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Research paper

Experimental and Theoretical Testing Results of Reinforced Concrete Columns under Biaxial Bending

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Abstract

The experimental tests data of reinforced concrete columns of a rectangular profile made of heavy concrete on biaxial bending with axial force are presented. The inclination angle of the external load plane to the vertical axis of inertia of the section varied in the range from 14º to 66º. The tests were conducted to study the work of the biaxial bended elements under load and to verify the developed method for strength analysis of such elements. It has been confirmed that the ultimate compressed fibrous strains of concrete depend not on the shape of the section, but on the shape of the concrete compressed area. The general method of strength analysis of biaxial bended reinforced concrete columns is developed. The problem of difficulty applying nonlinear deformation model in the study of biaxial deformed elements is successfully solved by the introduction of the stress-strain relations for concrete and reinforcement and extremal strength criterion. The results of the tests have good correspondence with theoretical calculations, which proves the expediency of using the developed method for the strength analysis.

Keywords: *column; reinforced concrete; biaxial bending; axial force; strength; test.*

1. Introduction

As the experience of building structures exploitation shows, the biaxial bending with axial force is experienced by the columns of industrial and civil framework buildings, the supports of bridges, overpasses, elevated high-ways, bunkers, silos and water towers, separate elements of masts, cooling towers, deformers, frame foundations under turbo-generators of electric rosters, and many other structures also.

The most of the reinforced concrete elements of buildings with structural systems, built for notypical and individual projects, operate in the case of biaxial bending.

Unconsole Uncapital Ungirder structural system of buildings can serve as an example of these systems [1], which has become widespread in the Ukrainian construction industry. This system, in particular, is characterized by wide possibilities of planning decisions. The edge and middle columns of Unconsole Uncapital Ungirder frame undergo biaxial deformation, which arises due to the peculiarities of overcolumned floor slab to column joints arrangement [2]. The absence of consoles in the joints of overcolumned floor slabs with columns causes biaxial bending.

All reinforced concrete elements under axial load sustain biaxial bending to a greater or lesser extent, since it arises not only because of the load application, but also because other causes and reasons: uneven distribution of temperature deformations, technological inaccuracies in the manufacture and installation of structures, mechanical damage [3] and changes of sections during the reconstruction of buildings and structures as a whole, and their parts in particular.

Technological inaccuracies in the manufacture of reinforced concrete structures include the displacement of the reinforcement from the projected position with its enclosure in the formwork, the discrepancy between the shape and size of the design data due to shuttering and other factors. As a result of these reasons, the displacement of the points of application of resultant forces in compressed (N_c) or tensioned (N_s) areas of the cross section from the main central axes of its inertia occur, that is, the element undergoes biaxial bending.

Today, reinforced concrete structures working under biaxial bending are often designed on axial loading and bending moment in the orthogonal principal inertia planes. Designers have to resort to this simplification, since there is not yet a perfect and at the same time sufficiently detailed method for designing of biaxial bended elements. And this, of course, leads to a distortion of the actual picture of the work of the design and, as a consequence, to overpay the materials and even – to accidents.

The analysis of recent publications [4 – 7] shows that it is most expedient to improve the existing proposals for calculating the bearing capacity of biaxial bended elements by applying a nonlinear deformation model, an extreme strength criterion, and stress-strain diagrams of materials.

Experimental studies of many scientists [8 – 10] suggest that there is a volumetric redistribution of stresses within the compressed part of reinforced concrete elements section at the ultimate limit state.

Characteristic features of such redistribution are both the stress reduction and the increase of the strains $\varepsilon_{c(1)}$ to the limit values ε_{cu1} in the ultimate distant from the neutral axis concrete deformed fibers. As a result, when the reinforced concrete element is de-

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stroying, the limit fibrous strains of the concrete compressed area ε_{cu1} essentially exceed the critical values of the concrete strains (≈2 ‰) ε_{c1} in standard concrete prism models under axial loading [6 – 8]. But the phenomenon of the supercritical state and the corresponding model of the subcritical stage of the deformation mode of reinforced concrete elements in Eurocode 2 is not yet used, and the scientific works devoted to the application of such a state of reinforced concrete elements in their calculations have a lot of directions [9 – 12].

The foregoing underlines that for using in practice, as well as it used to be earlier, there is a problem of developing a deformation model for a deformation mode of reinforced concrete element at ultimate limit state using a complete diagram of the concrete state. This is especially true for the reinforced concrete elements that work under conditions of biaxial bending.

In order to solve the existing problem the experimental based features of its deformation during the load increasing are laid in the basis of the deformation mode model of the biaxial bended reinforced concrete element at ultimate limit state.

2. Construction of experimental samples

In order to determine the strength of reinforced concrete columns subjected to biaxial bending and axial force, to study their work under load, and to obtain experimental values of the parameters characterizing the deformation mode of the column during structural failure, experimental tests were carried out. The influence of the angle β of inclination of the external load plane to the vertical axis of inertia of the cross section, as well as the number and location of the principal reinforcement on the strength of the columns, the position of the neutral axis, and the fibre strains of the concrete of the compressed area were studied.

Three series of experimental columns (Fig. 1) are tested depending on the number and location of the longitudinal principal reinforcement.

