

# Adaptive Phase-Shifted PWM based on Particle Swarm Optimization for Cascaded H-Bridge Inverter with Unequal Dc-link Voltages

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## Abstract

The cascaded H-Bridge Multilevel Inverter has been found a promising technology in industrial applications because of its higher voltage with less distortion production. Various PWMs techniques have been proposed to push the harmonics frequencies higher than the switching frequency and thus reduces the THD as compared to non-carrier control technique based upon grid frequency. The Phase-Shifted PWM technique has an advantage over others PWM techniques because its harmonics orders are multiples of switching frequency and also depend on the number of levels of the inverter. The phase shifting angle is uniform when the equal voltage sources are adopted. However, in applications where sets of different voltage source levels feed the H-Bridge cells, the Phase Shifted PWM suffers its high order harmonics elimination capability. As a solution to alleviate this problem, an adaptive variable angle approach is proposed in this paper using Particle Swarm Optimization (PSO) algorithm to eliminate desired higher order harmonics. The algorithm is used to minimize the cost function based on high order sideband harmonics elimination equations. The results through MATLAB/Simulink environment shown in this paper confirm the reduction of sideband harmonics of higher orders, and the overall THD.

**Keywords:** Cascaded H-Bridge Multilevel Inverter; Phase-Shifted PWM; Particle Swarm Optimization.

## 1. Introduction

The In industrial high-power medium voltage industrial applications, Multilevel Inverters (MLI) appear to be the convenient solution as they provide less distorted high voltage levels[1]-[2]. They come as an alternative in order to overcome the limited rating of power semi-conductor switches suffered by the conventional two level inverter. [3]-[4]. Their beauty includes not only the voltage stress reduction in each switch but also, they are capable to transfer the energy in an efficient way with higher output voltage. The MLI can be operated with low to high range of frequency. Because of these features, it can be used in various applications like grid-connected power systems, motor drives and Flexible AC Transmission Systems (FACTS)[5]. The main topologies of Multilevel inverters are diode-clamped inverter, capacitor-clamped inverter and cascaded inverter which is a family of cascaded H-Bridge multilevel inverters (CHB-MLI) and Modular Multilevel Converters (MMC) [5]-[6]. Among them, a great interest has been granted to CHB-MLI for not only its simplicity and modularity but also because it requires a smaller number of component and is more reliable than others. In the control part, its advantage over others is the possibility of applying independent modulation for each H-Bridge while its main drawback is its requirement of separate isolated dc supplies for each H-Bridge.

However, this property can be used as an opportunity in applications where different types of voltage sources are available at the input. Among numerous modulation techniques adapted to it, the PS-PWM has been demonstrated very suitable for its

modulation as it presents advantages like equal power distribution among H-Bridges, equal power switches utilization and for  $N$  number of bridges inverter, harmonics cancellation up to  $2N^{th}$  carrier multiples [7]-[8]. However, in applications where a set of different types of voltage sources of different voltage levels feed the H-Bridge cells like in Hybrid Electric Vehicles (HEV) applications, the Phase Shifted PWM suffers its high order sideband harmonics elimination capability [9]. As a solution to alleviate this problem, an adaptive variable angle Phase-Shifted PWM based on Particle Swarm Optimization (PSO) algorithm is proposed to eliminate desired higher order harmonics. The algorithm is used to minimize the cost function based on high order sideband harmonics elimination equations to find the right phase angles for the desired high order sideband harmonics elimination[10]-[13].

## 2. Cascaded H-Bridge Inverter with the Conventional Phase-Shifted PWM

### 2.1. Cascaded H-Bridge Inverter

The cascaded multilevel inverter is made up of multiple units of single-phase H-Bridge cells connected in cascaded series through their output to produce a staircase output voltage of the converter. An H-bridge inverter consists of an isolated DC voltage source; four switches along with four antiparallel connected diodes. The output voltage  $V_{ai}$  is obtained from the combination of the four switches S1, S2, S3 and S4 states where two of the same leg never conduct at a time, to produce three levels +Vdc, -Vdc and zero voltage. It should be recalled that S1,S2 and S3,S4 are

