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Research paper

Calculation of the stability of the equilibrium form of discrete systems

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Abstract

On the basis of the displacement method, together with the methods of iterations and half-division, an algorithm for stability calculation of the first kind equilibrium form of compressed reinforced concrete columns with hinged fixing at the ends, considering the stiffness changing was developed. The use of the above methods makes it possible to determine the minimum critical load or stress at the first bifurcation and their stability loss corresponding form. The use of matrix forms contributes to simplification of high order stability loss equation. This approach allows to obtain the form of stability loss that corresponds to the critical load.

Keywords: *equilibrium form, compressed reinforced concrete columns, software complex, stability loss equation*.

1. Introduction

When calculating the first kind equilibrium stability form of compressed reinforced concrete columns with hinged fixing at the ends, considering the stiffness changing (damage to the column sections) provided that the initial modulus of elasticity is constant, it is assumed the necessity of solving the stability loss equation which is non-linear transcendental and, as you know, does not have an analytical solution [1 – 4, 7 – 12].

In addition, this equation elements are complex mathematical dependencies, which contain Zhukovsky functions in its composition, which also greatly complicates the equation solution.

To solve this engineering task, the teachers of the Department of Structural and Theoretical Mechanics of the Poltava National Technical Yuri Kondratyuk University have developed the software complex "Persist".

In recent years, the software complex "Persist" has been tested and successfully implemented in the training of specialists for the building industry [5, 6].

2. Main body

We obtain stability loss equation of the equilibrium form of reinforced concrete columns with hinged fixing at the ends considering stiffnesses changing due to the displacements method in expanded form provided that compression and stretch deformations are ignored [5, 6, 13, 14].

Output data: $h_1 = m_1 \cdot l;\ \ h_2 = m_2 \cdot l;\ \ h_3 = m_3 \cdot l;\ \ h_4 = m_4 \cdot l$.

$E = const \neq \infty$.

$i_j = \left(\frac{EJ}{l}\right)_j \left(j = \overline{1,4}\right)$

We have to calculate: the value of the minimum critical longitudinal force $N_{cr}^{\min}$ at stability loss of the first kind equilibrium form considering damages on any column sections (Figure 1).

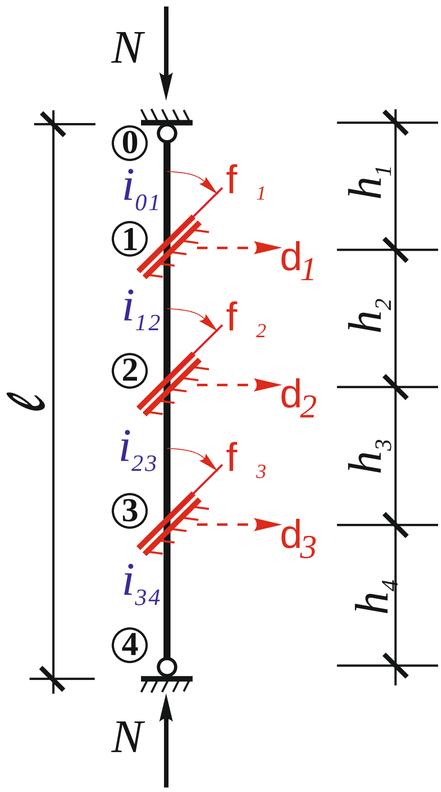

Fig. 1: The column calculation scheme

We accept, $i = i_{01}$, $J_{01} = J$ and we express all rigid stiffnesses on the bend of the column sections due to these parameters and the first section length $h_1 = m_1 \cdot \ell$.

$$i_{01} = \frac{EJ_{01}}{h_1};\ i_{12} = \frac{EJ_{12}}{h_2};\ i_{23} = \frac{EJ_{23}}{h_3};\ i_{34} = \frac{EJ_{34}}{h_4} \quad (1)$$

where

$$J_{01} = J;\ J_{12} = C_2 \cdot J;\ J_{23} = C_3 \cdot J;\ J_{34} = C_4 \cdot J. \quad (2)$$

