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Research paper

Various Separation Axioms on λ_g^δ-Closed Sets

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Abstract

The idea behind this article is to introduce and study the notions of λ_g^δ-compactness, λ_g^δ-connectedness and $\lambda_g^\delta G_i$-axioms. These notions are characterized using various spaces and different types of continuity.

Keywords: *Regular open sets, δ-open sets, λ_g^δ-open sets, λ_g^δ-compactness, λ_g^δ-connectedness.*

1. Introduction

The conceptualization of δ-closed sets was made by Velicko[10] during 1968. Georgiou et al.[1] dealt with the idea of (Λ, δ)-closed sets amid 2004. The notation of the so called λ_g^δ-closed sets[4] was made known in the year 2016. This definition was a generalization of δ-closed sets. Consequently, many concepts related to λ_g^δ-closed sets are being studied[5][6][7][8][9]. This work consists of some interesting axioms like λ_g^δ-compactness, λ_g^δ-connectedness and $\lambda_g^\delta G_i$-axioms. These concepts are analyzed through various forms of continuity and separation spaces.

2. Some Fundamentals

Definition 2.1: Let (P, τ) be a topological space. Then a subset Z of (P, τ) is known as

(1) **regular closed**[3] if Z = cl (int (Z)).

(2) **δ-open**[10] if Z is the union of regular open sets. The collection of all δ-open sets in (P, τ) is denoted by δO(P, τ).

(3) **Λ_δ-set**[1] if $\Lambda_\delta(Z)=Z$, where $\Lambda_\delta(Z)=\cap\{O\in\delta O(P,\tau) \mid Z\subseteq O\}$.

(4) **(Λ, δ)-closed**[1]if Z = T$\cap$C, where T is a $\Lambda\delta$ -set and C is a δ-closed set.

(5) **λ_g^δ-closed set**[4] if cl(Z) $\subseteq$ R whenever Z $\subseteq$ R and R is (Λ, δ)-open in P.

Definition 2.2 :[7] Let (P, τ) be a topological space. Then a subset Z is said to be a **λ_g^δ-neighborhood** of p $\in$ P iff $\exists$ a λ_g^δ-open set Q $\ni$ p $\in$ Q $\subseteq$ Z.

Definition 2.3: A map ψ : (P, τ) $\longrightarrow$ (Q, σ) is called

(1) **λ_g^δ-continuous[5]** if the inverse image of every open set in (Q, σ) is λ_g^δ-open in (P, τ).

(2) **quasi λ_g^δ-continuous[9]** if the inverse image of every λ_g^δ-open set in (Q, σ) is open in (P, τ).

(3) **perfectly λ_g^δ-continuous[9]** if the inverse image of every λ_g^δ-open set in (Q, σ) is clopen in (P, τ).

(4) **contra λ_g^δ-continuous[9]** if the inverse image of every open set in (Q, σ) is λ_g^δ-closed in (P, τ).

(5) **totally λ_g^δ-continuous[9]** if the inverse image of every open subset of (Q, σ) is λ_g^δ-clopen in (P, τ).

(6) **strongly λ_g^δ-continuous[9]** if the inverse image of every subset of (Q, σ) is λ_g^δ-clopen in (P, τ).

(7) **λ_g^δ-irresolute[9]** if the inverse image of every λ_g^δ-open set in (Q, σ) is λ_g^δ-open in (P, τ).

Definition 2.4 :[6]A space (P, τ) is known as a **λ_g^δ T_δ -space** if every λ_g^δ-closed subset of (P, τ) is δ-closed in (P, τ).

3. λ_g^δ-Compactness

Definition 3.1 :A collection $\mathcal{A}$ of a topological space (P, τ) is said to cover P (or) to be a covering of P if the union of elements of $\mathcal{A}$

is equal to P. $\mathcal{A}$ is said to be a **λ_g^δ-open covering of P** if its elements are λ_g^δ-open sets of (P, τ).

Definition 3.2 : A non-empty collection $\{Z_i \mid i\in I\}$ of λ_g^δ-open sets in (P, τ) is said to be an **λ_g^δ-open cover** of a subset B of (P, τ) if $B \subseteq \cup\{Z_i \mid i\in I\}$.

Definition 3.3 : A topological space (P, τ) is called **λ_g^δ-compact** if every λ_g^δ-open cover of P has a finite subcover.

