



# Various Characteristics of a Few Topological Indices for $\gamma$ -Stable Graphs

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## Abstract

In this manuscript we have given various characteristics of some topological indices for  $\gamma$  stable graphs and computation of Wiener index using python program for Complex graph structures which frequently appear in all chemical Structure.

**Keywords:** bipartite index; Computation index;  $\gamma$  stable graphs; topological index.

## 1. Introduction

Mixture of elements bonded jointly to form basic building of compounds are Molecules. An undirected graph symbolize the topological configuration of a substance mix, when the arrangement of vertices specify the element of the substance, the arrangement of edges specify the links among the elements and F is a corresponding function of edges and vertices. Hence this graph is of a substance mix is called molecular graph.[1-5]

Calculating Wiener index for composite graph structure is given by simple formula and python program. Concerned readers can check this in the web <https://repl.it/repls/ShabbyFirmCheief>.

Wiener manifestation nature of  $\gamma$  stable graph [7]also declared in this paper.

## 2. Definition:

### 2.1 Wiener Index [9]:

The Wiener manifestation  $W(G)$  of a linked graph  $G$  with set of vertices  $V(G)$  is described as

$W(G)$  is equal to  $\sum_{u,v \in V(G)} d_G(u, v)$

Where  $d_G(u, v)$  represents the shortest path among  $u$  and  $v$  in  $G$ .

### 2.2 Theorem:

The Wiener index of the Cartesian product  $K_{4,4} \times P_r = G$  is given by

$$W(G) = dW(K_{4,4}) + [2W(K_{4,4}) + 64](d - 1) + \sum_{i=1}^{n-i-1} (n-i-1)[2W(K_{4,4}) + 64 + i64]$$

Where  $d$  is number of copies of bipartite graphs,  $i$  ranges from 1 to till the summation reaches zero.

### 2.3 Python Program for Wiener index calculation of Cartesian Product $K_{4,4} \times P_r$

Common = 40  
Increment= 64

```

Connection=144
Output=0
Print("Enter the number of elements :")
Count = int(input())
Value=0
Output=(count*common)
For i in range (1,count):
Value=(count-i)*connection
Connection=connection+increment
Output=ouput+value
Print (output)
    
```

### 2.4 Table for Wiener Index of Cartesian Product $K_{4,4} \times P_r$ :

S.No	Number of $K_{4,4}$	Wiener Index	Number of $K_{4,4}$	Wiener Index
1	n=1	W(G)=40	n=6	W(G)=3680
2	n=2	W(G)=224	n=7	W(G)=5544
3	n=3	W(G)=616	n=8	W(G)=7936
4	n=4	W(G)=1280	n=9	W(G)=10920
5	n=5	W(G)=2280	n=10	W(G)=14560

## 3. General Formula

### 3.1 Theorem:

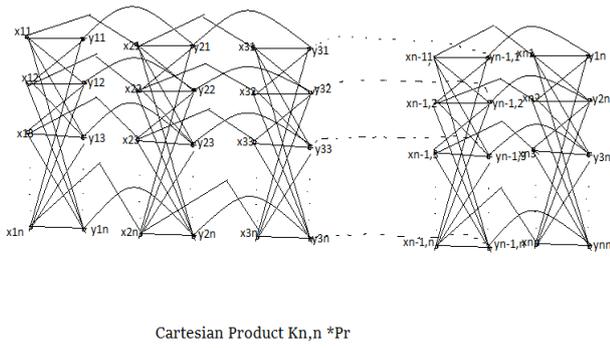
The Wiener index of the Cartesian product  $K_{m,n} \times P_r = G$  is given by th

$$W(G) = dW(K_{m,n}) + [2W(K_{m,n}) + (m+n)^2](d - 1) + \sum_{i=1}^{n-i-1} (n-i-1)[2W(K_{m,n}) + (2n)^2 + i(2n)^2]$$

Where  $W(K_{m,n}) = (m+n)^2 - (m-n) - mn$

Where  $d$  is number of copies of bipartite graphs,  $i$  ranges from 1 to till the summation reaches non negative value.

Proof:



Cartesian Product  $K_{n,n} \times P_r$

Fig.1: Cartesian Product  $K_{n,n} \times P_r$

The Wiener index of the Cartesian product  $K_{2,2} \times P_r = G$  is given by  $W(G) = dW(K_{2,2}) + [2W(K_{2,2}) + 16](d-1) + \sum_{i=1}^{d-1} (n-i-1)[2W(K_{2,2}) + 16 + i16]$

The Wiener index of the Cartesian product  $K_{3,3} \times P_r = G$  is given by  $W(G) = dW(K_{3,3}) + [2W(K_{3,3}) + 36](d-1) + \sum_{i=1}^{d-1} (n-i-1)[2W(K_{3,3}) + 36 + i36]$

The Wiener index of the Cartesian product  $K_{4,4} \times P_r = G$  is given by  $W(G) = dW(K_{4,4}) + [2W(K_{4,4}) + 64](d-1) + \sum_{i=1}^{d-1} (n-i-1)[2W(K_{4,4}) + 64 + i64]$  where d is number of copies of bipartite graphs, i ranges from 1 to till the summation reaches zero.