The values of the coefficients of the rigid stiffness ratio on bend, expressed through the first section rigid stiffness, will be equal to:

$$K_{01} = \frac{i_{01}}{i} = 1;\ K_{12} = \frac{i_{12}}{i} = \frac{C_2 m_1}{m_2};\ K_{23} = \frac{i_{23}}{i} = \frac{C_3 m_1}{m_3};\ K_{34} = \frac{i_{34}}{i} = \frac{C_4 m_1}{m_4}. \quad (3)$$

The rigid stiffness on each of the four sections bend will be expressed as:

$$J_{01} = J;\ i_{01} = K_{01} \cdot i;\ i_{12} = K_{12} \cdot i;\ i_{23} = K_{23} \cdot i;\ i_{34} = K_{34} \cdot i \quad (4)$$

Considering the above expressions, the rigid stiffness final value on the bend of each of the four sections will look like:

$$i_{01} = \frac{EJ_{01}}{m_1 l};\ i_{12} = \frac{EJ_{12}}{m_2 l};\ i_{23} = \frac{EJ_{23}}{m_3 l};\ i_{34} = \frac{EJ_{34}}{m_4 l} \quad (5)$$

Thus, this approach makes it possible to change the flexural stiffness of each section of the column (see calculation scheme, Figure 1) by varying the moments of inertia at the corresponding axes, provided that the initial elastic modulus remains constant.
Determination of the minimum critical force ($N_{cr}^{\min}$) is obtained using the displacement method expanded form [5, 6, 9].
We write down the displacement method equilibrium equation in accordance with the column calculation scheme (Figure 1):

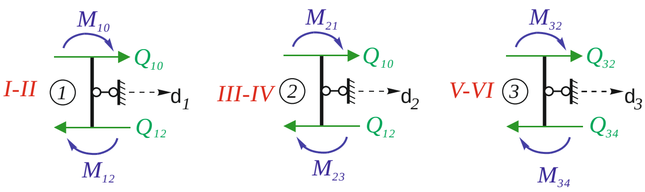

Fig. 2: Calculation schemes of the column nodes

Displacement method equilibrium equations will take the form (Figure 2):

$$\begin{cases} M_{10} + M_{12} = 0; \\ Q_{10} - Q_{12} = 0; \\ M_{21} + M_{23} = 0; \\ Q_{21} - Q_{23} = 0; \\ M_{32} + M_{34} = 0; \\ Q_{32} - Q_{34} = 0. \end{cases} \quad (6)$$

We will write down the same equations in a matrix form:

$$r \cdot \vec{Z} = 0 \quad (7)$$

where r – stiffness matrix of column sections elements (reactive forces in fictitious joints from unknown vector Z unit values),

$$r = \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} \\ r_{21} & r_{22} & r_{23} & r_{24} & r_{25} & r_{26} \\ r_{31} & r_{32} & r_{33} & r_{34} & r_{35} & r_{36} \\ r_{41} & r_{42} & r_{43} & r_{44} & r_{45} & r_{46} \\ r_{51} & r_{52} & r_{53} & r_{54} & r_{55} & r_{55} \\ r_{61} & r_{62} & r_{63} & r_{64} & r_{65} & r_{66} \end{pmatrix} \quad (8)$$

According to the reciprocity of reactions theorem $r_{ik} = r_{ki}\ (i,k = \overline{1,6})$. Therefore, the stiffness matrix will be represented only by the values of its upper triangle parameters.

$$r = \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} \\ & r_{22} & r_{23} & r_{24} & r_{25} & r_{26} \\ & & r_{33} & r_{34} & r_{35} & r_{36} \\ & & & r_{44} & r_{45} & r_{46} \\ & & & & r_{55} & r_{55} \\ & & & & & r_{66} \end{pmatrix} \quad (9)$$