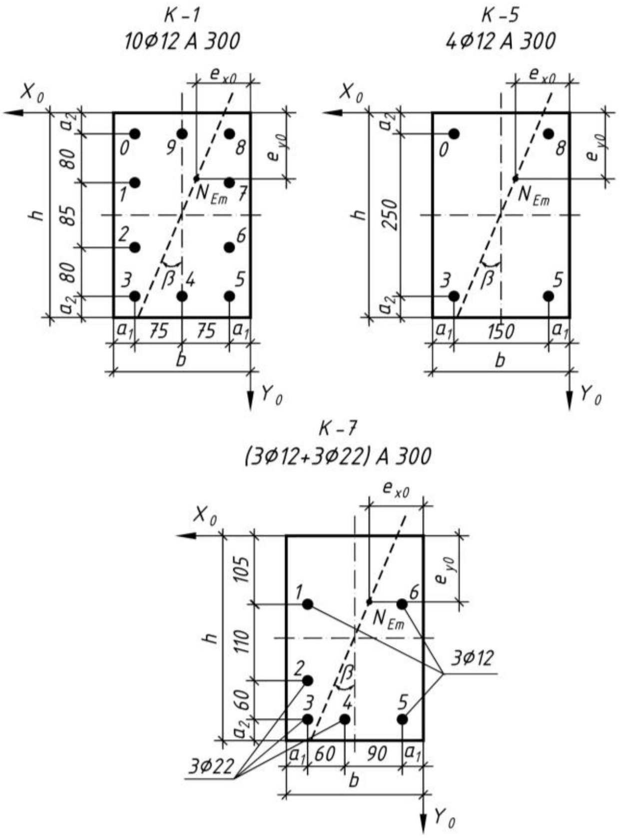

Fig. 1: The construction of experimental column samples.

In order to determine the physicomechanical characteristics of the reinforcement, the specimens (l = 600 mm) were cut from each reinforcing bar, which were then tested for tension on the UIM-50 press. As a result of the tests, the characteristic yield strength of the reinforcing steel f_{yk} have been determined (Table 1).

To determine the properties of concrete, compression tests were carried out on concrete prisms (150x150x600 mm) and cubes (150x150x150 mm) made of a concrete mix that was used for columns. The tests were carried out on a hydraulic press 2PG-125. As a result of the tests, the secant modulus of elasticity of concrete E_{cm} and mean value of concrete prismatic compressive strength $f_{cm,prism}$ have been determined (Table 1).

Table 1: Characteristics of experimental column samples

Sample code	Section dimensions		Eccentricities of N_{Em} position		Characteristics of concrete		Characteristics of reinforcement	
	h, mm	b, mm	e_{xo}, mm	e_{yo}, mm	$f_{cm,prism}$, MPa	$E_{cm}\cdot10^{-3}$, MPa	Diameter, class	f_{yk}, MPa
K-1-15	294	204	82.0	100.0	24.8	28.8	12A300	343
K-1-1	308	204	72.0	83.0	19.2	27.0	12A300	346
K-1-3	296	194	67.0	83.0	19.6	27.0	12A300	340
K-1-4	312	202	61.0	62.5	20.1	27.0	12A300	346
K-1-6	303	201	50.5	31.5	20.1	27.0	12A300	342
K-1-5	301	202	41.0	6.5	20.1	27.0	12A300	343
K-1-11	290	204	32.0	-24.0	26.1	29.3	12A300	346
K-1-20	303	204	82.0	132.0	15.7	25.3	12A300	379
K-1-19	312	206	73.0	126.0	15.7	25.3	12A300	378
K-1-16	304	203	61.5	112.0	26.0	29.3	12A300	349
K-1-13	300	196	48.0	100.0	24.8	28.8	12A300	341
K-1-2	300	204	32.0	80.0	20.1	27.1	12A300	350
K-1-9	296	204	22.0	68.0	24.4	28.7	12A300	349
K-1-7	298	205	2.0	49.0	24.4	28.7	12A300	350
K-1-14	300	205	56.0	130.0	24.2	28.6	12A300	350
K-1-10	295	205	31.0	117.5	24.2	28.6	12A300	345
K-1-12	294	205	8.0	107.0	24.2	28.6	12A300	344
K-5-3	324	204	92.0	138.0	11.3	22.8	12A300	331
K-5-4	310	220	80.0	108.0	11.9	23.2	12A300	333
K-5-1	304	204	72.0	80.0	11.9	23.2	12A300	333
K-5-2	299	201	61.0	56.0	11.9	23.2	12A300	336
K-7-2	311	205	82.0	108.0	11.5	22.8	12A300 22A300	390 360
K-7-4	314	202	71.0	87.0	11.5	22.9	12A300 22A300	378 355
K-7-1	315	204	72.0	86.0	20.5	28.1	12A300 22A300	397 355
K-7-3	309	202	61.0	60.0	18.9	26.6	12A300 22A300	395 355

3. Forms of the compressed zone of the experimental columns section

During testing the experimental column samples under biaxial bending at loading levels in the range from 0.7 to 1, and at the plastic deformation of the reinforcement in the tensioned zone, a neutral line in the section moves insignificantly towards compressed area parallel to the top of the crack in tensioned concrete. It has been established in the experiments that, in the case of biaxial bending in reinforced concrete elements of a rectangular profile during the loading process, only three cases of compressed area forms are formed (Fig. 2): pentagonal (*a*), trapezoidal (*b*) and triangular (*c*).

In the general case since the beginning of the loading of the reinforced concrete element and until the moment it reaches the supercritical state, there is only one sequence of the formation of the forms of the compressed area, regardless of the number of possible combinations of factors of influence on the formation process: a pentagon is transformed into a trapezoid, and then a triangle forms from the trapezoid.

