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Research paper

Extended state observer based control for DC motors

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Abstract

In this paper, an extended state observer (ESO) based control is derived for the position control of permanent magnet direct current (PMDC) motor. The implementation of the proposed control law requires only position sensor. The estimation of states and disturbance is achieved by ESO. The closed loop stability of the proposed scheme is derived. The performance of observer-controller is verified in the presence of uncertainties and load disturbance.

Keywords: *DC Motor Control; Extended State Observer; Nonlinear ESO; Position Control.*

1. Introduction

The electrical motors based motion systems has attracted a lot of attention of scientist in various field of applications [1] - [3]. It is possible to achieve precision motion control of such systems by designing a robust controller. However, the design of precision motion control for electrical motor based system is not an easy task. The design of controller has to consider modeling uncertainties i.e. parametric uncertainties, unknown external disturbance, load disturbance [4] and unstructured uncertainties i.e. dead zone, saturation, backlash, friction [5], [6]. In the presence of such uncertainties control performance may deteriorate and lead to undesirable tracking accuracy and in worst case it may lead to instability [7]. In such situation, it is desirable of control engineer to design a controller which performs as per desire in the presence of such uncertainties. If nonlinearities are known, a feedback linearization control law can be implemented. However, there are two major challenges, have to address for the design of feedback linearization control law i.e. (1) the control law required a full state vector to be available for all time, (2) capturing the exact nonlinearities and parametric uncertainties for implementation [8]. It is almost impossible task to capture all nonlinearities and parametric uncertainties accurately for exact compensation in the control law.
In the literature, many researchers have developed controllers to cater the effect of such uncertainties [9]. The adaptive control law has been designed to compensate uncertainties, but over adaptation may lead to very high gain of controller or in worst case it leads to instability of the system. The robust controller has been reported [10] in the literature to compensate the effect of disturbances. In another approach, an active disturbance rejection control (ADRC) has been developed [11] to deal with such type of uncertainties and disturbances. It has two components (1) an extended state observer and (2) a tracking differentiator. The extended state observer not only estimates uncertainties but also the internal states of the system for control law implementation. The tracking differentiator utilizes the estimated states and disturbance for the control law design. It is worth noting that the ESO estimates generalized disturbances, which includes structured uncertainties as well as unstructured nonlinearities, thus it does not requires accurate model information [12].
In this paper, the feedback linearization control law based on the linear ESO and nonlinear ESO is developed for the position control of DC motor. The dynamics of current loop is also considered in the design of control law. The unmeasurable states and generalized disturbance are estimated using the ESO. To verify the performance of the proposed scheme, various uncertainties and load disturbance conditions have been introduced for the control of a DC motor.
The rest of the paper is organized as: Section 2 deals with the modeling of DC motor, Section 3 discusses the extended state observer and Section 4 derives the control based on the extended state observer followed by the derivation of stability of the closed loop system in Section 5. Section 6 gives the simulation results with various scenarios and Section 7 concludes the paper.

2. Modeling of DC motor

In this paper, the problem of position control of a permanent magnet DC motor is considered.

2.1. A PMDC motor dynamics

The dynamics of a PMDC motor is represented by [13],

$$\dot{\theta} = \omega \tag{1}$$

$$J_m\ddot{\theta} = K_t\psi - B_m\dot{\theta} - T_l \tag{2}$$

$$L_m\dot{\psi} = -K_b\dot{\theta} - R_m\psi + V_m \tag{3}$$

Where θ is a motor shaft position, ω is the motor angular velocity, ψ is the armature current, V_m is the voltage across armature, T_l is a load torque applied mechanically on the motor shaft, J_m is the equivalent moment of inertia, B_m is the equivalent viscous friction, R_m is the equivalent armature resistance, L_m is the equivalent

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armature inductance, K_b is the back emf constant and K_t is the torque constant.