Z – vector of the displacement method unknown in accordance with the column calculation scheme (Figure 1).

$$\vec{Z} = \begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{Bmatrix} = \begin{Bmatrix} \varphi_1 \\ \delta_1 \\ \varphi_2 \\ \delta_2 \\ \varphi_3 \\ \delta_3 \end{Bmatrix} \quad (10)$$

The system of equilibrium equations (6) is a system of linear algebraic homogeneous equations that has the following properties:
a) the system has a zero (trivial) solution (the physical content of the vector $\vec{Z} = 0$, that is, the column maintains a straightforward form - there is no equilibrium form stability loss);
b) the system will have no zero solution if $\det(r) = D = 0$. In this case, we have a problem of eigenvalues, that is, system (6) has many solution roots. Physical content: vector $\vec{Z} \neq 0$ (there are angular and linear displacements of the system nodes, indicating the appearance of a new deformation form - the longitudinal bending, i.e., the system loses the equilibrium form stability (the first kind stability loss). Thus, det(r) there is an analytical condition for the equilibrium stability loss (stability loss equation), the solution of which will be able to determine $N_{cr}^{\min}$ with the methods of iterations and half-division.
Compressed column equilibrium form stability loss equation has the following general form:

$$\det(r) = D = \begin{vmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} \\ & r_{22} & r_{23} & r_{24} & r_{25} & r_{26} \\ & & r_{33} & r_{34} & r_{35} & r_{36} \\ & & & r_{44} & r_{45} & r_{46} \\ & & & & r_{55} & r_{56} \\ & & & & & r_{66} \end{vmatrix} = 0 \quad (11)$$

Equilibrium form stability loss equation (11) is a transcendental equation, that is, has many solution roots. To determine the smallest value of the critical load (critical longitudinal force $N_{cr}^{\min}$), we use numerical methods of iterations and half-division.

In the equation (11), the compressive force N is an unknown value and is expressed by Zhukovsky functions ($\alpha,\beta,\gamma,\bar{\alpha},\bar{\gamma},...$), which are expressed through the parameter t, which in turn is expressed in longitudinal force N terms. These functions consider the deformed scheme of the system when the stability of the equilibrium form is lost.

From the stability loss equation (11) the critical value base parameter t_{cr} is determined the value of which must be multiplied by the number of sections along the column length. The value must be considered in the final definition formula

$$N_{cr,j}^{\min} = \left(\frac{(t_{cr} \cdot n)^2 \cdot i}{l} \right)_j$$

By preliminary calculating the relationship between the parameters t in the compressed column through the base parameter t_0, we determine the critical parameters t in each compressed element, and then their critical longitudinal forces and critical load.

$$t_j^{\,2} = \left(\frac{Nl^2}{EI} \right)_j ; \quad t_j^{\,2} = \left(\frac{Nl}{i} \right)_j \tag{12}$$

$$N_{j,cr} = \left(\frac{t^2{}_{cr} EI}{l^2} \right)_j = \left(\frac{t^2{}_{cr} \cdot i}{l} \right)_j \tag{13}$$

In this case, the critical forces in each section of the column should be the same. This will be $N_{cr}^{\min}$ for the column.

Finally, the upper triangle of the loss of stability equation of the equilibrium form will look like:

$$D = \begin{vmatrix}
\bar{\alpha}_{01} + \frac{2C_2m_1}{m_2}\alpha_{12} & \left(\frac{1}{m_1}\bar{\alpha}_{01} - \frac{2C_2m_1}{m_2^2}(\alpha+\beta)_{12} \right) & \frac{2C_2m_1}{m_2}\beta_{12} & \frac{2C_2m_1}{m_2^2}(\alpha+\beta)_{12} & 0 & 0 \\
 & \left(\frac{1}{m_1^2}\bar{\gamma}_{01} + \frac{2C_2m_1}{m_2^3}\gamma_{12} \right) & -\left(\frac{2C_2m_1}{m_2^2}(\alpha+\beta)_{12} \right) & \frac{2C_2m_1}{m_2^3}\gamma_{12} & 0 & 0 \\
 & & \frac{2C_2m_1}{m_2}\alpha_{12} + \frac{2C_3m_1}{m_3}\alpha_{23} & \frac{2C_2m_1}{m_2^3}\gamma_{12} + \frac{2C_3m_1}{m_3^3}\gamma_{23} & \frac{2C_3m_1}{m_3}\beta_{23} & \frac{2C_3m_1}{m_3^2}(\alpha+\beta)_{23} \\
 & & & \frac{2C_2m_1}{m_2^3}\gamma_{12} + \frac{2C_3m_1}{m_3^3}\gamma_{23} & \frac{2C_3m_1}{m_3^2}(\alpha+\beta)_{23} & \frac{2C_3m_1}{m_3^3}\gamma_{23} \\
 & & & & \frac{C_4m_1}{m_4}\bar{\alpha}_{34} + \frac{2C_3m_1}{m_3}\alpha_{23} & \frac{C_4m_1}{m_4^2}\bar{\alpha}_{34} - \frac{2C_3m_1}{m_3^2}(\alpha+\beta)_{23} \\
 & & & & & \frac{C_4m_1}{m_4^3}\bar{\gamma}_{34} + \frac{2C_3m_1}{m_3^3}\gamma_{23}
\end{vmatrix} = 0 \tag{14}$$

The equation (14) will be calculated if the stiffness of the system elements changes. To do this, you need to set the appropriate correlation of the relationship parameters: m_1, m_2, m_3, m_4 and C_1, C_2, C_3, C_4 .

So, for the given calculation scheme of the column (Figure 1), the relationship between the parameters t of each section is:

As the base parameter we take $t = t_{01}$, then:

$$t = t_{01} = t_1 = \sqrt{\frac{N \cdot h_1}{i_{01}}} = \sqrt{\frac{N \cdot m_1 \cdot l}{i}} \qquad (15)$$

$$t_{12} = t_2 = \sqrt{\frac{N \cdot h_2}{K_{12} \cdot i}} = \sqrt{\frac{N \cdot m_2^2 \cdot l}{C_2 \cdot m_1 \cdot i}} \qquad (16)$$

$$t_{23} = t_3 = \sqrt{\frac{N \cdot h_3}{K_{23} \cdot i}} = \sqrt{\frac{N \cdot m_3^2 \cdot l}{C_3 \cdot m_1 \cdot i}} \qquad (17)$$

$$t_{34} = t_4 = \sqrt{\frac{N \cdot h_4}{K_{34} \cdot i}} = \sqrt{\frac{N \cdot m_4^2 \cdot l}{C_4 \cdot m_1 \cdot i}} \qquad (18)$$

$$\zeta_1 = \frac{t_{01}}{t} = 1. \qquad (19)$$

$$\zeta_2 = \frac{t_{12}}{t} = \frac{t_2}{t} = \frac{m_2}{m_1}\sqrt{\frac{1}{C_2}} \qquad (20)$$

$$\zeta_3 = \frac{t_{23}}{t} = \frac{t_3}{t} = \frac{m_3}{m_1}\sqrt{\frac{1}{C_3}} \qquad (21)$$

$$\zeta_4 = \frac{t_{34}}{t} = \frac{t_4}{t} = \frac{m_4}{m_1}\sqrt{\frac{1}{C_4}} \qquad (22)$$

Thus,

$$t_{01} = t_1;\ t_{12} = \zeta_2 \cdot t;\ t_{23} = \zeta_3 \cdot t;\ t_{34} = \zeta_4 \cdot t \qquad (23)$$

To determine $N_{cr}^{\min}$ (calculation of the equation of stability loss of equilibrium form (4)) we use the software complex "Persist" specially developed by the authors for the corresponding output data (m_j, C_j, ζ_j), which algorithm is based on numerical methods of iterations and half-division [5, 6]. The iteration method makes it possible to determine the subinterval with the minimum value of the base critical parameter t_{cr}, and the half-division method to specify its value to the predefined accuracy.