Definition 3.4 : A subset B of a topological space (P, τ) is called **λ_g^δ-compact relative to P** if for every collection $\{Z_i \mid i \in I\}$ of λ_g^δ-open sets of (P, τ) $\ni B \subseteq \cup\{Z_i \mid i\in I\}$ $\exists$ a finite subset I_0 of I $\ni B \subseteq \cup\{Z_i \mid i\in I_0\}$.

Theorem 3.5 : Every λ_g^δ-closed subset of a λ_g^δ-compact space P is λ_g^δ-compact relative to P.

Proof : Let Z be a λ_g^δ-closed subset of a λ_g^δ-compact space P. Then P\Z is λ_g^δ-open in P. Let S = $\{V_i \mid i \in I\}$ be a λ_g^δ-open cover of Z in P. Then $S^* = S \cup \{P\backslash Z\}$ is a λ_g^δ-open cover of P. Since P is λ_g^δ-compact, S^* has a finite subcover of P, say $P = V_{i1} \cup V_{i2} \cup ... \cup V_{im} \cup Z^c$, where $V_{ik} \in S$. But Z and P\Z are disjoint and hence $Z \subseteq V_{i1} \cup V_{i2} \cup ... \cup V_{im}$, where $V_{ik} \in S$. This implies that any λ_g^δ-open cover S of Z contains a finite sub-cover. Therefore Z is λ_g^δ-compact relative to P.

Theorem 3.6 : A surjective λ_g^δ-continuous image of a λ_g^δ-compact space is compact.

Proof : Let $\psi : P \longrightarrow Q$ be a surjective λ_g^δ-continuous function from a λ_g^δ-compact space P to Q. Let $\{V_i \mid i\in I\}$ be an open cover of Q. Since ψ is λ_g^δ-continuous, $\{\psi^{-1}(V_i) \mid i\in I\}$ is a λ_g^δ-open cover of P. Since P is λ_g^δ-compact, $\exists$ a finite subcover $\{\psi^{-1}(V_1), \psi^{-1}(V_2),..., \psi^{-1}(V_n)\}$ of $\{\psi^{-1}(V_i) \mid i\in I\}$. Since ψ is surjective, $\{V_1, V_2,...,V_n\}$ is a finite open cover of Q. Hence (Q, σ) is compact.

Theorem 3.7 : A surjective, quasi λ_g^δ-continuous image of a compact space is λ_g^δ-compact.

Proof : Let $\psi : (P, \tau) \longrightarrow (Q, \sigma)$ be a surjective, quasi λ_g^δ-continuous function and $\{V_i \mid i\in I\}$ be a λ_g^δ-open cover of Q. Since ψ is quasi λ_g^δ-continuous, $\{\psi^{-1}(V_i) \mid i\in I\}$ is an open cover of P. Since P is compact, $\exists$ a finite open subcover $\{\psi^{-1}(V_1), \psi^{-1}(V_2),..., \psi^{-1}(V_n)\}$ of $\{\psi^{-1}(V_i) \mid i\in I\}$. Since ψ is surjective, $\{V_1, V_2, ...,V_n\}$ is a finite λ_g^δ-open subcover of Q and hence Q is λ_g^δ-compact.

Corollary 3.8: A surjective, perfectly λ_g^δ-continuous image of a compact space is λ_g^δ-compact.

Proof : Since every perfectly λ_g^δ-continuous function is a quasi λ_g^δ-continuous function, the result follows.

Theorem 3.9: If $\psi : (P, \tau) \longrightarrow (Q, \sigma)$ is λ_g^δ-irresolute and $B \subseteq P$ is λ_g^δ-compact relative to P then the image, $\psi(B)$ is λ_g^δ-compact relative to Q.

Proof : Let $\cup\{Z_i \mid i\in I\}$ be a λ_g^δ-open cover of $\psi(B)$ i.e., $\psi(B) \subseteq \cup\{Z_i \mid i \in I\} \Longrightarrow B \subseteq \cup\{\psi^{-1}(Z_i) \mid i \in I\}$. Since B is λ_g^δ-compact relative to P, $\{\psi^{-1}(Z_i) \mid i\in I\}$ has a finite subcover $\cup\{\psi^{-1}(Z_i) \mid i \in I_0\}$(say) $\ni B \subseteq \{\psi^{-1}(Z_i) \mid i \in I_0\} \Longrightarrow \psi(B) \subseteq \cup\{Z_i \mid i \in I_0\} \Longrightarrow \cup\{Z_i \mid i \in I_0\}$ is a finite subcover of $\cup\{\psi^{-1}(Z_i) \mid i \in I\}$. Therefore $\psi(B)$ is λ_g^δ-compact relative to Q.