2.2. State space representation

In this paper, it is assumed that the motor shaft position ($y = \theta$) is the only measurable signal of the system and considered as an output of the system. Selecting the motor shaft position ($x_1 = \theta$), the shaft velocity ($x_2 = \omega$) and the shaft acceleration ($x_3 = \dot{\omega}$) as state variables and the armature voltage ($u = V_m$) as a control input of the system. The system (1) − (3) can be written into state space form as

$$\dot{x} = Ax + Bu + Bd$$

$$y = Cx \tag{4}$$

Where, $x = [\theta \quad \omega \quad \dot{\omega}]^T$ is the state vector, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\frac{K_t K_b}{J_m L_m} & -\frac{B_m}{J_m} \end{bmatrix}$ is the system parameter matrix, $B = \begin{bmatrix} 0 & 0 & \frac{K_t}{J_m L_m} \end{bmatrix}^T$ is the input matrix, $C = [1 \quad 0 \quad 0]$ is the output matrix and $d = -\frac{J_m L_m}{K_t}\left(\frac{K_t R_m}{J_m L_m}\psi + \frac{\dot{T}_l}{J_m} + \xi(x,t)\right)$ is the disturbance acting on the system (4), which includes parametric uncertainties and external disturbances acting on the system.

3. Extended state observer

Consider an $n-$th order, single input-single output nonlinear dynamical system described by [11]

$$x^n = a(x, \dot{x}, \dots, x^{n-1}, w) + bu \tag{5}$$

Where $a(.)$ represents the dynamics of the plant and the disturbance, $w(t)$ is an unknown disturbance, u is the control signal, and x is the measured output. Let $a(.) = a_o(.) + \triangle a$ and $b = b_o + \triangle b$ where ao(.) and b_o are the best available estimates of a and b respectively and $\triangle a$ and $\triangle b$ are their associated uncertainties. Defining the uncertainty to be determined as $d = \triangle a + \triangle bu$ and designating it as an extended state, x_{n+1}, the dynamics (5) can be re-written in a state-space form as,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\vdots$$

$$\dot{x}_n = x_{n+1} + a_o + b_o u$$

$$\dot{x}_{n+1} = h$$

$$y = x_1 \tag{6}$$

Where h is the rate of change of the uncertainty and an unknown external disturbance, i.e., $h = \dot{d}$ and is assumed to be an unknown but bounded function. By making d as a state, however, it is now possible to estimate it by using a state estimator.
Writing the dynamics of (6) into compact form as,

$$\dot{x} = A_0 x + B_0 u + Eh$$

$$y = C_0 x \tag{7}$$

Where,

$$A_0 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{0_1} & a_{0_2} & a_{0_3} & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, B_0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b_0 \\ 0 \end{bmatrix}, E = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, C_0 = [1 \quad 0 \quad 0 \quad \cdots \quad 0]$$

3.1. Linear ESO [8]

Consider a linear extended state observer of the form,

$$\dot{\hat{x}}_1 = \hat{x}_2 + \beta_1 e$$

$$\dot{\hat{x}}_2 = \hat{x}_3 + \beta_2 e$$

$$\vdots$$

$$\dot{\hat{x}}_n = \hat{x}_{n+1} + \beta_n e + a_o + b_o u$$

$$\dot{\hat{x}}_{n+1} = \beta_{n+1} e \tag{8}$$

Where $e = y - \hat{x}_1 = x_1 - \hat{x}_1$ and $\hat{x}_{n+1}$ is an estimate of the uncertainty. Writing the observer dynamics (8) into compact form as

$$\dot{\hat{x}} = A_0 \hat{x} + B_0 u + \beta e$$

$$\hat{y} = C_0 \hat{x} \tag{9}$$

Where, $\hat{x} = [x_1 \quad x_2 \quad \cdots \quad x_n \quad x_{n+1}]^T$ is the state and disturbance estimation vector, $\beta = [\beta_1 \quad \beta_2 \quad \beta_3 \quad \cdots \quad \beta_n \quad \beta_{n+1}]^T$ is the observer gain matrix and $e = y - \hat{y}$ is the output estimation error of the linear ESO. The error dynamics of linear ESO and criteria for selection of the gain matrix β are discussed in the Section 5.