At the final stage of the shaping process in the biaxial bended reinforced concrete elements, which coincides with the stage of subcritical deformation mode, the shape of the compressed area with the same frequency can take the form of all three shapes: a triangle, a trapezoid, and a pentagon. The frequency of occurrence of this or that form depends mainly on the value of the external longitudinal compressive force application eccentricity.

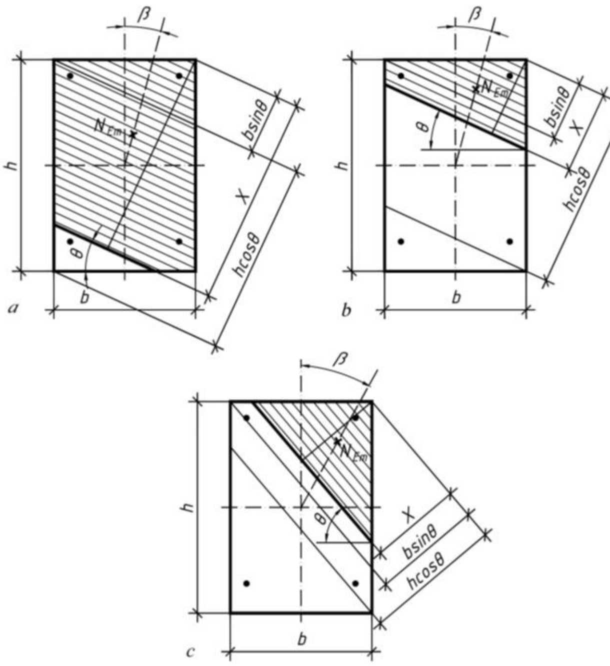

Fig. 2: Concrete compressed area forms of the experimental columns: (*a*) pentagonal; (*b*) trapezoidal; (*c*) triangular.

In the biaxial bended elements, the pentagonal form of the compressed area is peculiar to force deformation from the action of the longitudinal compressive force applied with the small eccentricities. Under force deformation of reinforced concrete elements by the longitudinal compressive force applied with large eccentricities, the trapezoidal and triangular form of the compressed area may occur with the same frequency. Although it should be noted that the appearance of the triangular shape of the compressed area of the cross-section in the design of reinforced concrete, working under biaxial bending with axial force, should be avoided.

The analysis of the change of the neutral axis position both in the process of loading and at the moment of destruction of the experimental samples shows that, in the case of biaxial bending, it in the general case is not perpendicular to the plane of external forces action. Its position in the normal section of the element has a multifactor dependence, which in the degree of influence includes the external force plane inclination angle, the shape of the cross-section, the quantity and location of the reinforcement, the class of concrete, the level of loading by external load, the presence of cracks. In spite of such a large number of different factors being present at any one of their combinations, it is possible to achieve orthogonality between the neutral axis and the external load plane by the reinforcement location.

4. Strains of concrete and reinforcement of the experimental columns

In the columns, which were tested with constant loading rate, after reaching, respectively, the extreme value of moment and longitudinal force, that is, in the supercritical stage, there was a rapid destruction of them regardless of the shape of the compressed area and the concrete class. The destruction had brittle or plastic character depending on the value of the reinforcement ratio.

In the columns that were tested at a constant rate of deformation, the subsequent growth of concrete strains in the range ε_{c1} ... ε_{cu1}, after the perception the maximal value of the destructive N_{Em} force by the test elements, was accompanied by a steady decrease in its magnitude. The length of the decay area of $N_{Em} - \varepsilon_c$ diagram depended on the percentage of reinforcement, the shape of the compressed area, the class of concrete.

In the columns, the compressed area of which at the time of their destruction was a triangle, the corresponding values of the fibrous strains ε_{cu1} were greater than in the columns with a rectangular form of the compressed area under all other identical conditions. For the same physical-mechanical and geometric parameters, the ultimate values of the fibrous concrete strains ε_{cu1} in the loaded elements with the trapezoidal form of the compressed area had intermediate values, that is, those that is located between the values of the ultimate fibrous concrete strains ε_{cu1} for elements with triangular and rectangular forms of the compressed area.

The value of the ultimate fibrous concrete strains ε_{cu1} in the most distant from the neutral line of the point of the compressed area in the biaxial bended elements of the smaller class concrete was greater than in the elements of the concrete of the larger classes (Table 2).

Table 2: Comparison of theoretical results of the ultimate concrete strains level of biaxial bended reinforced concrete columns with experimental data