complementary; the +Vdc level is produced when S1 and S4 conduct, the -Vdc level when S2 and S3 conduct and the zero voltage is obtained when S1 and S3 conduct or S2 and S4 conduct. For m-level multilevel inverter, the phase output voltage has m levels while the line output voltage has (2m-1) levels. The number of levels of the output voltage is given by  $m=2N+1$ , where N is the total number of H-bridge cells connected in series cascaded. The total number of active switches required for m-level converter is given by  $N_{sw}=2(m-1)$ . Fig.1 shows a structure of a single-phase n-level cascaded H-Bridge Inverter. The phase output voltage of m-level cascaded H-bridge's equation is given in equation (1)

$$V_{an} = V_{a1} + V_{a2} + V_{a2} + \dots + V_{an} \quad (1)$$

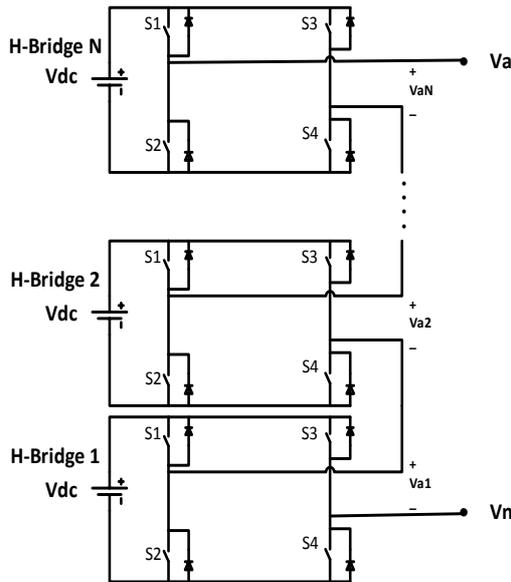


Fig.1. Structure of a single-phase n-level CHB-MLI

### 2.2. Conventional Phase-Shifted PWM

The PS-PWM technique is based on unipolar PWM or bipolar PWM applied to all H-Bridges with the same reference signals while the carriers are consecutively phase shifted one from another by  $\theta_{cr} = 360^\circ / (m-1)$  where m is the inverter's levels count[8]. For m-level inverter it is required (m-1) triangular carrier signals of same frequency and same peak to peak amplitude. The H-Bridges reference signals are sinusoidal waves of frequency  $f_m$  and 180° phase shifted. The switching frequency that is the same as the carrier frequency is given by  $f_{cr} = f_m \cdot m_f$ , where  $f_m$  is the modulation signal frequency and  $m_f$  the frequency modulation index that is chosen by the converter designer. For the harmonics study, the analytic solution of the output voltage is expressed in form of double summation Fourier series which is given in equation (2) [14]–[15].

$$V_{an}(t) = \begin{cases} NMV_{dc} \cos(\omega_0 t) + \frac{2V_{dc}}{\pi} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{m} J_{2n-1}(Nm\pi M) \cdot \\ \cos[(Nm+n-1)\pi] \cos[2Nm\omega_c t + (2n-1)\omega_0 t] \end{cases} \quad (2)$$

where N is the H-Bridges count, M is the modulation index,  $V_{dc}$  the dc voltage of each bridge,  $\omega_0$  and  $\omega_c$  respectively the pulsation of the modulation signal and the carrier signal,  $J_{2n-1}$  is the Bessel function of order (2n-1); n and m are respectively the order of

modulation signal and carrier harmonics. This output voltage has two components, the first being the fundament wave's and the second being the carriers along with their sideband harmonics. It is observable that the sideband harmonics that are multiples of the fundamental frequency  $\omega_0$  are placed around 2N multiples of carrier frequency  $\omega_c$ . But in unequal dc-link voltages condition, the inverter loses the disposition of all harmonics around multiple of carriers because of the presence of multiples of two low-order sideband harmonics. Fig.2 shows the block diagram for PS-PWM generation for two cells CHB-MLI.