3.2. Non-linear ESO [11]

The non-linear extended state observer is represented as

$$\dot{\hat{x}}_1 = \hat{x}_2 + \beta_1 g_1(e_{0_1})$$

$$\dot{\hat{x}}_2 = \hat{x}_3 + \beta_2 g_2(e_{0_1})$$

$$\vdots$$

$$\dot{\hat{x}}_n = \hat{x}_{n+1} + \beta_n g_n(e_{0_1}) + a_o + b_o u$$

$$\dot{\hat{x}}_{n+1} = \beta_{n+1} g_{n+1}(e_{0_1}) \tag{10}$$

The quantities β_i are the observer gains while $g_i(.)$ are the set of suitably constructed nonlinear gain functions satisfying $e_{0_1} g_i(e_{0_1}) > 0, \forall e_{0_1} \neq 0$ and $g_i(0) = 0$. If one choses the nonlinear functions, $g_i(.)$, and their related parameters properly, the estimated state variables $\hat{x}_i$ are expected to converge to the respective states of the system $\hat{x}_i$, i.e., $\hat{x}_i \longrightarrow x_i$, $i = 1,2,\dots,n+1$. One nonlinear function became popular in the design of nonlinear ESO is expressed as [11],

$$g_i(e_{0_1}, \alpha_i, \delta) = \begin{cases} |e_{0_1}|^{\alpha_i} sign(e_{0_1}), & |e_{0_1}| > \delta \\ \frac{e_{0_1}}{\delta^{1-\alpha_i}}, & |e_{0_1}| \leq \delta \end{cases} \tag{11}$$

Where $\delta > 0$ and $0 < \alpha_i < 1$. This function has high gain when the error is small and has low gain when the error is large. Writing the observer dynamics (10) into compact form as

$$\dot{\hat{x}} = A_0 \hat{x} + B_0 u + \beta g_i(.)$$

$$\hat{y} = C_0\hat{x} \tag{12}$$

Where the vector of nonlinear function $g_i(.)$ is defined in (11). The selection of observer gain vector $\beta g_i(.)$ is discussed in the Section 5.

4. The controller

In this section, the control law is derived for the system (4). If the states and uncertainties are available, the control law can be implemented as [8]

$$u = -B^{-1}[K(x - R) - \ddot{r}] - d \tag{13}$$

Where, the inverse of input matrix B has not null i.e. $B^{-1} \neq 0$ and reference vector R is given by $R = [r \quad \dot{r} \quad \ddot{r}]^T$. To implement the control law, the states and uncertainty are estimated using an ESO. The ESO based control law can be implemented as

$$u = -B^{-1}[K(\hat{x} - R) - \ddot{r}] - \hat{d} \tag{14}$$

The control law (14) is implementable using ESO designed in (10). The block diagram of the closed loop scheme is shown in Fig.1. The closed loop stability of the system (4) in the next section.

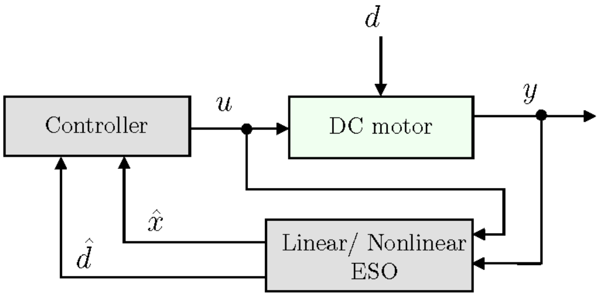


Fig. 1: The Block Diagram of the Overall Scheme.

5. Stability analysis

In this section, the closed loop stability of the system (4) with the nonlinear ESO based controller derived in (10) and (14). The ultimate boundedness of the closed-loop system is proved [14].

5.1. Observer error dynamics

Defining the state observer error dynamics as,

$$\tilde{x} = x - \hat{x}$$

$$\tilde{y} = C\tilde{x} \tag{15}$$

The observer error dynamics is obtained by subtracting (9) from (7) as,

$$\tilde{x} = (A_0 - \beta C)\tilde{x} + Eh \tag{16}$$

Assuming that the pair (A_0, C_0) is observable and the rate of change of disturbance h is bounded. By selecting the eigenvalue of vector $(A_0 - \beta C)$ in the left hand plane at appropriate location, the observer error dynamics can be stabilized.
On the similar lines for nonlinear observer dynamics can be obtained as

$$\tilde{x} = A_0\tilde{x} - \beta\ g(C\tilde{x}) + Eh \tag{17}$$

Next the closed-loop stability of the system is derived.

