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Research paper

Hybrid Estimator of Kumaraswamy Distribution Parameter

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Abstract

This study suggest a hybrid estimator to estimate the shape parameter $\beta$of the Kumaraswamy distribution. The suggested approach based on consisting between the MED estimator of exponential distribution and the statistical property median (MED)of Kumaraswamy distribution. The output estimation will called (Hybrid Estimator). So, how are we will know if this estimator is a best? To answer this question, it is necessary to compute the numerical results and then compare them. The results, which will obtain from simulation study, they will give us the decision. To find out which estimator is better, it must run a system of numerical simulation. Then, the results will be compared using a standard tool like MEA Square Error (MSE). We will use Matlab to implement the simulation steps. Finally, the results show that the hybrid method give the best estimated values and near to the proposed values.

Keywords: *Parameter Estimation, Kumaraswamy distribution, Maximum Likelihood Estimator, Simulation, MATLAB.*

1. Introduction

The distribution of Kumaraswamy is one of the most recent important statistical distributions that have used mainly in hydrological random variables such as daily rainfall, while most of the distributions failed the rest. It has continues random variable. The experiment of Poondi Kumaraswamy (1933-1988) gives some important conclusions. One of these is developing this new distribution. [1]

Recently, there have been many interests in studies concerning the inference of statistics in many distributions has increased, and the study of many of the parameters in terms of standard values has become more important. Especially the classical estimation for the unknown parameters of the Kum distribution.[2]

Some mathematical statistical properties were introduced by [3] such as the moment generating function mgf of ..... We will use some of these properties in this study to introduce a new estimator of Kum distribution.

2. Kumaraswamy distribution

One of the family of continuous statistical distributions is distribution Kumaraswamy Distribution. It is the most appropriate distribution in the simulation study because it is specified in the closed interval [1]. This distribution has a probability density function defined as follows[1]:

$$f(x;\alpha,\beta)=\alpha\beta x^{\alpha-1}(1-x^{\alpha})^{\beta-1}$$

Where, x is the random variable and α, β are the scale and shape parameter respectively.

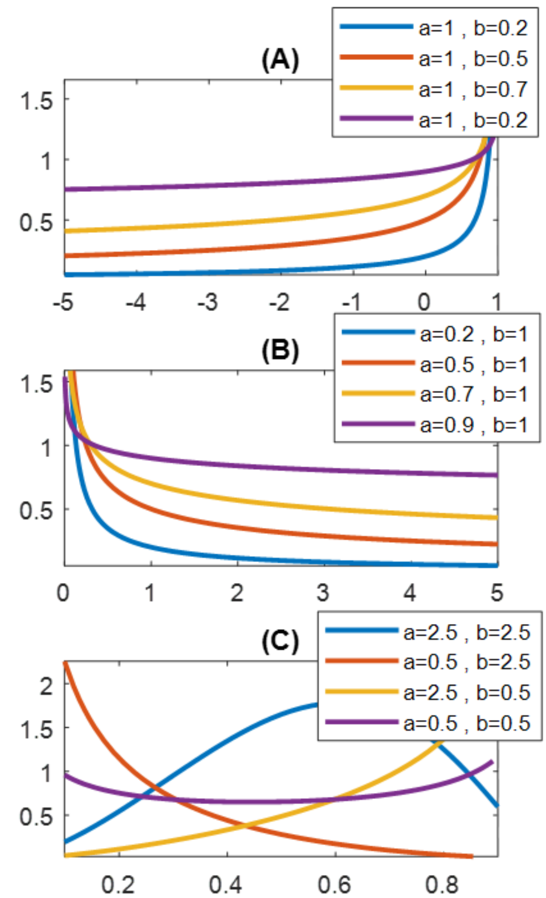

Fig.1: Plots of Kum Distribution Densities for Some Values of Parameters

2.1 maximum likelihood estimator

To find the estimator $\hat{\beta}_{ML}$ by Maximum Likelihood Estimator we should follow the next steps[4]:
To obtain the likelihood function L:

$$L = \alpha^n \beta^n \prod_{i=1}^{n} x_i^{\alpha-1} \prod_{i=1}^{n} (1 - x_i^{\alpha})^{\beta-1}$$

$$\log L = n \log \alpha + n \log \beta +$$

$$(\alpha - 1)\sum_{i=1}^{n} \log x_i + (\beta - 1)\sum_{i=1}^{n} (1 - x_i^{\alpha})$$

$$\frac{\partial \log L}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n} \log(1 - x_i^{\alpha})$$

$$\therefore \hat{\beta}_{ML} = \frac{-n}{\sum_{i=1}^{n} \log(1 - x_i^{\alpha})}$$

We will consider the scale parameter α= 1. So, the estimators methods will apply to find the estimation of the parameter β.