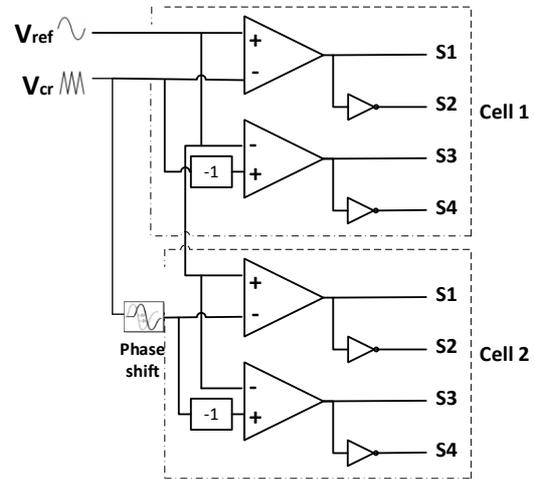


Fig.2. PS-PWM generator diagram of two cells CHB-MLI

## 3. Variable-angle Phase-Shifted PWM Based on PSO Algorithm

### 3.1. Objective Function for Harmonics Elimination

This paper proposes a variable angle PS-PWM adapted to the naturally sampled PWM. Based on equation (3), an equation that considers different carrier phases and dc-link voltages of order  $i$  can be written as followed:

$$V_{an}(t) = \begin{cases} M \sum_{i=1}^N V_{dc,i} \cos(\omega_0 t) + \\ \frac{2}{\pi} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{m} J_{2n-1}(Mm\pi) \cos[(m+n-1)\pi] \cdot \\ \sum_{i=1}^N V_{dc,i} \cos[2m\omega_c t + (2n-1)\omega_0 t + 2m\theta_i] \end{cases} \quad (3)$$

$V_{dc,i}$  and  $\theta_i$  are respectively the dc-link voltage and the carrier phase of the  $i^{th}$  H-Bridge. The objective of elimination sideband harmonics can be achieved by considering the equation (4) below derived from an analysis of equation (3):

$$\sum_{i=1}^N V_{dc,i} \cos[2m\omega_c t + (2n-1)\omega_0 t + 2m\theta_i] = 0 \quad (4)$$

The equation (4) can be simplified into equation (5) below:

$$\begin{cases} \sum_{i=1}^N V_{dc,i} \cos(2m\theta_i) = 0 \\ \sum_{i=1}^N V_{dc,i} \sin(2m\theta_i) = 0 \end{cases} \quad (5)$$

Where  $2m\theta_i$  represents the phase of all the  $m^{th}$  sideband harmonics produces by the  $i^{th}$  H-Bridge. Setting the first bridge phase angle  $\theta_1 = 0$ , equation (5) becomes a system of  $N-1$  equations when  $N$  is odd or  $N-2$  equations when  $N$  is even. Solving these equations is quite complex because of their transcendental non-linear nature and as a solution in this paper, Particle Swarm Optimization (PSO) is proposed with less computational burden and complexity. In the case study of this paper, an 11-level inverter is considered and the system of equations to solve is given by equation (6) that is the objective function to minimize the most possible using the algorithm.

$$\begin{cases} f_1 = V_{dc,1} + V_{dc,2} \cos 2\theta_2 + V_{dc,3} \cos 2\theta_3 + V_{dc,4} \cos 2\theta_4 + V_{dc,5} \cos 2\theta_5 = 0 \\ f_2 = V_{dc,2} \sin 2\theta_2 + V_{dc,3} \sin 2\theta_3 + V_{dc,4} \sin 2\theta_4 + V_{dc,5} \sin 2\theta_5 = 0 \\ f_3 = V_{dc,1} + V_{dc,2} \cos 4\theta_2 + V_{dc,3} \cos 4\theta_3 + V_{dc,4} \cos 4\theta_4 + V_{dc,5} \cos 4\theta_5 = 0 \\ f_4 = V_{dc,2} \sin 4\theta_2 + V_{dc,3} \sin 4\theta_3 + V_{dc,4} \sin 4\theta_4 + V_{dc,5} \sin 4\theta_5 = 0 \end{cases} \quad (6)$$

