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Research paper

Spiral Batik with Rational Bezier Curve

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Abstract

Spiral is one of the commercial designs that is used in designing Batik patterns. There are many types of spiral motifs that can be designed for beautiful batik patterns. In this paper, one of the spiral shapes was chosen as a reference figure to regenerate using quadratic and cubic rational Bezier curves. This curve is well known as areas of research in Computer Aided and Geometric Design (CAGD). This paper deals with generating the spiral batik design and manipulation of weight magnitude to regenerate the design that is closer to the reference figure and produce new and variety designs by manipulating the value of weight in the curve. From the result, it shows that the cubic curve generated the spiral batik design was closer to the reference figure and produced more designs of the spiral Batik compared to the quadratic curve. The result also shows that quadratic Bezier curve is suitable to be used in generating the design since it involves simple and less cost method by manipulating a variable which is the weight in the curve. Lastly the result shows. Conclusively, this method improves the batik industry by using CAGD in designing the motif.

Keywords: *Spiral Batik; Rational; Quadratic; Cubic; Bezier curve*

1. Introduction

Bezier curves were generated in 1962 by a French automotive engineer and mathematician named Pierre Bezier [3,4]. Pierre Bezier used them to design cars bodies. Since then the use of Bezier curves has expanded into many areas such as computer-aided design (CAD), computer graphics and animation, and the design of typographic fonts [5].

According to [5], the Bernstein polynomial basis has been used as the main function in Bezier model. This Bernstein Polynomial foundation demonstrates the performance of the model which also helps in designing, detecting and drawing certain curves of different arbitrative objects with different sizes depending on a group of control points. Bezier curves of any degree n, has $n+1$ control points whose merging functions are denoted $B_i^n(t)$. This paper was focused on Rational Bezier curve for degree 2 (quadratic) and 3 (cubic). This curve has a variable that called as a weight, w_i that can manipulate the shape of the curves without changing the position of the control points. This property makes it easy to edit, modify and reshape the curve, which is one reason for its popularity [5].

Batik is one of the famous traditional handicraft areas in Malaysia. It is produced through a process of layering wax and dye. According to [17], the word batik means 'wax writing'. It is a way of decorating cloth by covering a part of it with a coat of wax and then dyeing the cloth. The waxed areas keep their original colour and when the wax is removed the contrast between the dyed and non-coloured areas makes the pattern. The exact origin of batik making process is lost to history. Recent research shows that early proof of batik has been found from Far East up through Central Asia, Middle East, and also Africa. The origin of batik may be uncertain, but if we look at the word "Batik" is of Indonesian word which in Indonesia, the process of making batik was developed into one of the great art form [17].

Malaysia Batik is one of the arts of Malaysia, especially on the east coast. There are many popular motifs batik such as flowers, leaves and geometrical designs like spirals [15]. This paper uses quadratic and cubic rational Bezier curve to draw the spiral curve of Batik design.

The ancient ways to produce Batik motifs require a longer time. This will surely consume more time and cost to produce Batik. By applying rational Bezier curve in Batik motifs design process, time taken to finish certain design will decrease thus it can decrease the cost. The objectives of this paper are to apply the rational Bezier curve on a chosen Batik design, to show the effect of weight manipulation on the curve and to compare the best results between cubic and quadratic of rational Bezier curve on Batik design.

2. Literature Study

Computer aided geometric design (CAGD) concerns the mathematical description of shapes that is used in computer graphics, manufacturing, or analysis. It draws upon the fields of geometry, computer graphics, numerical analysis, approximation theory, data structures and computer algebra [5]. Thus, CAGD property is used as an alternative of computational method in generating the design of Spiral Batik.