5.2. Observer error dynamics

Defining the tracking error as $e_t = x - R$, the closed loop dynamics is given by,

$$\dot{e}_t = (A - BK)e_t - BK\tilde{x} - B\tilde{d} \tag{18}$$

Where, $\tilde{d} = d - \hat{d}$ is the disturbance estimation error. By assuming the pair (A, B) is controllable, the closed loop stability can be assured by placing the eigenvalue of $(A - BK)$ in the left half plane at appropriate location. Combining (18) and (17) gives

$$\begin{bmatrix}\dot{e}_t \\ \dot{\tilde{x}}\end{bmatrix} = \begin{bmatrix}A - BK & -[BK\ B] \\ 0 & A_0 - \beta C\end{bmatrix}\begin{bmatrix}e_t \\ \tilde{x}\end{bmatrix} + \begin{bmatrix}0 \\ E\end{bmatrix}h \tag{19}$$

By placing the observer and controller poles appropriately in the left half plane. Thus, the closed-loop stability of the system is proved.

5.3. How to select the parameters of nonlinear ESO

One can implement first the linear ESO with eigenvalues in the left half plane. Once the performance of the linear ESO is satisfactorily, the nonlinear function can be introduced by keeping gain vector same. The parameter δ decides a region inside which the observer act as a high gain observer and outside this region it acts as a nonlinear observer. Thus the parameter δ should be small enough to grantee the robust performance. The parameters α_i can be selected as $1 > \alpha_1 > \alpha_2 > \cdots > \alpha_{n+1} > 0$.

6. Simulation results

In this section, the simulations were carried out on the position control of PMDC motor to verify the performance of proposed observer-controller scheme. The nominal parameters of PMDC motor considered for simulations are given in Table 1. The efficacy of the proposed scheme is shown by considering different cases i.e.

- Case-I: Parametric uncertainty
- Case-II: Constant load disturbance
- Case-III: Sinusoidal load disturbance
- Case-IV: Performance comparison of linear ESO and nonlinear ESO

For all cases, the poles of controller are set at $[-10 \quad -10 \quad -10]^T$ and the observer poles are set at $[-140 \quad -140 \quad -140 \quad -140]^T$. For all cases the plant initial conditions set to $x(0) = [0.1 \quad 0 \quad 0]^T$. In the first case, the initial condition of observer is set to $\hat{x}(0) = [0 \quad 0 \quad 0 \quad 0]^T$ and for rest of the cases it is set to $\hat{x}(0) = [0.1 \quad 0 \quad 0 \quad 0]^T$. For all cases, the reference vector is set to $R = [0 \quad 0 \quad 0]^T$.

Table 1: Nominal Parameters of the PMDC Motor

Parameters	Nominal value	Unit
R_m	6.898	Ω
L_m	27	mH
J_m	0.032	kg/m^2
B_m	0.0022	Nm/rad
K_b	1.073	
K_t	1.073	

6.1. Case-I

In this case, $+10\%$ of parametric uncertainty is introduced in the nominal parameters of the plant. The tracking performance of the system with the proposed scheme is shown in Fig. 1. The plots of internal states and disturbance with their estimation are shown in Fig. 1 – Fig. 1. The estimation performance of the observer is shown in Fig. 3. The estimation errors go to zero from their initial mismatch. The performance of the proposed scheme is verified with different level of uncertainties and received similar performances, the results are omitted here.

6.2. Case-II

In this case, a constant load disturbance $T_l = 1$ is introduced in the system. The tracking performance of the proposed scheme and the control input are shown in Fig. 4. In-spite of load disturbance, the output is tracked accurately by the observer and the output goes to zero from its initial condition. The control input is adjusted according to load disturbance value. The plots of other states and their estimations are omitted here to save space.

6.3. Case-III

In this case, a sinusoidal load disturbance $T_l = 0.1\sin(t)$ is introduced in the system. The tracking of output is shown in Fig. 4 and the control input is shown in Fig. 4. The rate of change of disturbance in this case is non-zero, thus system is driven by h. The performance is still satisfactory in the presence of sinusoidal disturbance. The reason for satisfactory performance is accurate estimation of states and disturbance by the ESO. The performance can be improved by implementing the higher order ESO or nonlinear ESO. The control input is adjusted as load disturbance is varied.