2.2 some statistical properties

In statistics and probability there are a family of continues distributions. The kum distribution does not seem to be very familiars distributions. It likes other distributions, it has several statistical and mathematical properties. Some of them are:[5,6]

a) Cumulative distribution function

$$F(x) = [1 - (1 - x^{\alpha})^{\beta}]$$

b) Mean

$$E(X) = \frac{\beta \Gamma(1 + \frac{1}{\alpha})\Gamma(\beta)}{\Gamma(1 + \frac{1}{\alpha} + \beta)}$$

c) Median

$$Med(X) = (1 - 2^{\frac{-1}{\beta}})^{\frac{1}{\alpha}}$$

2.3 hybrid estimator

Suppose the scale parameter α= 1. Then the modified Kumaraswamy distribution will has a pdf:

$$f(x; \beta) = \beta(1 - x)^{\beta-1}, x \in [0,1], \beta > 0$$

Simplify this function to :

$$f(x) = \beta(1 - x)^{-\beta(1-\frac{1}{\beta})}$$

Let $(1 - x)^{(1-\frac{1}{\beta})} = e^x$

Then, the considered pdf is:

$$f(x) = \beta e^{-\beta x}$$

The statistical property Median when α= 1 will be:

$$med(x) = 1 - 2^{-\frac{1}{\beta}}$$

Now, the proposed estimator for the shape parameter β using hybrid method will be:

$$\hat{\beta}_H = med(x) * \frac{1}{n^2}$$

So, the hybrid β estimation is :

$$\hat{\beta}_H = \frac{1 - 2^{(-\frac{1}{\beta})}}{n^2}$$

3. Simulation and results

To obtain the results that make us be able to choose the better estimation approach, that is representing a simulation study. The simulation study considered by generating random data in the interval [1]. This data is representing the random variable X. The estimation methods that used are the maximum likelihood and proposed hybrid estimator. These methods are applied to estimate just the shape parameter B, while the scale parameter a is considered as 1 in all steps of the simulation algorithm. We assumed that the initial values for β = 0.5, 0.7, 1, and 1.5. The hole simulation is carried out in different sample sizes, namely n = 10 30 50 100 1000. Mean square error (MSE) is the standard tool that used to compare between the methods of estimation. If the method has less MSE then it is better estimator. Simulation steps were implemented in the MATLAB, which can be represented by the flowchart steps in Fig.2:

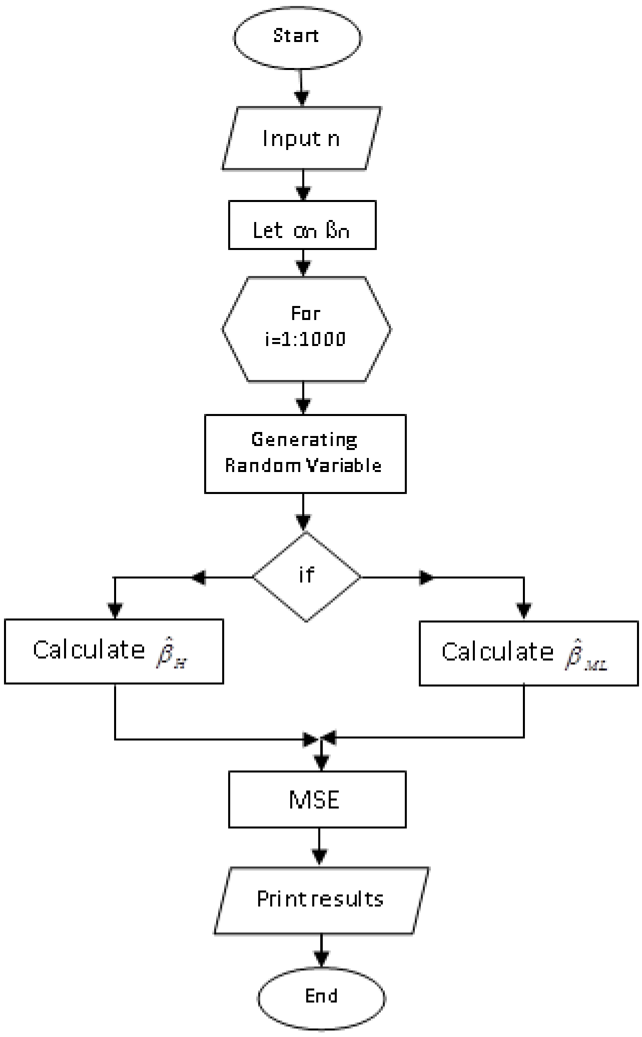

Fig. 2: Flowchart of Simulation Strategy

After carrying out the strategy of simulation study. We obtained on the results in the next Table:

Table 1. Estimation Results from Simulation When α= 1

n	β	$\hat{\beta}_{ml}$	MSE	$\hat{\beta}_H$	MSE
10	0.5	0.9975	0.2475	0.6922	0.0369
	0.7	1.3120	0.1632	0.9963	0.0878
	1	1.5728	0.3280	**0.9950**	**$2.5*10^{-5}$**
	1.5	1.0962	0.1630	1.3303	0.0288
30	0.5	1.0812	0.3378	0.9997	0.2497
	0.7	1.0812	0.1453	0.9996	0.0898
	1	1.0822	0.0066	**0.9994**	**$3.0864 * 10^{-7}$**
	1.5	1.0107	0.2394	1.0812	0.1754
50	0.5	0.9999	0.2499	0.9110	0.1689
	0.7	0.9830	0.0801	0.9793	0.0780
	1	0.9830	$2.8768*10^{-4}$	**0.9998**	**$4*10^{-8}$**
	1.5	0.9830	0.2672	0.9997	0.2503
100	0.5	0.9928	0.2428	0.9718	0.2226
	0.7	1.0070	0.0943	0.9718	0.0739
	1	0.9718	$7.9411*10^{-4}$	**1.0000**	**0**
	1.5	0.9718	0.2790	0.9999	0.2501
1000	0.5	1.0130	0.2631	0.9731	0.2238
	0.7	0.9925	0.0856	1.0130	0.0980
	1	1.0130	$1.6845*10^{-4}$	**1.0000**	**0**
	1.5	1.0031	0.2470	1.0130	0.2372

4. Discussion

After obtaining the results shown in Table 1. The hybrid method gave the best results in the sample sizes 10, 30, 50, 100 and 1000, especially when B = 1 where the MSE= 0.
In sample size 10 the nearest estimated value was

- The sample size is 10 the closest guesses were B=1 and b hat = 0.9950 with MSE= $2.5*10^{-5}$.
- The best result for hybrid estimation is when B= 1 in sample sizes 100 and 1000 whereas the $\hat{\beta}_H = \beta$. That is mean this is a sufficient and unbiased estimator. In other words, the mean square MSE = 0.

5. Conclusion

This paper was based on the method of comparing the estimations and selecting the best estimator that has the lowest error. The study supposed the maximum likelihood estimator as a known classical. It could be compared with the hybrid estimator. The estimation of proposed hybrid was proposed based on one of the characteristics of the Kumaraswamy distribution. This is median property [7, 8]. After conducting, a simulation study on the random variable randomization generated in the simulation experiment and obtained the numerical results. It was revealed by the values of estimations that the proposed estimator reflected the best results in most sample sizes and most of the values adopted for the parameter β.These good results were when β = 1. This means that there is a relationship between the assumed parameter and the parameter to be estimated. In other words, the sooner the shape and scale parameters were closer, the results were better. Finally, the hybrid method in this paper gave the unbiased and coefficient estimation for the shape parameter β of the Kumaraswamy distribution [9, 10].

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