2.1. Rational Bezier Curve

A rational Bezier curve is one for which each control point P_i is assigned a scalar weight, w_i. The blending functions are rational polynomials, or the ratio of two polynomials. Parabola can be representing by polynomial of quadratic Bezier and circle can be representing exactly by rational Bezier curves [7].A circular arc and an ellipse can be expressed by quadratic rational Bezier curves in CAD system, but not by any polynomial Bezier curves. The quadratic rational Bezier curves are usually used, for example, to design the bodies of aircraft, car bodies, to design fonts and others shape of curves. Many papers have been published on the rational Bezier curves with topics such as curvature continuous interpolation problems, expression as conics, contact order, and high accuracy approximation. [1]. Majority of properties of Bezier curves still hold for rational Bezier curves. In particular, rational Bezier curves have end-point interpolation, prescribed tangent lines at the endpoints, convex hull property for the positive value weights, and variation diminishing property. The property of symmetry was stated the same curve that was obtained by reversing the order of the sequence of control points was still holds also, but it is important to understand that the weights are associated to the projected points [4] in the curves. When all the weights are set to 1, rational Bezier reduces to the non-rational Bezier. The weights serve as additional parameters and provide fine, accurate control of the shape of the curves and the surface that plotted [5]. This property is advantage to generate the curve easy and flexible.

2.2. Batik

Geometrical designs are one of the famous batik in Malaysian. Normally it uses more on flora designs such as leaves and flower designs instead of fauna designs likes animals or human forms. However, in this research we focus on spiral batik design rather than other designs. Batik can be found in many forms, such as a batik shawl, skirt, bag and even a shirt. In fact, majority of Malaysians also wear batik for formal occasions in Malaysia. The Malaysian government has endorsed Malaysian batik as a national dress code, and many Malaysian designers have come up with creative and new batik designs that reflected Malaysia in their latest collections [16]. Besides, batik designs also can be seen on a variety of items, not just silk shirts, tops and skirts and it's also can be used in designing others products such as handbags, motif on the vase, glass, as decoration and others [17]. Most of batik makers are still maintaining the traditional way of making batik using traditional tools. Although batik now has been promoted aggressively both locally and internationally, there are still not so many efforts on developing, or improving the batik making process. By introducing this computational printing batik hopefully, it can help those who have involved in batik industry [15]. Computational printing is the alternative way to go for batik. It opens a lot more possibilities, with a massive range of patterns and colours which are not possible with hand-drawn batik before. With a computer, design possibilities are unlimited, and it can create an endless variant of batik designs in a short period of time [17]. Thus, that makes this field exciting and can attract new generation to preserve this art.

3. Methodology

3.1. Rational Bezier curve

A rational Bezier curve is one in which each control point is assigned a scalar weight. The blending functions are rational polynomials, or the ratio of two polynomials. If all weights are 1 or same, a rational Bezier curve reduces to a polynomial Bezier curve [5]. Rational Bezier curve have few advantages over polynomial Bezier curve since it provides more control over the shape of a curve. Rational Bezier curves are needed to exactly express all conic sections [6, 7].
The equation of the Bezier basis function with degree n is given in (1), [5]:

$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i} \qquad (1)$$

$$\binom{n}{i} = \frac{n!}{i!(n-i)!}, \quad \text{for } i = 0, 1, \ldots, n \quad and \quad 0 \le t \le 1$$

This paper used quadratic and cubic curve to design Spiral Batik. The equation, *P(t)* of rational Bezier curve is given below, [5]:

$$P(t) = \sum_{i=0}^{n} P_i \left(\frac{w_j B_i^n(t)}{\sum_{j=0}^{n} w_j B_i^n(t)} \right), \qquad 0 \le t \le 1 \qquad (2)$$

with P_i as the control points.

3.2. Quadratic Rational Bezier

Quadratic Rational Bezier curve is the curve of degree two and has three basis functions. The basis was derived from equation (1) with $n = 2$ as follows:

$$B_0^2(t) = (1-t)^2 \qquad (3)$$

$$B_1^2(t) = 2t(1-t)^2 \qquad (4)$$

$$B_2^2(t) = t^2 \qquad (5)$$

The graph the Bezier quadratic basis functions is shown in Fig. 1.