6.4. Case-IV

In this case, the state and disturbance estimation performance of linear ESO is compared with nonlinear ESO. The controller and observer gain vectors are kept same as previous cases set for linear ESO. The additional parameters of nonlinear ESO are set to $\delta = 0.01, \alpha_1 = 0.25, \alpha_2 = 0.15, \alpha_3 = 0.1, \alpha_4 = 0.0001$. The comparative performance of the nonlinear ESO and linear ESO is shown in Fig. 6. The state and disturbance estimation is improved with nonlinear ESO as compare to linear ESO. The reason for improvement in nonlinear ESO's performance is the nonlinear function present in the structure. Similar results of improvement are obtained with nonlinear ESO in the presence of parametric uncertainties and load disturbances. The results are not shown here to save space.

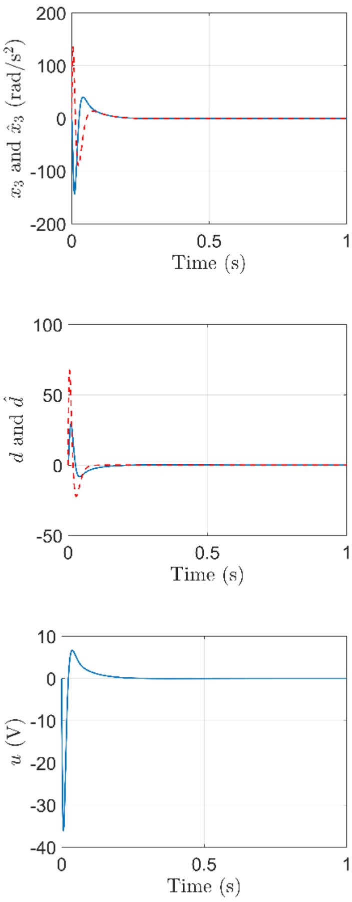
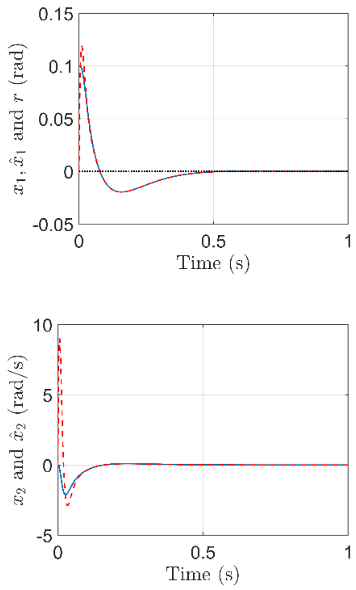
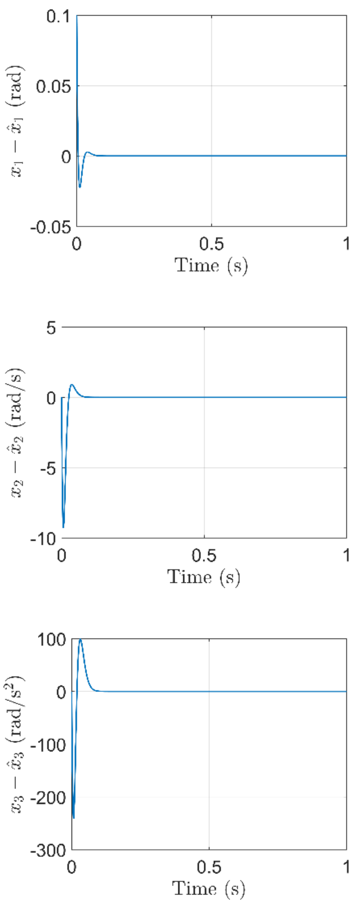


Fig. 2: Case-I: Plant States and Disturbance, Estimation and Control Input: (A) x_1 (Solid Line), $\hat{x}_1$ (Dashed Line) and r (Dotted Line), (B) x_2 (Solid Line) and $\hat{x}_2$ (Dashed Line), (C) x_3 (Solid Line) and $\hat{x}_3$ (Dashed Line), (D) d (Solid Line) and $\hat{d}$ (Dashed Line), (E) u.