### 3.2 PSO algorithm for Variable Angle Phase-Shifted PWM

Introduced for the first time in 1995 by Kennedy and Eberhart, PSO is a classic algorithm that is inspired by the ways nature solves its problems [16]. It attempts to accurately simulate the behavior of flocks of birds or fish known as particles. It has been found very efficient in solving numerical optimization problems. Its principle is to put particles into a search space, giving them behaviors that depend on the fitness function and each particle  $i$  is defined by its position  $X_i$  and its velocity vector  $V_i$  and each particle get involved by following two fundamental rules as followed:

- Each particle follows the best on and steers towards the best position in its own vicinity.
- Each particle updates its parameters by following equation (7) and (8).

$$V_i(k+1) = W(k)V_i(k) + c_1r_1(P_{Best,i} - X_i(k)) + c_2r_2(G_{Best} - X_i(k)) \quad (7)$$

$$X_i(k+1) = X_i(k) + V_i(k+1) \quad (8)$$

Where  $W$  is the inertia weight chosen according to the problem's criteria;  $C_1$  and  $C_2$  are learning factors within the range of 1 and 2;

$r_1$  and  $r_2$  are random numbers.  $P_{Best,i}$  and  $G_{Best}$  are respectively the current best position of the particle  $i$  and the current global best position. For the algorithm to perform better, in this paper the inertia weight is being linearly damped when the algorithm iterates by the equation (9) [17]-[19].

$$W(k) = W_{max} - \frac{k}{k_{max}}(W_{max} - W_{min}) \quad (9)$$

For the case study of this paper, the multi-objective function in equation (6) is converted into a single objective function as

equation (10) and the population size is 30; the number of iterations is 500;  $W_{max}=0.9$ ;  $W_{min}=0.2$  and  $C_1=C_2=2$ .

$$f_{obj} = (f_1)^2 + (f_2)^2 + (f_3)^2 + (f_4)^2 \quad (10)$$

The variable angles PS-PWM based on PSO's algorithm's flowchart shown in Fig.3 [19]-[20].

### 3. Simulation Results and Analysis

Three convergent curves of the algorithm are shown in Fig.4. where the best result is obtained on the second run. The phase angle computed from the algorithm are shown in TABLE.I for a set of dc-link voltages shown in the same table. For the simulation, modulation signals of 50Hz frequency with modulation index  $M=0.92$  are considered.

TABLE I. A set of an 11-level CHB-MLI dc-link voltages and the H-Bridge units phase angles  $\theta_i$

	H-Bridge 1	H-Bridge 2	H-Bridge 3	H-Bridge 4	H-Bridge 5
$V_{dc,i}$	15V	18V	21V	24V	27V
$\theta_i$ For PS-PWM	$0^\circ$	$32^\circ$	$72^\circ$	$108^\circ$	$144^\circ$
$\theta_i$ For PSO based PS-PWM	$0^\circ$	$30.010^\circ$	$65.661^\circ$	$102.963^\circ$	$143.881^\circ$