Sample code	k*	Load plane inclination angle, β^o			Ultimate concrete strains level, η_u**		
		Theor.	Exp.	$\beta^o_{Theor.}$ / $\beta^o_{Exp.}$	Theor.	Exp.	$\eta_{u,Theor.}$ / $\eta_{u,Exp.}$
K-1-15	2.20	20.8	23.0	0.90	1.625	1.75	0.93
K-1-1	2.46	26.3	24.0	1.10	1.870	1.37	1.36
K-1-3	2.46	27.9	27.0	1.03	1.870	1.66	1.13
K-1-4	2.38	27.0	22.0	1.23	1.870	1.24	1.51
K-1-6	2.38	24.4	24.0	1.02	1.870	1.64	1.14
K-1-5	2.38	23.3	22.0	1.06	1.875	1.24	1.51
K-1-11	2.16	21.1	21.0	1.00	1.875	1.73	1.08
K-1-20	2.65	43.7	32.0	1.37	1.750	1.56	1.12
K-1-19	2.65	43.5	32.0	1.36	1.875	1.40	1.34
K-1-16	2.17	41.9	44.0	0.95	1.750	1.20	1.46
K-1-13	2.20	43.8	42.0	1.04	1.750	1.98	0.88
K-1-2	2.39	42.8	45.0	0.95	1.875	1.32	1.42
K-1-9	2.22	43.5	44.0	0.99	1.875	1.35	1.39
K-1-7	2.22	44.8	42.0	1.07	1.875	1.20	1.56
K-1-14	2.22	60.2	61.0	0.99	1.625	1.54	1.06
K-1-10	2.22	62.5	61.0	1.02	1.750	1.27	1.38
K-1-12	2.22	65.5	66.0	0.99	1.750	1.36	1.29
K-5-3	3.00	12.3	14.0	0.88	1.750	1.46	1.20
K-5-4	2.94	15.6	19.0	0.82	1.750	1.46	1.20
K-5-1	2.94	18.2	19.0	0.96	2.130	1.63	1.31
K-5-2	2.94	19.8	23.0	0.86	2.000	1.64	1.22
K-7-2	2.97	37.0	22.0	1.68	1.875	2.40	0.78
K-7-4	2.97	31.4	27.0	1.16	2.000	2.50	0.80
K-7-1	2.45	34.4	27.0	1.27	1.875	1.760	1.07
K-7-3	2.45	29.1	25.0	1.16	1.875	2.010	0.93
Expectation				1.0749			0.8982
Arithmetical mean deviation				0.1385			0.0475
Dispersion				0.0363			0.0035
Root mean square deviation				0.1905			0.0589
Variation coefficient				0.1772			0.0656

* $k = 1{,}05\, E_{cm} \cdot \varepsilon_{c1} / f_{cm,prism}$;
** $\eta_u = \varepsilon_{cu1} / \varepsilon_{c1}$.

In the overcritical stage, the stresses and strains in the reinforcement (Table 3) and in the concrete in the sections between cracks and in the areas with cracks were distributed unevenly: they varied from the maximum in the crack section to the minimum in the section in the middle between cracks. An explanation for this phenomenon is the effect of the overcracked part of tensioned concrete on the work of the biaxial bended reinforced concrete element, which can be taken into account by the coefficient ψ_s. The value of this coefficient for biaxial bending depend on the values of the load plane inclination angle and the eccentricities of the longitudinal force application. The value of this coefficient, within the limits of the change in the level of loading from 0.5 to 0.9 in the usual reinforced concrete element, varied in the range of 0.824 ... 0.92 and by the physical essence for the elements at the normal loading corresponds to its essence for the biaxial bended elements.

Table 3: Comparison of theoretical results of the reinforcement bars strains of biaxial bended reinforced concrete columns with experimental data

Sample code	f_{yk}, MPa	Bars numbers in the section (Fig. 1)									
		0	1	2	3	4	5	6	7	8	9
		$\sigma_{si,exp}$									
		$\sigma_{si,theor}$									
K-1-15	343	343	192	-123	-240	-144	58.4	188	343	343	343
		296	148	-9	-156	-27	101	248	343	343	343
K-1-1	346	180	42	-197	-336	-138	163	346	346	346	346
		230	60	-120	-291	-125	40	210	346	346	346
K-1-3	340	255	65	-156	-320	-136	129	241	340	340	340
		229	60	-120	-289	-112	63	233	340	340	340
K-1-4	346	187	62	-104	-346	-128	-45	142	346	346	346
		170	-15	-214	-346	-209	-18	167	346	346	346
K-1-6	342	103	-89	-342	-342	-342	-106	144	342	342	342
		146	-80	-319	-342	-324	-104	121	342	342	342
K-1-5	343	58.3	-68.6	-343	-343	-343	-223	89	343	343	343
		112	-133	-343	-343	-343	-172	73	334	343	343
K-1-11	346	132	145	-346	-346	-346	-346	-41.5	346	346	346
		77	211	-346	-346	-346	-298	-10	295	346	330
K-1-20	379	303	178	102	-34	201	379	379	379	379	379
		213	133	48	-31	121	275	355	379	379	366
K-1-19	378	151	68	-26	-174	45	313	378	378	378	378
		167	70	-32	-129	55	241	338	378	378	352
K-1-16	349	56	87	-230	-293	-52	310	349	349	349	349
		127	4.5	-12.6	-250	-23	203	327	349	349	349
K-1-13	341	191	-44	-130	-245	-17	170	341	341	341	341
		72	-60	-201	-334	-84	165	298	341	341	323
K-1-2	350	42	-154	-318	-350	-98	182	301	350	350	318
		-13	-149	-320	-350	-195	92	253	350	350	302
K-1-9	349	17	-84	-265	-349	-168	42	164	349	349	220
		38	-210	-349	-349	-259	46	219	349	349	269
K-1-7	350	-213	-350	-350	-350	-350	42	203	350	350	136
		-111	-294	-350	-350	-336	0.97	183	350	350	223
K-1-14	350	109	-31.5	-94.5	-63	38.5	231	276.5	350	350	231
		34	-36	-112	-183	61	306	350	350	350	280
K-1-10	345	59	-38	-128	-207	-59	245	335	345	345	166
		-85	-175	-278	-345	-55	261	345	345	345	231
K-1-12	344	-248	-272	-344	-344	-127	309	344	344	344	124
		-183	-290	-344	-344	-153	202	309	344	344	172
K-5-3	331	331			39.7		271			331	
		297			11		186			331	
K-5-4	333	333			-106		127			333	
		255			-123		134			333	
K-5-1	333	153			-233		153			333	
		220			-278		59			333	
K-5-2	336	121			-336		33.6			336	
		164			-336		6			336	
K-7-2	Ø12 390		199	-72	-122	-79	140	390			
	Ø22 360		193	-7.1	-94	-21	90	360			
K-7-4	Ø12 378		189	-117	-248	-78	75.6	378			
	Ø22 355		167	-36	-147	-60	68	78			
K-7-1	Ø12 397		147		-344		-32	397			
	Ø22 355		127		-277		47	355			
K-7-3	Ø12 395		67	-249	-348	-252	-7.9	395			
	Ø22 355		137	-98	-226	-115	-50	355			