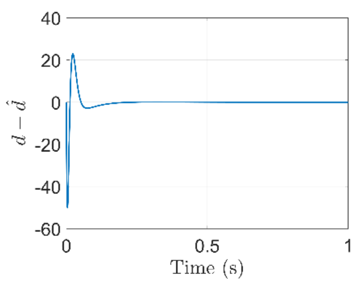


Fig. 3: Case-I: State and Disturbance Estimation Errors: (A) $x_1 - \hat{x}_1$, (B) $x_2 - \hat{x}_2$, (C) $x_3 - \hat{x}_3$, (D) $d - \hat{d}$.

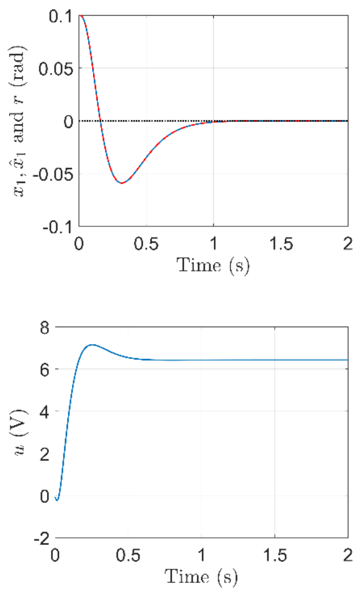


Fig. 4: Case-II: Motor Position and Its' Estimation and Control Input with Constant Load Disturbance: (A) x_1 (Solid Line), $\hat{x}_1$ (Dashed Line) and r (Dotted Line), (B) r.

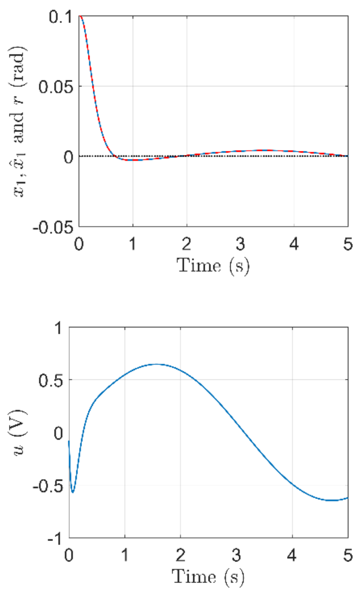


Fig. 5: Case-III: Motor Position and Its' Estimation and Control Input with Sinusoidal Load Disturbance: (A) x_1 (Solid Line), $\hat{x}_1$ (Dashed Line) and r (Dotted Line), (B) u.

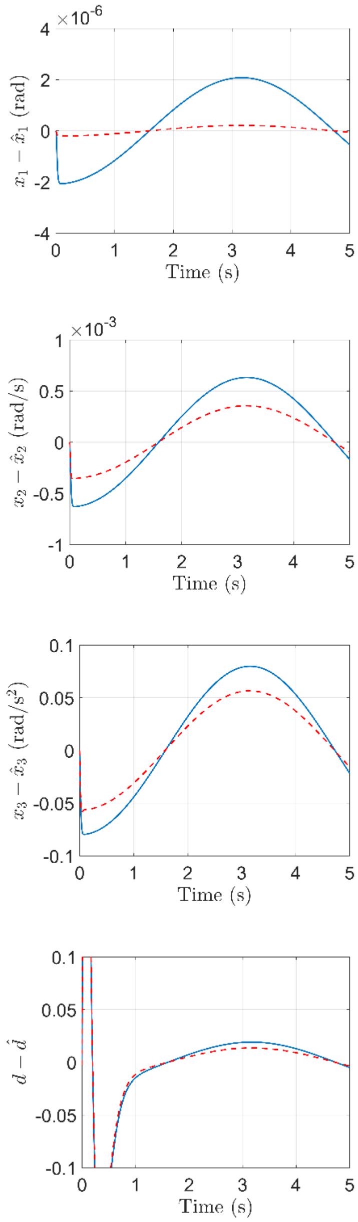


Fig. 6: Case-IV: Comparative Performance of Linear ESO (Solid Line) and Nonlinear ESO (Dashed Line) in the State and Disturbance Estimation Errors: (A) $x_1 - \hat{x}_1$, (B) $x_2 - \hat{x}_2$, (C) $x_3 - \hat{x}_3$, (D) $d - \hat{d}$.