Theorem 3.10 : A topological space P is λ_g^δ-compact iff each family of λ_g^δ-closed subsets of P with the finite intersection property has a non-empty intersection.

Proof : Given a collection G of subsets of P, let H = {P\G | G ∈ G } be the collection of its complements. Then we have,

G is a collection of λ_g^δ-open sets iff H is a collection of λ_g^δ-closed sets.

The collection G covers P iff the intersection $\bigcap_{H\in\mathcal{H}} H$ of all elements of H is non-empty.

The finite sub-collection $\{G_1, G_2, ... , G_n\}$ of G covers P iff the intersection of the corresponding elements $H_i = P \setminus G_i$ of H is empty.

Statement (i) is obvious whereas (ii) and (iii) follow from DeMorgan's law: $P \setminus \bigcup_{\alpha\in J} A_\alpha = \bigcap_{\alpha\in J} (P \setminus A_\alpha)$. Now we prove the theorem by contra positive approach which is equivalent to the following:

Let G be any collection of λ_g^δ-open sets in P. If no finite subcollection of G covers P, then G does not cover P. Now applying (i) to (iii), we observe that this statement is equivalent to the following:

Given any collection H of λ_g^δ-closed sets, if every finite intersection of elements of H is non-empty then intersection of all elements of H is non-empty.

Definition 3.11 : A topological space (P, τ) is **λ_g^δ-Lindelof** if every λ_g^δ-open cover of P contains a countable subcover.

Theorem 3.12 : Every λ_g^δ-compact space is λ_g^δ-Lindelof.

Theorem 3.13 : A surjective, λ_g^δ-irresolute image of a λ_g^δ-Lindelof space is λ_g^δ-Lindelof.

Proof : Let $\psi : P \longrightarrow Q$ is a λ_g^δ-irresolute, surjection and P be a λ_g^δ-Lindelof space. Let $\{R_i \mid i \in I\}$ be an λ_g^δ-open cover of Q. Then $\{\psi^{-1}(R_i) \mid i \in I\}$ is a λ_g^δ-open cover of P. Since P is λ_g^δ-Lindelof, it has a countable subcover namely $\{\psi^{-1}(R_1), \psi^{-1}(R_1),..., \psi^{-1}(R_n),...\}$. Since ψ is surjective, $\{R_1, R_2,...,R_n,...\}$ is a countable subcover of Q. Hence Q is λ_g^δ-Lindelof.

Theorem 3.14 : A surjective λ_g^δ-continuous image of a λ_g^δ-Lindelof is Lindelof.

Proof : Let $\psi : P \longrightarrow Q$ be a surjective, λ_g^δ-continuous function from a λ_g^δ-Lindelof space P to Q. Let $\{R_i \mid i\in I\}$ be an open cover of Q. Since ψ is λ_g^δ-continuous, $\{\psi^{-1}(R_i) \mid i\in I\}$ is a λ_g^δ-open cover of P. Since P is λ_g^δ-Lindelof, $\exists$ a countable subcover {$\psi^{-1}(R_1)$, $\psi^{-1}(R_2)$,..., $\psi^{-1}(R_n)$,...} of {$\psi^{-1}(R_i)$ | i∈ I}. Since ψ is surjective, {R_1, R_2,...,R_n,...} is a countable subcover of Q. Hence (Q, σ) is Lindelof.

Theorem 3.15 : A surjective, quasi λ_g^δ-continuous image of a Lindelof space is λ_g^δ-Lindelof.