Investigations of the biaxial bended reinforced concrete elements also showed that the strains of concrete in the extreme compressed fibers in the sections between the cracks, as well as the deformations in the reinforcement, are distributed unevenly. Consideration of this phenomenon should be made by factor of the uneven distribution of strains in compressed concrete ψ_c, the essence of which is similar to that when studying reinforced concrete elements under simple types of loads.

Experimental studies have shown that if it is necessary to take into account in practical calculations the phenomenon of uneven distribution of strains in reinforcement and concrete in the element in the sections between cracks it is sufficient to take the values of the coefficients ψ_s and ψ_c constant and, accordingly, equal to 0.9 and 0.7.

5. Theoretical analysis results

The theoretical research on the work of biaxial bended reinforced concrete elements under load causes significant difficulties in describing their deformation mode. And first of all, this is due to the fact that the compressed-tensioned form of the section depends on many factors: the external load plane inclination angle, the reinforcement location, the ratio of section dimensions, reinforcement method and reinforcement ratio, physical and mechanical characteristics of reinforcement and concrete, the loading level, etc.

All given above suggest that the deformation mode model of biaxial loaded reinforced concrete elements should be taken in general form based on experimental research data. Using this approach as a base, the deformation mode model of biaxial bended reinforced concrete elements at ultimate limit state in the section between the cracks is taken as shown in Figure 3.

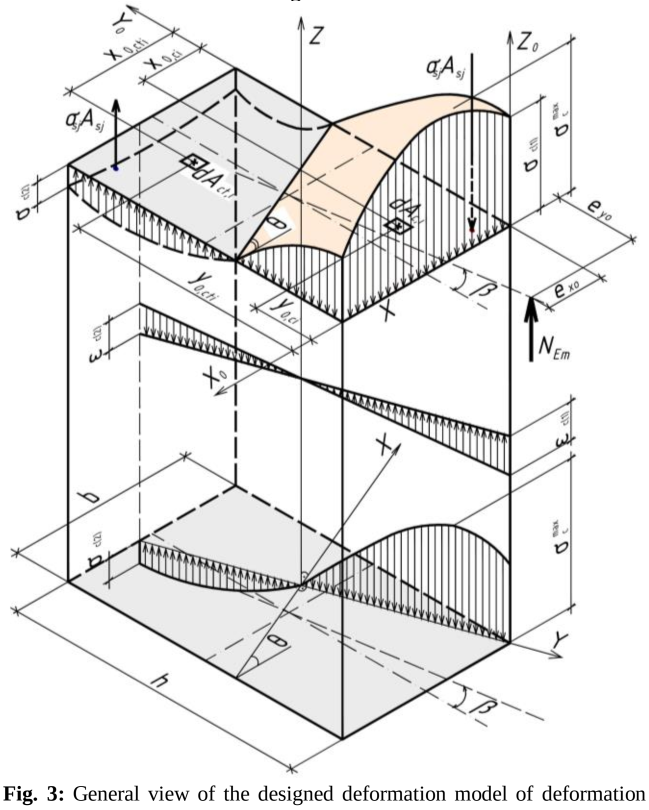

Fig. 3: General view of the designed deformation model of deformation mode of the biaxial bended reinforced concrete element.

The theoretical models in this work are considered on the example of biaxial bended elements of a rectangular profile with a not overreinforced cross-section. In this case, the coordinate system for representing the surface of stress distribution in the section is located so that its beginning coincides with the point that is outermost from the neutral axis.

The strains distribution law in the reinforcement and in the solid part of the concrete is taken linear.

Intermediate values of stresses in reinforcement and concrete correspond to the stress-strain relation for reinforcement and concrete for structural analysis according to Eurocode 2.

In theoretical research of the reinforced concrete elements, which operate under biaxial bending, the general system of equilibrium equation according to the represented in Fig. 3 physical model of the deformation mode at ultimate limit state, adopted in the matrix

$$\left[A_{ij}\right]\{B\}=\{C\}. \qquad (1)$$

In the applied mathematical model of the deformation mode for reinforced concrete elements, the component {B} is a vector of deformation parameters and in expanded form, using the concrete stress-strain relation for structural analysis at $k = 2$, includes the following elements:

$$\{B\}=\begin{Bmatrix} 2f_{cm}\eta_{c(1)}/X \\ f_{cm}\eta_{c(1)}^2/X^2 \\ E_s/\rho \end{Bmatrix}. \qquad (2)$$