At the same time, since everything was expressed due to the basic rigid stiffness i and the length of the rod l, the final value $N_{cr}^{\min}$ should be multiplied by the rigid stiffness i and divided by the length of the rod l.

Let us consider calculation example, when the stiffness of all column elements is constant (Figure 1). In this case, the source data will be:

$$h_1 = h_2 = h_3 = h_4 = \frac{l}{4};\ E = const \neq \infty;$$
$$J = J_{01} = J_{12} = J_{23} = J_{34};\ i = i_{01} = i_{12} = i_{23} = i_{34} \qquad (24)$$

$$C_1 = C_2 = C_3 = C_4 = 1;\ m_1 = m_2 = m_3 = m_4 = \frac{1}{4} \qquad (25)$$

The relation between the parameters t will be written as:

$$t_0 = t_{01} = t_{12} = t_{23} = t_{34}, \qquad (26)$$

$$\zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 1. \qquad (27)$$

The stability loss equation for such output data (24-27) will look like:

$$D = \begin{vmatrix} \bar{\alpha}_{01} + 2\alpha_{12} & 4\bar{\alpha}_{01} - 8(\alpha+\beta)_{12} & 2\beta_{12} & 8(\alpha+\beta)_{12} & 0 & 0 \\ & 16\bar{\gamma}_{01} + 32\gamma_{12} & -8(\alpha+\beta)_{12} & -32\gamma_{12} & 0 & 0 \\ & & 2\alpha_{12} + 2\alpha_{23} & 8(\alpha+\beta)_{12} - 8(\alpha+\beta)_{23} & 2\beta_{23} & 8(\alpha+\beta)_{12} \\ & & & 32\gamma_{12} + 32\gamma_{23} & -8(\alpha+\beta)_{23} & -32\gamma_{23} \\ & & & & \bar{\alpha}_{34} + 2\alpha_{23} & -4\bar{\alpha}_{34} + 8(\alpha+\beta)_{23} \\ & & & & & 16\bar{\gamma}_{34} + 32\gamma_{23} \end{vmatrix} = 0 \qquad (28)$$

To further calculating the column for the equilibrium form stability we use the software complex "Persist".
Here is a step-by-step algorithm for entering input data and necessary parameters for the software complex "Persist":
1. Enter the relation for the parameter t:

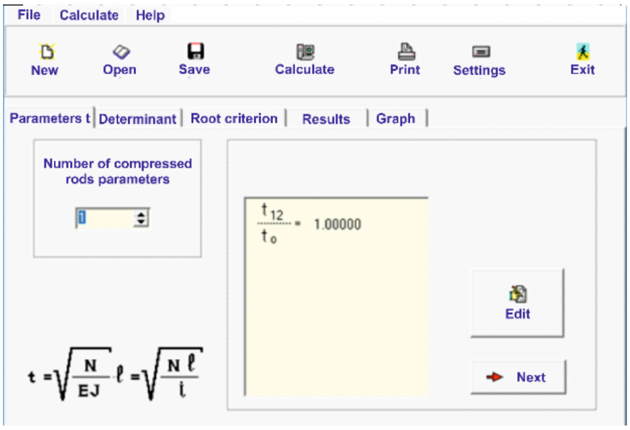

Fig. 3: Recording correlations for parameters t

2. We enter the elements of the upper triangle of equilibrium stability loss equation (28).
For example, we enter an element r_{11}. To do this, it is necessary to select element 1.1, and press the "Build" button:

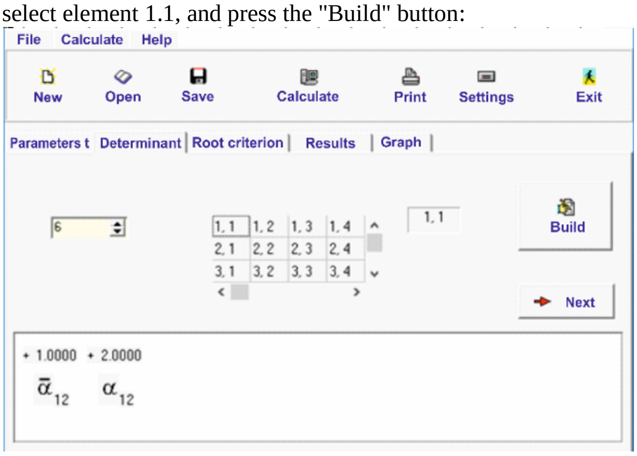

Fig. 4: Entering values for an element r_{11}

At the next step, we enter the expression value for the element in the menu item "Determinant", the button "Build". Similarly, we introduce all other elements of the upper triangle (28).

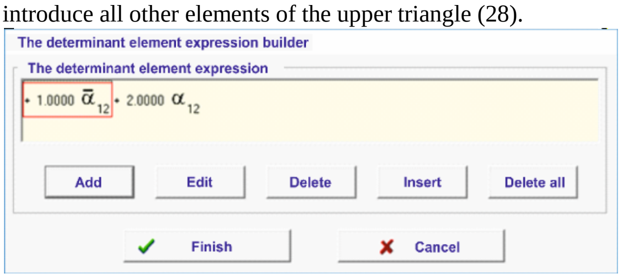

Fig. 5: Writing the expression for the element r_{11}

Expression for an element r_{66}

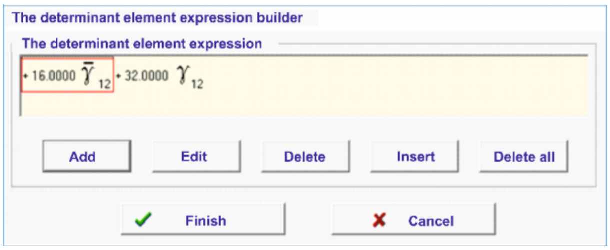

Fig. 6: Writing the expression for the element r_{66}

3. We choose the desired accuracy for calculating the root of the stability loss equation t_0 (the determinant minimum value, 10^{-4}). Also, we set the base parameter initial value (0), and also basic parameter to changing step t_0 (0,1). If necessary, this step can be reduced.