Proof : Let ψ : (P, τ) ⟶ (Q, σ) be a surjective, quasi λ_g^δ-continuous function and {R_i | i ∈ I} be a λ_g^δ-open cover of Q. Since ψ is quasi λ_g^δ-continuous, {$\psi^{-1}(R_i)$ | i ∈ I} is an open cover of P. Since P is Lindelof, ∃ a countable subcover {$\psi^{-1}(R_1)$, $\psi^{-1}(R_2)$,..., $\psi^{-1}(R_n)$,...} of {$\psi^{-1}(R_i)$ | i ∈ I}. Since ψ is surjective, {R_1, R_2, ..., R_n,...} is a countable subcover of Q and hence Q is λ_g^δ-Lindelof.

Corollary 3.16 : A surjective, perfectly λ_g^δ-continuous image of a compact space is λ_g^δ-compact.

Proof : The proof follows since every perfectly λ_g^δ-continuous function is a quasi λ_g^δ-continuous function.

4. λ_g^δ-Compactness

Definition 4.1: A subset Z of a topological space (P, τ) is called

λ_g^δ-regular closed if Z = λ_g^δcl(λ_g^δint(Z)).

λ_g^δ-regular open if Z = λ_g^δint(λ_g^δcl(Z)).

λ_g^δ-regular if it is both λ_g^δ-regular closed and λ_g^δ-regular open.

Definition 4.2 :[8] Let (P, τ) be a topological space. Then a subset Z of (P, τ) is known as **λ_g^δ-Frontier** (briefly, λ_g^δFr(Z)) is defined as λ_g^δFr(Z) = λ_g^δcl(Z)\$\lambda_g^\delta$int(Z).

Theorem 4.3 : A subset Z of a topological space (P, τ) is λ_g^δ-regular iff λ_g^δFr(Z) = ϕ.

Proof : ***Necessity :*** Let Z be λ_g^δ-regular then (i) Z = λ_g^δcl(λ_g^δint(Z)) and (ii) Z = λ_g^δint(λ_g^δcl(Z)). Now, (i) ⟹ λ_g^δcl (Z) = λ_g^δcl(λ_g^δcl(λ_g^δint(Z))) = λ_g^δcl(λ_g^δint(Z)) = Z and (ii) ⟹ λ_g^δint(Z) = λ_g^δint(λ_g^δint(λ_g^δcl(Z))) = λ_g^δint(λ_g^δcl(Z)) = Z. Thus λ_g^δFr(Z) = λ_g^δcl(Z)\$\lambda_g^\delta$int(Z) = ϕ.

Sufficiency : Let λ_g^δFr(Z) = ϕ. This implies λ_g^δcl(Z) = λ_g^δint(Z) which means λ_g^δ int(Z) = Z = λ_g^δ cl(Z). Thus we have λ_g^δcl(λ_g^δint(Z)) = λ_g^δcl(Z) = Z and λ_g^δint(λ_g^δcl(Z)) = λ_g^δint(Z) = Z. Hence Z is λ_g^δ-regular.

Definition 4.4: A topological space (P, τ) is called **λ_g^δ-connected** if P cannot be expressed as a union of two disjoint, non-empty, λ_g^δ-open sets.

Theorem 4.5: For a topological space (P, τ), the following are equivalent:

P is λ_g^δ-connected.

P and ϕ are the only λ_g^δ-regular subsets of P.

Each λ_g^δ-continuous function of P into a discrete space Q with atleast two points is a constant function.

Every non-empty proper subset has a non-empty λ_g^δ-Frontier.

Proof :(i) ⟹ (ii) Let R be a λ_g^δ-regular subset of P. Then P\R is both λ_g^δ-open and λ_g^δ-closed in P. Since P is the disjoint union of λ_g^δ-open sets R and P\R, P is not λ_g^δ-connected which is a contradiction to (i) and hence one of these must be empty. That is

R = ϕ or R = P.

(ii) ⟹ (i) Suppose P = Z ∪ B, where Z and B are non-empty, λ_g^δ-open sets. Then Z = P\B is λ_g^δ-closed. Then Z is a non-empty, proper subset that is λ_g^δ-regular. This is a contradiction to (ii). Hence P is λ_g^δ-connected.