The system (1) uses a matrix vector {C} of external loads

$$\{C\}=\begin{Bmatrix} N_{Em} \\ N_{Em}e_y \\ N_{Em}e_x \end{Bmatrix}, \qquad (3)$$

and a matrix of geometric characteristics

$$\left[A_{ij}\right]=\begin{Bmatrix} S_{c,x} & -I_{c,x} & S_{s,x} \\ I_{c,x} & \bar{S}_{c,x} & I_{s,x} \\ I_{c,xy} & -I_{c,y^2x} & I_{s,xy} \end{Bmatrix}. \qquad (4)$$

In the matrix (4) elements $S_{c,x}$, $S_{s,x}$, $I_{c,x}$, $I_{c,xy}$, $I_{s,x}$, $I_{s,xy}$ – respectively for concrete (with index c) and reinforcement (with index s) static moments (S) and moments of inertia (I); $\bar{S}_{c,x}$, $\bar{I}_{c,y^2x}$ – static moment and moment of inertia of higher orders in accordance with the area of the compressed area of concrete and reinforcement, mm^5.

According to the analysis of experimental studies, it is concluded that the surface of stress distribution over the section can be taken as close as possible to the cylindrical with the generators located along the curve $z = f(y)$ parallel to the neutral axis (Fig. 3). In the derivation of the functional dependence of the stress distribution $\sigma_c = f(x_0, y_0)$ on the section in the coordinate system $Y_0O_0Z_0$ it is necessary to first obtain the equation of the curve $z = f(y)$, which in essence is a functional dependence of the stress distribution in the coordinate plane *YOZ*. The coordinate plane *YOZ* runs orthogonally to the neutral axis, and through the most remote (critically deformed) point from the neutral axis in the compressed zone. Considering that in the coordinate system YOZ the axis Z will correspond to the coordinates of the current values of stress σ_c, then the graph of the desired curve on the physical content is a stress-strain diagram of the concrete $\sigma_c - \varepsilon_c$ with the downward branch in the system of coordinates *YOZ*. When applying the concrete stress-strain relation for structural analysis according to Eurocode 2 (3.14), the relation of stress distribution in the *YOZ* plane is given by the following conditions:

$$\sigma_c=\frac{f_{cm}\eta_{c(1)}y\left(kX-\eta_{c(1)}y\right)}{X\left(X+k\eta_{c(1)}y-2\eta_{c(1)}y\right)}, \qquad (5)$$

where $\eta_{c(1)} = \varepsilon_{c(1)} / \varepsilon_{c1}$ – the level of concrete compressive strain at the point most distant from the neutral axis;
y – current coordinate value, which according to Fig. 3 equals y_{ci};
X – neutral axis depth.

The function of distributing stresses in the cross section in the system of coordinates $X_0Y_0Z_0$ from (5) can be obtained by moving to this coordinate system from the coordinate system *YOZ*. Formulas for reordering coordinates will be made by expressions:

$$y=y_1+X\ ;\ y_1=x_0\sin(\pi+\theta)+y_0\cos(\pi+\theta). \qquad (6)$$

Using (5) the required function in the coordinate system $X_0Y_0Z_0$ in general form for the concrete compressed area is presented as follows:

$$\sigma_c=f_{cm}\left(\frac{k\eta_{c(1)}y}{X}-\frac{\eta_{c(1)}^2y^2}{X^2}\right)\Bigg/\left(1+(k-2)\frac{\eta_{c(1)}y}{X}\right). \qquad (7)$$

The value of the stress σ_{ctm} in the concrete tensioned area can be determined using directly the function (7), after replacing the value of f_{cm} to f_{ctm}. When solving the problem of strength it may be taken $\sigma_{ctm} = 0$.

The analysis of the dependence (1) shows that the deformation mode of biaxial bended reinforced concrete elements is uniquely determined by three parameters – X, θ, $\eta_{c(1)}$. Knowing these parameters can be argued that the deformation mode is quite certain. From this, it follows that solving the problem in the first stage is to find the components of the system of equations that describe the deformation mode with the help of unknown parameters. At the second stage, it is necessary to solve this system of equations with respect to the specified parameters of deformation mode. But, as it is not difficult to see, the number of unknowns will be on one more than equations. So unknown, which can not be determined with the help of existing equations, is the ultimate value of the deformation level $\eta_{c(1)}$ at the most distant from the neutral axis point of compressed concrete. This value by physical nature is the determining parameter of the subcritical stage of the deformation mode. In other words, it is necessary to find the value $\eta_{c(1)}$, which corresponds to the carrying capacity of the reinforced concrete element at fully exhausted possibilities of the materials. It is when the stresses in the reinforcement reached the limits of its strength, in the concrete compressed zone the process of its stabilization in the most distant from the neutral axis fibers is completed, the value of the internal bending moment at the same time at the maximum. This value of $\eta_{c(1)}$ can be found from the equation when all the necessary conditions are fulfilled

$$\frac{dN_{Rm}\left(\eta_{c(1)},...\right)}{d\left(\eta_{c(1)},...\right)}=0, \qquad (8)$$

which is the consequence of the use of the concept of strength criterion with regard to the biaxial loaded reinforced concrete elements in the form:

$$N_{Rm}\left(\eta_{c(1)},\theta_{c(1)}\right)=max\left(N_{Rm}\left(\eta_u,\theta_u\right)\right), \qquad (9)$$

where η_u – the ultimate value of concrete strain level in the composition of the reinforced concrete elements, which depends on the external loading type, the coefficient of reinforcement, the physical-mechanical characteristics of the concrete, the form of the compressed concrete area, and others (hence $\eta_u \neq$ const) and corresponds to the maximum value of the internal moment.