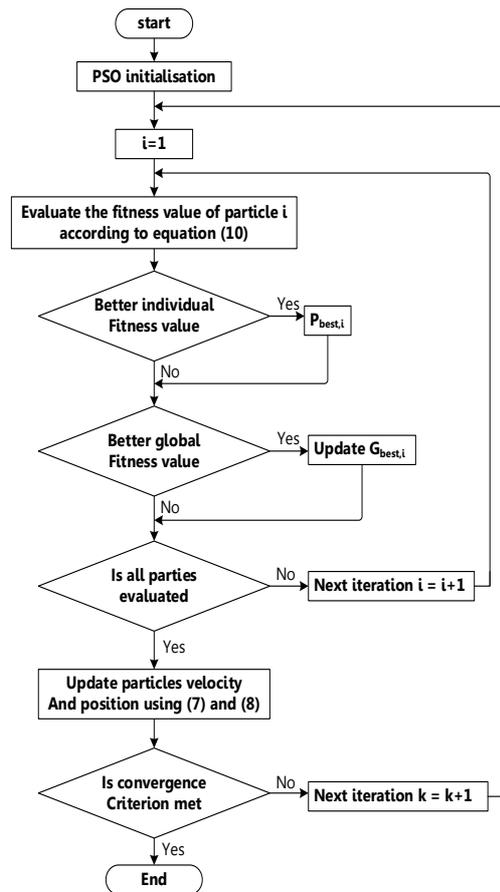


Fig.3. Flowchart of the PSO algorithm for variable-angle PS-PWM

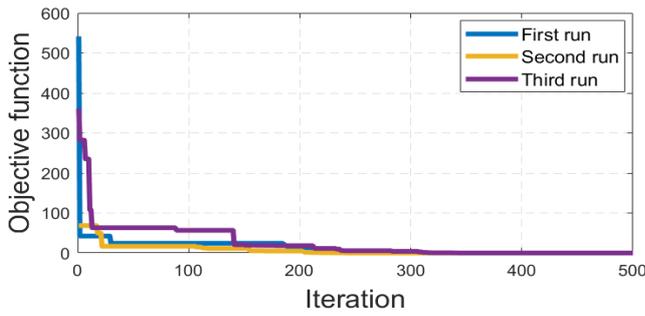


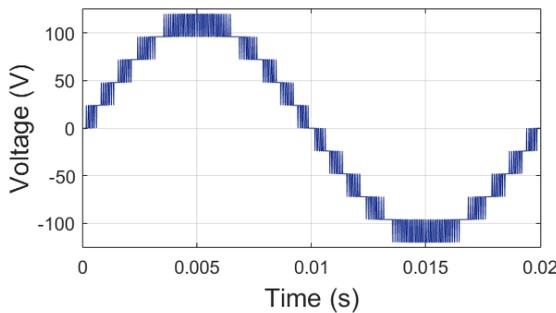
Fig.4. Cost function vs number of iterations of the PSO algorithm

Fig.5 shows the output voltage and harmonic spectrum of the CHB-MLI with equal dc-link voltages and it is shown in the harmonic spectrum that the first sideband harmonic appears around 10kHz that confirm the theory in section 2 and the THD in this case, considering harmonics up to 5kHz is 1.51%. Fig.6

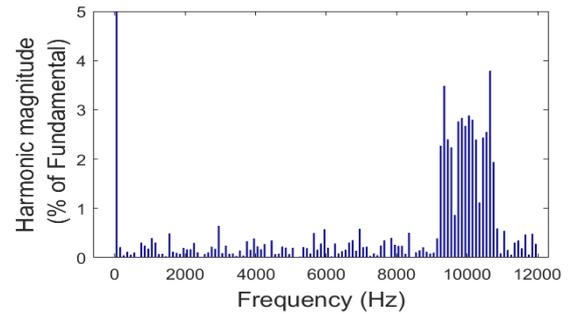
shows the output voltage and the harmonic spectrum when the same conventional modulation technique is used but with unequal dc-link voltages. It is observed the presence of sideband harmonics around 2kHz, 4kHz, 6kHz and 8kHz which give an increase to the output voltage THD up to 6.17%. Fig.7 shows the same parameters when the phase angles computed off-line from the PSO algorithm are applied to the H-Bridge units and the output voltage THD becomes 3.47%. It is clearly observed that the sideband harmonics around 2kHz and 4kHz have been considerably reduced. The sideband harmonics magnitudes have been reduced about 5.63% around 2kHz and about 2.49% around 4kHz and the THD has been reduced up to 43% of the THD when the conventional PS-PWM is used. It is also observable a slight increase of sideband harmonics around 6kHz and 8kHz but does not influence that much the inverter performance because of their location at higher frequencies. Table II. Show the magnitudes of the highest harmonics of sideband harmonics around 2kHz and 4kHz of the conventional PS-PWM and the PSO based variable angle PS-PWM applied.

Table 2: Magnitudes of sideband harmonics around 2kHz and 4kHz

Magnitude of sideband harmonics around 2kHz and 4kHz								
Modulation technique	1850Hz	1950Hz	2050Hz	2150Hz	3850Hz	3950Hz	4050Hz	4150Hz
Conventional PS-PWM	2.41%	3.29%	3.09%	2.52%	0.36%	1.18%	0.99%	0.37%
PSO based PS-PWM	0.62%	0.69%	0.81%	0.68%	0.47%	0.54%	0.69%	0.32%