(ii) ⟹ (iii) Let ψ : (P, τ) ⟶ (Q, σ) be a λ_g^δ-continuous function and Q be a discrete space with at least two points. Then for each q ∈ Q, {q} is both open and closed. Since ψ is λ_g^δ-continuous, ψ^{-1}{q} is λ_g^δ-open as well as λ_g^δ-closed in P and P = ∪{ψ^{-1}{q}| q ∈ Q}. By hypothesis ψ^{-1}{q} = ϕ or P for each q ∈ Q. If ψ^{-1}{q} = ϕ, for all q ∈ Q then ψ will not be a function. If ψ^{-1}{q} = P, for a single point q ∈ Q then there cannot exist another point q_1∈ Q ∋ ψ^{-1}{q_1} = P. Hence ∃ only one q ∈ Q ∋ ψ^{-1}{q} = P and ψ^{-1}{q_1} = ϕ, where q_1 ∈ Q and q_1 ≠ q. This proves that ψ is a constant function.

(iii) ⟹ (ii) Let R be a λ_g^δ-regular subset in P. We wish to prove that the only λ_g^δ-regular subsets are ϕ and P. Suppose R ≠ ϕ then we claim R = P. Let q_1, q_2 ∈ Q. Define ψ : P ⟶ Q by

$$\psi(p) = \begin{cases} q_1, & p \in U \\ q_2, & \text{otherwise} \end{cases}$$

Then for any open set S in Q,

$$\psi^{-1}(V) = \begin{cases} R & \text{if} \quad S \ \text{contains} \ q_1 \ \text{only} \\ P \setminus R & \text{if} \quad S \ \text{conatins} \ q_2 \ \text{only} \\ P & \text{if} \quad S \ \text{contains} \ q_1,\ q_2 \\ \varphi & \text{otherwise.} \end{cases}$$

In all the cases, ψ^{-1}(S) is λ_g^δ-open in P. Also, ψ is a non-constant, λ_g^δ-continuous function. This is a contradiction. Hence the only λ_g^δ-clopen subsets of P are ϕ and P.

(ii) ⟹ (iv) Let Z be a non-empty, proper subset of P. Suppose λ_g^δFr(Z) = ϕ. Then Z is both λ_g^δ-open and λ_g^δ-closed which is a contradiction to (ii).

(iv) ⟹ (ii) Suppose that Z is a non-empty, proper subset of P which is both λ_g^δ-closed and λ_g^δ-open. This implies Z is λ_g^δ-regular and hence by Theorem 4.3, λ_g^δFr(Z) = ϕ, which is a contradiction.

Theorem 4.6 : A surjective, λ_g^δ-continuous image of a λ_g^δ-connected space is connected.

Proof : Let ψ : (P, τ) ⟶ (Q, σ) be a surjective, λ_g^δ-continuous function. Suppose Q is not connected. Then Q = Z ∪ K, where Z and K are two disjoint, non-empty, λ_g^δ-open subsets of Q. Since ψ is surjective & λ_g^δ-continuous, P = ψ^{-1}(Z) ∪ ψ^{-1}(K) where

$\psi^{-1}(Z)$ and $\psi^{-1}(K)$ are disjoint, non-empty and λ_g^δ-open sets in (P, τ). But this is a contradiction to the fact that P is λ_g^δ-connected. Hence Q is connected.

Theorem 4.7 : If ψ : P ⟶ Q is a surjective, contra λ_g^δ-continuous function and P is λ_g^δ-connected then Q is connected.

Proof : Let S be a clopen subset of Q. Since ψ is contra λ_g^δ-continuous, $\psi^{-1}(S)$ is λ_g^δ-regular. As P is λ_g^δ-connected, $\psi^{-1}(S) = \phi$ or P. Since ψ is surjective, S = ϕ or Q. Hence Q is connected.

Theorem 4.8 : Let ψ : (P, τ) ⟶ (Q, σ) be a surjective, λ_g^δ-irresolute function. If P is λ_g^δ-connected then Q is λ_g^δ-connected.

Proof : Let S be a λ_g^δ-regular subset of Q. Since ψ is λ_g^δ-irresolute, $\psi^{-1}(S)$ is λ_g^δ-regular in P. As P is λ_g^δ-connected, $\psi^{-1}(S) = \phi$ or P. Since ψ is surjective, S = ϕ or Q. Hence Q is λ_g^δ-connected.

Theorem 4.9 : Let ψ : P ⟶ Q be a λ_g^δ-open, λ_g^δ-closed (resp. δ-open, δ-closed) injection. If Q is λ_g^δ-connected then P is also λ_g^δ-connected.