7. Conclusion

In this paper, the position control of permanent magnet DC motor is derived. The feedback linearization control law has been made implementable based on the extended state observer. The performance of proposed observer−controller scheme is verified with different types of load disturbances and uncertainties. The results show effectiveness of the proposed scheme. The performance of nonlinear ESO is compared with the linear ESO. The noticeable performance improvement has been observed in the states and the disturbance estimation with the nonlinear ESO as compare to the linear ESO.

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References

[1] M. Iwasaki, K. Seki, and Y. Maeda, “High-precision motion control techniques: A promising approach to improving motion performance,” *IEEE Industrial Electronics Magazine*, vol. 6, no. 1, pp. 32–40, 2012. https://doi.org/10.1109/MIE.2012.2182859.

[2] C. Hu, B. Yao, and Q. Wang, “Adaptive robust precision motion control of systems with unknown input dead-zones: A case study with comparative experiments,” *IEEE Transactions on Industrial Electronics*, vol. 58, no. 6, pp. 2454–2464, 2011. https://doi.org/10.1109/TIE.2010.2066535.

[3] A. Hughes and W. Drury, *Electric motors and drives: fundamentals, types and applications*. 1em plus 0.5em minus 0.4em Newnes, 2013. https://doi.org/10.1016/B978-0-08-098332-5.00001-2.

[4] J. Yao, Z. Jiao, D. Ma, and L. Yan, “High-accuracy tracking control of hydraulic rotary actuators with modeling uncertainties,” *IEEE/ASME Transactions on Mechatronics*, vol. 19, no. 2, pp. 633–641, 2014. https://doi.org/10.1109/TMECH.2013.2252360.

[5] S. Villwock and M. Pacas, “Time-domain identification method for detecting mechanical backlash in electrical drives,” *IEEE Transactions on Industrial Electronics*, vol. 56, no. 2, pp. 568–573, 2009. https://doi.org/10.1109/TIE.2008.2003498.

[6] W. Sun, Z. Zhao, and H. Gao, “Saturated adaptive robust control for active suspension systems,” *IEEE Transactions on Industrial Electronics*, vol. 60, no. 9, pp. 3889–3896, 2013. https://doi.org/10.1109/TIE.2012.2206340.

[7] G. Tao and F. L. Lewis, *Adaptive control of nonsmooth dynamic systems*. Springer Science & Business Media, 2013.

[8] S. E. Talole, J. P. Kolhe, and S. B. Phadke, “Extended-state-observer-based control of flexible-joint system with experimental validation,” *IEEE Transactions on Industrial Electronics*, vol. 57, no. 4, pp. 1411–1419, 2010. https://doi.org/10.1109/TIE.2009.2029528.

[9] A. Radke and Z. Gao, “A survey of state and disturbance observers for practitioners,” in *American Control Conference, 2006,* IEEE, 2006, pp. 6–pp.11. https://doi.org/10.1109/ACC.2006.1657545.

[10] S. Li, J. Yang, W.-H. Chen, and X. Chen, *Disturbance observer-based control: methods and applications*. CRC press, 2014.

[11] J. Han, “From pid to active disturbance rejection control,” *IEEE transactions on Industrial Electronics*, vol. 56, no. 3, pp. 900–906, 2009. https://doi.org/10.1109/TIE.2008.2011621.

[12] B.-Z. Guo and Z.-l. Zhao, “On the convergence of an extended state observer for nonlinear systems with uncertainty,” *Systems & Control Letters*, vol. 60, no. 6, pp. 420–430, 2011. https://doi.org/10.1016/j.sysconle.2011.03.008.

[13] M. W. Spong, S. Hutchinson, M. Vidyasagar *et al.*, *Robot modeling and control*. Wiley New York, 2006.

[14] B. Barmish and G. Leitmann, “On ultimate boundedness control of uncertain systems in the absence of matching assumptions,” *IEEE Transactions on Automatic Control*, vol. 27, no. 1, pp. 153–158, 1982. https://doi.org/10.1109/TAC.1982.1102862.