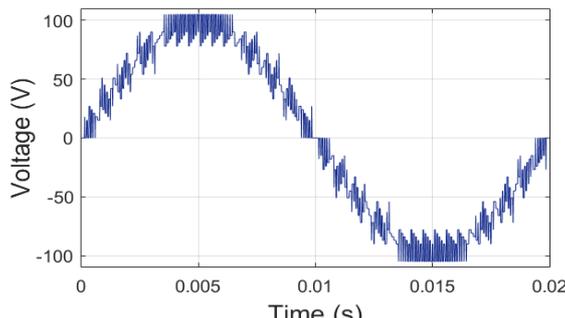


(a)

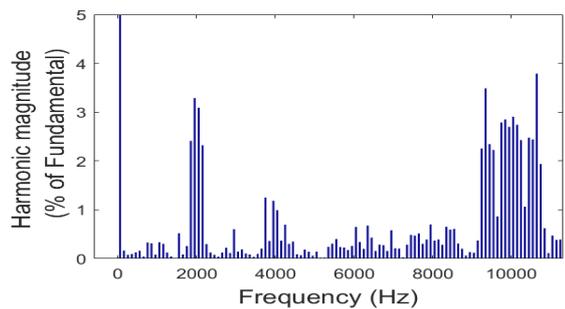


(b)

Fig.5. Result of the conventional PS-PWM with equal dc-link voltages for eleven level Inverter: (a) Output voltage waveform; (b) harmonic spectrum

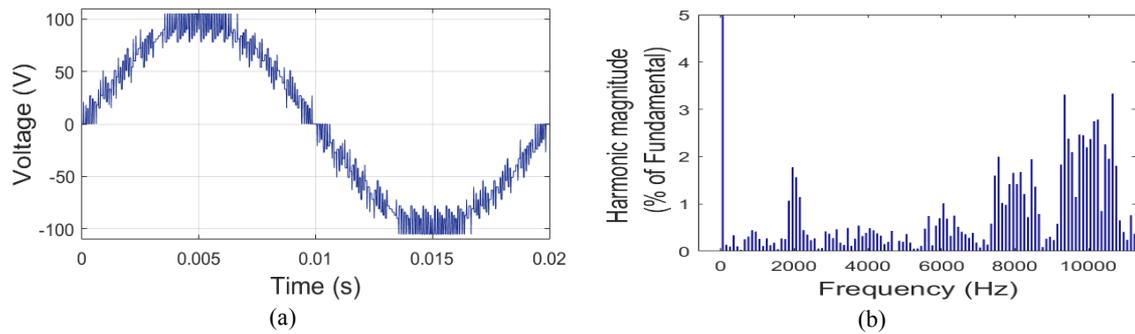


(a)



(b)

Fig.6. Result of the conventional PS-PWM with unequal dc-links voltage for eleven level Inverter: (a) Output voltage waveform; (b) harmonic spectrum.



**Fig.7.** Result of the adapted PS-PWM based on PSO algorithm with unequal dc-links voltage for eleven-level inverter: (a) Output voltage waveform; (b) harmonic spectrum.

## 4. Conclusion

In this paper, an eleven-level CHB-MLI with equal and unequal dc-link voltages have been analyzed with the conventional PS-PWM. It is shown that in case of unequal dc-link voltages the PS-PWM is not a good solution for inverter modulation because of the considerable presence of sideband harmonics closes to switching frequency. For that, an adapted PS-PWM is proposed in this paper, where the phase angles are computed off-line through PSO algorithm for reduction of desired sideband harmonics. Besides, this technique can be used for on-line estimation of phase shifting angles. The simulation results confirm the reduction of sideband harmonics with frequencies closes to twice and four times of switching frequency. The sideband harmonics magnitudes have been reduced about 5.63% around 2kHz and about 2.49% around 4kHz and the THD has been reduced up to 43% of the THD when the conventional PS-PWM is used. It is also observed a slight increase of sideband harmonics around 6kHz and 8kHz but does not influence that much the inverter performance because of their location at higher frequencies. Considering equal dc-link voltages, the algorithm has computed exactly the same phase angles as those used in the conventional PS-PWM. This makes the adapted technique suitable in case of unbalanced as well as balanced dc-link conditions.

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