The Wiener index of the Cartesian product  $K_{m,n} \times P_r = G$  is given by  $W(G) = dW(K_{m,n}) + [2W(K_{m,n}) + (m+n)^2](d-1) + \sum_{i=1}^{d-1} (n-i-1)[2W(K_{m,n}) + (2n)^2 + i(2n)^2]$

**3.2 Python Program for Wiener calculation of Cartesian Product  $K_{m,n} \times P_r$**

```
Common = 8
Vertex=4
Increment= vertex*vertex
Connection=(common*2)+increment
Output=0
Value=0
Print("Enter the number of elements :")
Count = int(input())
Output=(count*common)
For i in range (1,count):
Value=(count-i)*connection
Connection=connection+increment
Output=output+value
Print (output)
```

**3.3 Result**

For two cycles  $C_1$  and  $C_2$  with any number of vertices n connected by an single bridge the resulting graph G, Wiener manifestation is  $W(G)=2W(C_1)+(n+1)(n/2)$  or  $W(G)=W(C_1)+W(C_2)+(n+1)(n/2)$

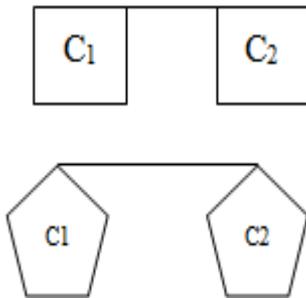


Fig.2: Graph G

**3.4 Result**

Consider any (G) connected Graph with any number of vertices n, join any two non adjacent vertices by an single edge the resulting graph  $G_R$ , Total eccentricity [8] manifestation does not change. That is  $\zeta(G) = \zeta(G_R)$



Fig.3: Graph G

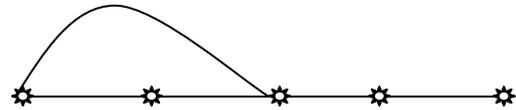


Fig.4: Graph  $G_R$

**4. Definition**

**4.1  $\gamma$ - stable graph [12]**

$\gamma$  stable graph is nothing but if  $\gamma(G_{xy}) = \gamma(G)$ ,  $\forall x$  and  $y \in V(G)$ , x and y are nonadjacent,  $G_{xy}$  indicate the graph acquire by identifying x and y [12].

**4.2 The Zagreb Index [7,10]:**

For any simple, connected graph G, the Zagreb manifestation of a graph G is clear by the following

$$M_1(G) = \sum_{v \in V(G)} d_G(v)^2 \text{ or } M_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]$$

$$M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v)$$

where  $d_G(v)$  is the vertex degree in G.

**4.3 The F-Index Index [7,10]:**

$$F(G) = \sum_{v \in V(G)} d_G(v)^3 \text{ or } F(G) = \sum_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2]$$

**4.4 Result**

Relation between Wiener, First Zagreb index, second Zagreb index and F-index of the following  $\gamma$  stable graph G and its complement Graph  $\bar{G}$  is given by

$W(G)$  less than  $M_1(G)$

$M_1(G)$  less than  $M_2(G)$

$M_2(G)$  less than  $F(G)$  and

$W(\bar{G})$  less than  $M_1(\bar{G})$

$M_1(\bar{G})$  less than  $M_2(\bar{G})$

$M_2(\bar{G})$  less than  $F(\bar{G})$

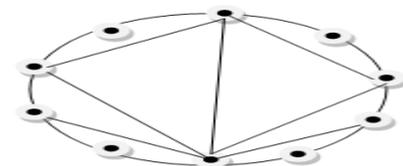


Fig.5:  $\gamma$ -Stable Graph G

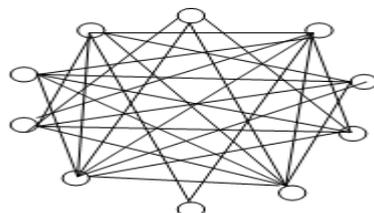


Fig.5:  $\gamma$ -Stable Graph  $G^c$

## 4.5 Result

From the Figure 2 and 3 ,  $\gamma(G)$  and  $\gamma(G^c)=2$ , and for all possible  $G_{xy}$  ,

(i)  $W(G_{xy}) < W(G)$ .

(ii)  $M_1(G_{xy}) > M_1(G)$

(iii)  $F(G_{xy}) > F(G)$

In common we get increased degree of vertex, when indentifying its end vertices. So in both (ii) and (iii) we get less index values for  $\gamma$ -stable graphs comparing with  $G_{xy}$ .

## 5. Conclusion:

The Wiener Topological index is established to the complex structure graph, that is Cartesian product of complete bipartite graph and path graph with two vertices, three vertices, four vertices and any number of vertices, and characteristics of topological indices for  $\gamma$ -stable graphs.

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