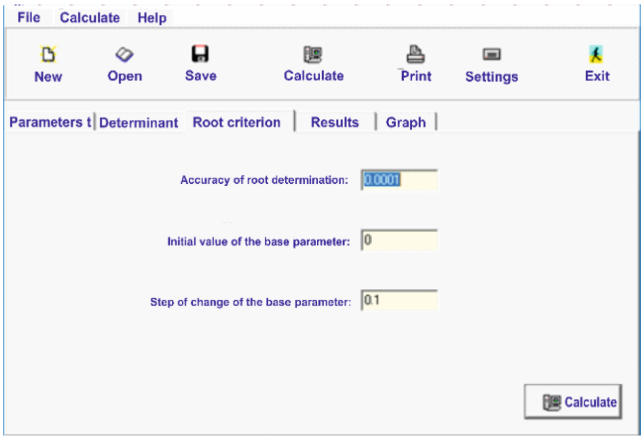

Fig. 7: Configuring accuracy for calculate the stability equation loss root

4. In the menu bar, press the "Calculate" button and go to the "Results" tab:

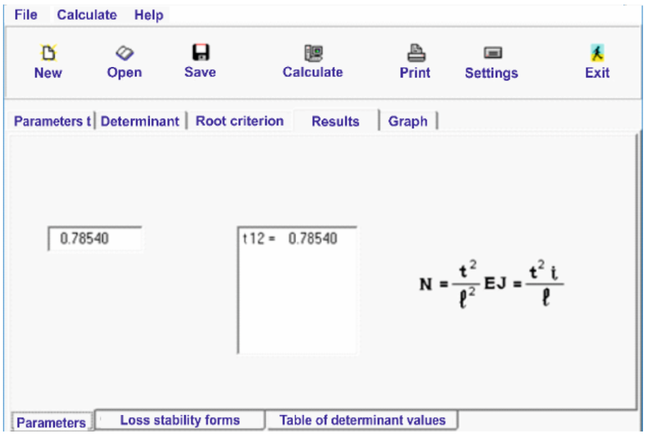

Fig. 8: View of the calculation results

If necessary, activate the tab "Table of determinant values", and you can view step-by-step calculation results by iteration and half-division methods:

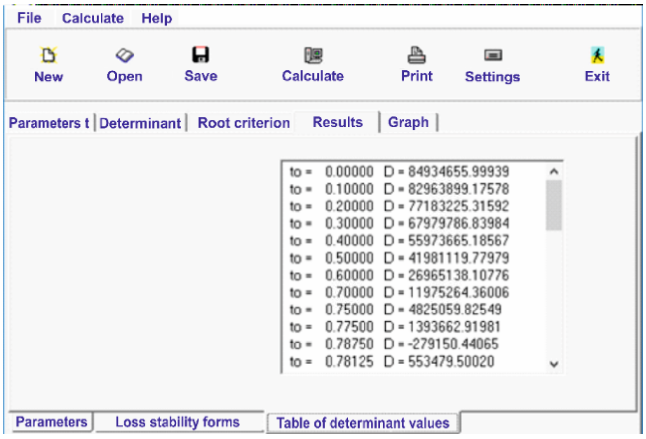

Fig. 9: Table of determinant values

If necessary, by activating the "Graph" tab, you can view a step-by-step graphic representation of the stability loss equation solution (28):

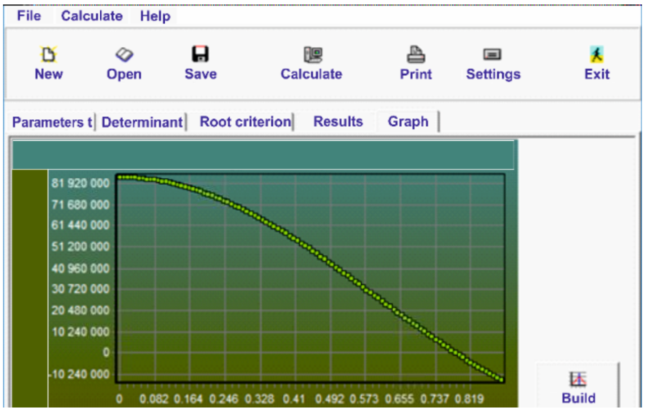

Fig. 10: Graphical representation of the of stability loss equation solution

This $N_{cr}^{\min}$ will be equal to:

$$N_{cr,j}^{\min} = \left(\frac{(t_{cr} \cdot n)^2 \cdot i}{\ell} \right) = \frac{(0,7854 \cdot 4)^2 \cdot i}{\ell} = \frac{3,1416^2 \cdot i}{\ell}$$

It should be noted that the dimension of the critical force depends on the given dimensions of the stiffening on bend *i* and the rod length *l*.

3. Conclusions

The method of calculation and algorithm are developed, which are implemented in the software complex "Persist". The program complex has been successfully implemented in the educational process in the study of the discipline "Structural Mechanics" at the educational-scientific Architecture and Civil Engineering Institute of the Poltava National Technical Yuri Kondratyuk University. This software can be used by students and engineer-designers for engineering calculations, including calculating the first kind equilibrium form stability of compressed reinforced concrete columns with hinged fixing at the ends, considering the stiffness changing.

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