Proof : Let Z be a λ_g^δ-regular set in P. Since ψ is λ_g^δ-open and λ_g^δ-closed, $\psi(Z)$ is λ_g^δ-regular in Q. Since Q is λ_g^δ-connected, $\psi(Z) = \phi$ or Q. Since ψ is an injection, Z = ϕ or P. Hence P is λ_g^δ-connected.

Theorem 4.10 : If ψ : P ⟶ Q is a totally λ_g^δ-continuous function from a λ_g^δ-connected space P to Q then Q has the indiscrete topology.

Proof : Let S be open in Q. Since ψ is a totally λ_g^δ-continuous function, $\psi^{-1}(S)$ is λ_g^δ-regular in P. Since P is λ_g^δ-connected, $\psi^{-1}(S) = \phi$ or P. Since ψ is an injection, S = ϕ or Q. Hence Q has the indiscrete topology.

Theorem 4.11 : If ψ : P ⟶ Q is a strongly λ_g^δ-continuous bijective function and Q is a topological space with atleast two points then P is not λ_g^δ-connected.

Proof : Let q ∈ Q. Then $\psi^{-1}(\{q\})$ is a non-empty proper subset of P which is λ_g^δ-regular, as ψ is strongly λ_g^δ-continuous. Therefore P is not λ_g^δ-connected.

Theorem 4.12 : If a topological space (P, τ) is almost weakly Hausdorff and connected then it is λ_g^δ-connected.

Proof : Suppose P is not λ_g^δ-connected. Then P = Z ∪ B, where Z and B are non-empty, disjoint, λ_g^δ-open sets of P. Since P is almost weakly Hausdorff, Z and B are open in P[9]. This contradicts the connectedness of P. Hence P is λ_g^δ-connected.

Theorem 4.13: Every topological space which is both $\lambda_g^\delta T_\delta$ and connected is λ_g^δ-connected.

Proof : Obvious.

5. $\lambda_g^\delta G_i$ - Axioms (i = 1, 2)

Definition 5.1 : Let (P, τ) be a topological space. It is said to be a **$\lambda_g^\delta G_1$-space** if for any point p ∈ P and any connected subset M of P with p ∉ M, ∃ λ_g^δ-open sets R and S ∋ p ∈ R, M ⊆ S, R ∩ M = ϕ and {p} ∩ S = ϕ.

Example 5.2 : Let P = {x, y, z, d} and τ = {P, ϕ, {x}}. Then (P, τ) is a $\lambda_g^\delta G_1$-space as for z ∈ M and a connected set M = {x, y} with z ∉ {x, y}, ∃ λ_g^δ-open sets R = {z} and S = {x, y} ∋ z ∈ {z}, {x, y} ⊆ {x, y}, {z} ∩ {x, y} = ϕ.

Theorem 5.3 : If every connected subset of P is λ_g^δ-closed then for any two disjoint connected subsets M and N of P, ∃ λ_g^δ-open sets R and S ∋ M ⊆ R, N ⊆ S, R ∩ N = ϕ and M ∩ S = ϕ.

Proof : Let M and N be any two disjoint connected subsets of P. Then by hypothesis, M and N are λ_g^δ-closed. This implies P\M and P\N are λ_g^δ-open sets containing N and M respectively, as M and N are disjoint. Now let R = P\N and S = P\M. Then N ∩ R = S ∩ M = ϕ.

Theorem 5.4 : If for any two disjoint connected subsets M and N of P, ∃ λ_g^δ-open sets R and S ∋ M ⊆ R, N ⊆ S, R ∩ N = ϕ and S ∩ M = ϕ then P is $\lambda_g^\delta G_1$.

Definition 5.5 : Let (P, τ) be a topological space and (Q, σ) be its subspace. Then a subset Z of Q is **λ_g^δ-open in Q** if Z can be written as Z = Q ∩ K where K is λ_g^δ-open in P.

Theorem 5.6 : Every δ-open subspace Q of a $\lambda_g^\delta G_1$-space P is $\lambda_g^\delta G_1$.