The solution (1) for the general case was carried out by a numerical method based on a specially designed algorithm, based on the method of statistical comparisons. The essence of the method lies in the fact that the solution of this system, based on its physical content as a model of the deformation mode of a biaxial loaded reinforced concrete elements, at any moment must satisfy the system of inequalities:

$$\left.\begin{array}{l} 0 < X < H \\ 0{,}5 \le \eta_{c(1)} \le 2{,}5 \\ 0 \cong \theta \cong \pi / 2 \end{array}\right\}, \qquad (10)$$

in which H is determined by the formula

$$H = b\sin\theta + h\cos\theta \,. \qquad (11)$$

Thus, using (10) the system (1) solution is based on the condition that for the particular case of a compressed-tensioned cross-sectional form at the given deformation level there will exist such values of X, θ and $\eta_{c(1)}$ that will simultaneously satisfy as (1), (8) and (10).

The process of performing the calculations is repeated until the value, for example, of the moment of the internal couple of forces, which is perceived by the section, does not reach its maximum value. The presence of the maximum value of such a moment is determined by the fact that, for some value $\eta_{c(1)}$, it begins to decrease, in contrast to its steady growth from the beginning of the loading. Once this decrease is fixed, it is necessary to proceed to the process of numerical differentiation in order to find the value of $\eta_{c(1)} = \eta_u$, which will meet the requirements of the extreme strength criterion (8) and which will determine the parameter of the ultimate limit stage of the deformation mode of the biaxial loaded reinforced concrete elements.

To determine all the supercritical state parameters of reinforcement elements, a method for implementing the proposed model on a PC is developed.

6. Comparative analysis of experimental and theoretical data

The results of comparisons between the experimental and calculated values of the neutral axis inclination angle θ, the neutral axis depth X, and the strength N_{Em} (ultimate axial force at definite eccentricities about vertical and horizontal axes of inertia of the column section) of the tested columns are given in Table 4.

Table 4: Comparison of the results of theoretical strength calculations in the normal cross-section of biaxial bended reinforced concrete columns with experimental data

Sample code	Neutral axis inclination angle θ^o			Neutral axis depth X, mm			Destructive force N_{Em}, kN		
	Theor.	Exp.	$\theta^o_{Theor.}/\theta^o_{Exp.}$	Theor.	Exp.	$X_{Theor.}/X_{Exp.}$	Theor.	Exp.	$N_{Em,Theor.}/N_{Em,Exp.}$
K-1-15	42.9	43.0	1.00	256	211	1.21	1108	1250	0.89
K-1-1	46.0	48.0	0.96	225	219	1.03	750	800	0.94
K-1-3	48.0	53.0	0.91	218	205	1.06	723	800	0.90
K-1-4	48.0	40.0	1.20	202	173.5	1.16	620	750	0.83
K-1-6	46.2	46.0	1.00	183	191	0.96	479	585	0.82
K-1-5	45.4	43.0	1.06	171	166	1.03	394	462	0.85
K-1-11	43.2	42.0	1.03	153	165	0.93	359	350	1.03
K-1-20	64.0	68.0	0.94	264	249	1.06	1011	1100	0.92
K-1-19	63.8	72.0	0.89	235	204	1.15	883	1073	0.82
K-1-16	63.0	71.0	0.89	208	163	1.28	798	865	0.92
K-1-13	63.5	62.0	1.02	187	169	1.11	1021	1000	1.02
K-1-2	62.2	64.0	0.97	172	146	1.18	539	642	0.84
K-1-9	62.2	62.0	1.00	160	150	1.07	520	600	0.87
K-1-7	62.9	64.0	0.98	148	135	1.10	409	425	0.96
K-1-14	74.8	79.0	0.95	189	186	1.02	997	1173	0.85
K-1-10	74.6	71.0	1.05	158	132	1.20	717	800	0.90
K-1-12	74.3	77.0	0.96	140	126	1.11	548	630	0.87
K-5-3	45.6	46.0	0.99	334	325	1.03	754	850	0.89
K-5-4	48.5	40.0	1.21	259	274	0.95	597	650	0.92
K-5-1	48.5	40.0	1.21	224	245	0.91	468	500	0.94
K-5-2	49.1	46.0	1.07	202	232	0.87	375	450	0.83
K-7-2	35.9	41.0	0.88	279	254	1.10	529	550	0.96
K-7-4	37.9	49.0	0.77	267	231	1.16	456	475	0.96
K-7-1	41.3	48.0	0.86	246	196	1.26	723	800	0.90
K-7-3	40.8	40.0	1.02	243	255	0.95	583	700	0.83
Expectation			0.9929			1.0746			0.8982
Arithmetical mean deviation			0.0770			0.0887			0.0475
Dispersion			0.0113			0.0119			0.0035
Root mean square deviation			0.1065			0.1092			0.0589
Variation coefficient			0.1073			0.1017			0.0656

As it can be seen from the obtained data, the method of strength analysis of biaxial bended elements proposed by the authors makes it possible to get sufficiently accurate values of the columns strength. The coefficient of variation with respect to the values of the destructive axial force is 6.56%.

Good convergence of the theoretical and experimental values of the neutral axis depth X and the neutral axis inclination angle θ should also be noted as indicated by the mean arithmetic deviation.

