified as fuzzy critical path.

## 4. Conclusion

The basic way distinguishing proof and task achievement span count is an essential task in project manager. The task achievement term is expressed in form of language variable. In this paper, the dialect factors have been changed to fuzzy numbers to be specific triangular bipolar fuzzy numbers and triangular fuzzy numbers. Two unique calculations are produced for tackling the basic way issue on a system with fuzzy circular segment lengths which encourages the undertaking supervisor to deal with the complex project in an effective way to beat the real life changes.

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