Proof : Let Z be a connected subset in Q. Then Z is connected in P as well. Let q ∈ Q ⊆ P ∋ q ∉ Z. Then by hypothesis, ∃ λ_g^δ-open sets R and S ∋ q ∈ R, Z ⊆ S, R ∩ Z = ϕ and {q} ∩ S = ϕ. By the definition of subspace topology, Q ∩ R and Q ∩ S are λ_g^δ-open sets in Q ∋ q ∈ Q ∩ R, Z ⊆ Q ∩ S and (Q ∩ R) ∩ Z = {q} ∩ (Q ∩ S) = ϕ. Hence Q is a $\lambda_g^\delta G_1$-space.

Theorem 5.7 : A bijective, continuous and λ_g^δ-irresolute image of a $\lambda_g^\delta G_1$-space is a $\lambda_g^\delta G_1$-space.

Proof : Let ψ : P ⟶ Q be a continuous function and M be a connected subset in P ∋ p ∉ M. then $\psi(M)$ is connected in Q. Since ψ is one to one and onto, $\psi(p) \notin \psi(M)$. Now since Q is $\lambda_g^\delta G_1$, ∃ λ_g^δ-open sets R and S in Q ∋ $\psi(p)$ ∈ R, $\psi(M)$ ⊆ S and R ∩ $\psi(M)$ = {$\psi(p)$} ∩ S = ϕ. Since ψ is λ_g^δ-irresolute, $\psi^{-1}(R)$ and $\psi^{-1}(S)$ are λ_g^δ-open sets in P with p ∈ $\psi^{-1}(R)$, M ⊆ $\psi^{-1}(S)$ and $\psi^{-1}(R)$ ∩ M = {p} ∩ $\psi^{-1}(S)$ = ϕ. Hence P is a $\lambda_g^\delta G_1$-space.

Definition 5.8 : A topological space (P, τ) is called **$\lambda_g^\delta G_2$-space** if for every connected set F and a point p ∉ F, ∃ λ_g^δ-open sets R and S ∋ p ∈ R, F ⊆ S and R ∩ S = ϕ.

Example 5.9 : Let P and □ be defined as in Example 5.2. Then (P, □) is a $\lambda_g^\delta G_2$-space as for z ∈ F and a connected set F = {x, y} with z ∉ {x, y}, ∃ λ_g^δ-open sets R = {z} and S ={x, y} ∋ z ∈ {z}, {x, y} ⊆ {x, y}, {z} ⋂ {x, y} = ϕ.

Theorem 5.10 : Every $\lambda_g^\delta G_2$-space is a $\lambda_g^\delta T_2$-space.

Proof : Let (P, □) be a $\lambda_g^\delta G_2$-space and p ≠ q ∈ P. Then p ∉ {q}, which is a connected set. By hypothesis, ∃ λ_g^δ-open sets R and S ∋ p ∈ R, {q} ⊆ S and R ⋂ S = ϕ. Therefore ∃ λ_g^δ-open sets R and S ∋ p ∈ R, q ∈ S. Hence (P, □) is a $\lambda_g^\delta T_2$-space.

Theorem 5.11 : A □-open subspace of a $\lambda_g^\delta G_2$-space is $\lambda_g^\delta G_2$.

Proof : Similar to Theorem 5.6.

Theorem 5.12 : If a topological space (P, □) is $\lambda_g^\delta G_2$ then for any point p ∈ P and any connected subset M not containing p, λ_g^δcl(R) ⋂ M = ϕ, where R is a λ_g^δ-open neighborhood of p.

Proof : Let M be a connected subset of P ∋ p ∉ M. Since P is a $\lambda_g^\delta G_2$-space, ∃ disjoint, λ_g^δ-open sets R and S ∋ p ∈ R, M ⊆ S. This implies R ⊆ P\S and hence λ_g^δcl(R) ⊆ λ_g^δcl(P\S) = P\S, as P\S is λ_g^δ-closed. Further λ_g^δcl(R) ⋂ M = ϕ, as M ⊆ S.

6. Conclusion

Some conditions for preserving λ_g^δ-compactness are derived. Results relating λ_g^δ-compactness with compactness are obtained. λ_g^δ-connectedness is related to connectedness through almost weakly Hausdorff space and $\lambda_g^\delta T_□$-space, even though λ_g^δ-open sets and open sets are independent of each other. It is interesting to note that any surjective, λ_g^δ-irresolute image of a λ_g^δ-connected space is λ_g^δ-connected. The nature of $\lambda_g^\delta G_1$-space is preserved by a bijective, continuous and λ_g^δ-irresolute function.

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