7. Conclusions

As a result of the columns samples tests under biaxial bending with axial load, the following is established:

1. Experimental studies on columns, which were tested with constant rate of deformation of the most compressed concrete fibers, found that after the element perception of the maximum value of the destructive N_{Em} force, it was steady with its decrease, with the simultaneous growth of the concrete strains in the cross-section extreme fiber.
2. It is established that the identical elements, when they reach the supercritical stage, collapse with the largest values of the compressive concrete fibrous strains ε_{cu1} in the triangular form of the compressed area and with the smallest in the rectangular form of the compressed zone for all other similar conditions. The ultimate values ε_{cu1} of the concrete fibrous strains in the biaxial loaded elements with the trapezoidal form of the compressed area are have intermediate values, that is, those that lie between the ultimate values ε_{cu1} for elements with triangular and rectangular forms of the compressed area.
3. In general, the nonlinear deformation model of the deformation mode of the biaxial bended reinforced concrete element in theoretical researches is the most easily implemented in the application of the analytic expression in a matrix form, which components are the matrix of geometric characteristics, the matrix-vector of parameters of strains and curvatures and a matrix-vector of external loads.
4. Analysis of the comparison of the theoretical values of the parameters of the deformation mode and the strength of the biaxial bended reinforced concrete columns with experimental data confirms the possibility of using in practice the method of the strength analysis developed by the authors.

References

[1] A.M. Pavlikov, S.M. Mykytenko, A.V. Hasenko, Effective structural system for the construction of affordable housing, *International Journal of Engineering & Technology*, Vol 7, No 3.2, (2018), pp. 291-298, https://doi.org/10.14419/ijet.v7i3.2.14422.

[2] O. Dovzhenko, V. Pohribnyi, L. Karabash, Experimental Study on the Multikeyed Joints of Concrete and Reinforced Concrete Elements, *International Journal of Engineering & Technology*, Vol. 7 (3.2), (2018), pp. 354-359, http://dx.doi.org/10.14419/ijet.v7i3.2.14552.

[3] M. Orešković , Ie. Klymenko , A. Aniskin , G.o Kozina, Analysis of Damaged Concrete Columns of Circular Cross-Section, *Technical gazette*, Vol. 25, No. 2, pp. 337-343. https://doi.org/10.17559/TV-20160621085905

[4] J.L. Bonet, M.H.F.M. Barros, M.L. Romero, Comparative study of analytical and numerical algorithms for designing reinforced concrete sections under biaxial bending, *Computers & Structures*, Vol. 84, No. 31-32, (2006), pp. 2184-2193, https://doi.org/10.1016/j.compstruc.2006.08.065.

[5] J.L. Bonet, M.L. Romero, P.F. Miguel, Effective flexural stiffness of slender reinforced concrete columns under axial forces and biaxial bending, *Engineering Structures*, Vol. 33, No. 3, (2011), pp. 881-893, https://doi.org/10.1016/j.engstruct.2010.12.009.

[6] Jin-Keun Kim, Sang-Soon Lee, The behavior of reinforced concrete columns subjected to axial force and biaxial bending, *Engineering Structures*, Vol. 22, No. 11, (2000), pp. 1518-1528, https://doi.org/10.1016/S0141-0296(99)00090-5.

[7] M.G. Sfakianakis, Biaxial bending with axial force of reinforced, composite and repaired concrete sections of arbitrary shape by fiber model and computer graphics, *Advances in Engineering Software*, Vol. 33, No 4, (2002), pp. 227-242, https://doi.org/10.1016/S0965-9978(02)00002-9.

[8] L. Pallarés, J.L. Bonet, P.F. Miguel, M.A. Fernández Prada, Experimental research on high strength concrete slender columns subjected to bending and biaxial bending forces, *Engineering Structures*, Vol. 30, No 7, (2008), pp. 1879-1894, https://doi.org/10.1016/j.engstruct.2007.12.005.

[9] Shuenn-Yih Chang, Experimental Studies of Reinforced Concrete Bridge Columns under Axial Load Plus Biaxial Bending, *Journal of Structural Engineering*, Vol. 136, No 1(12), (2010), pp. 12-18, https://doi.org/10.1061/(ASCE)0733-9445(2010)136:1(12).

[10] A. Pavlikov, M. Kosior-Kazberuk, O. Harkava, Experimental testing results of reinforced concrete beams under biaxial bending, *International Journal of Engineering & Technology*, Vol 7, No 3.2, (2018), pp. 299-305, https://doi.org/10.14419/ijet.v7i3.2.14423.

[11] Piskunov, V. G., Gorik, A. V., & Cherednikov, V. N. (2000). Modeling of transverse shears of piecewise homogeneous composite bars using an iterative process with account of tangential loads 2. resolving equations and results. Mechanics of Composite Materials, 36(6), 445-452. https://doi.org/10.1023/A:1006798314569

[12] Piskunov, V. G., Goryk, A. V., & Cherednikov, V. N. (2000). Modeling of transverse shears of piecewise homogeneous composite bars using an iterative process with account of tangential loads. 1. construction of a model.Mechanics of Composite Materials, 36(4), 287-296. doi:10.1007/BF02262807

[13] Shkurupiy, O., Lazariev, D., & Davydenko, Y. (2018). Strength design of compressed reinforced concrete elements by deformation method based on extreme criterion. *International Journal of Engineering and Technology(UAE)*, *7*(3), 334-338. https://doi.org/10.14419/ijet.v7i3.2.14430