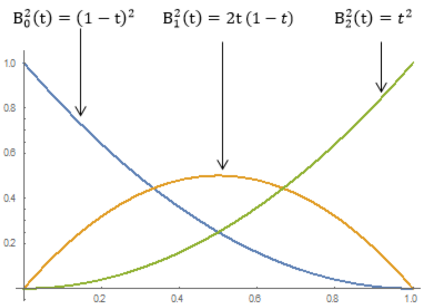
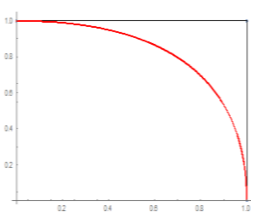
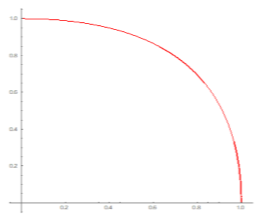


Fig 1: The Quadratic Bezier basis

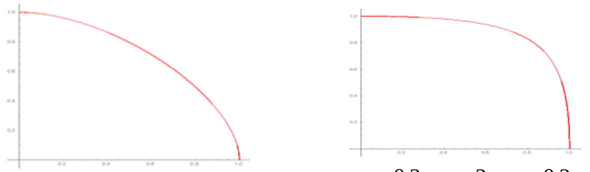
The quadratic rational Bezier curves need 3 control points and they are known as P_0, P_1 and P_2. These 3 points will form a control polygon. For rational Bezier curve, each point will be assigned a weight. This weight is used to modify the curve. Equation (2) will combine with the quadratic basis (3) to (5) to generate the curve. The quadratic Rational Bezier curves with different value of weight are presented as follows. The curves show the different curves can be plotted using different weight on the same control polygon.



Curve with control polygon



$w_0 = 0.7, w_1 = 2, w_2 = 2$



$w_0 = 0.7,\ w_1 = 0.3,\ w_2 = 0.6$ $w_0 = 0.2,\ w_1 = 2,\ w_2 = 0.2$

Fig 2: The Quadratic Bezier Curve

From Fig. 2, it is shown that the curve is pulled to the control point with higher value of *w*.

3.3. Quadratic Rational Bezier

Cubic Rational Bezier curve is the curve of degree three and have four basis functions. The basis also derived from equation (1) with value of $n = 3$. There are the basis functions and the basis graph of cubic:

$$B_0^3(t) = (1-t)^3 \quad (6)$$

$$B_1^3(t) = 3t(1-t)^3 \quad (7)$$

$$B_2^3(t) = 3t^2(1-t) \quad (8)$$

$$B_3^3(t) = t^3 \quad (9)$$

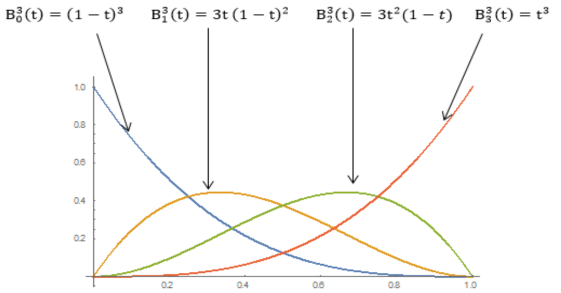
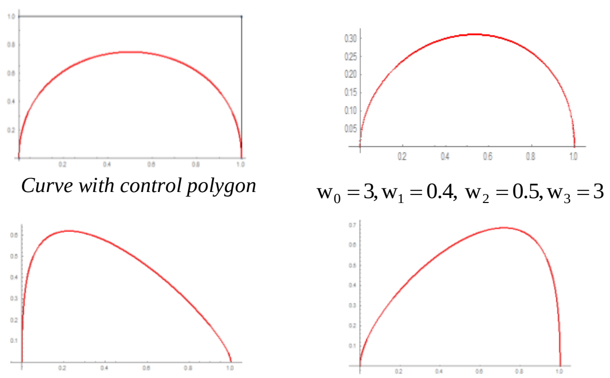


Fig 3: The Cubic Bezier basis

The cubic rational Bezier curve is degree three that has 4 control points that are called as P_0, P_1, P_2 and P_3. These 4 points will form a control polygon. For rational Bezier curve, each point will be assigned a weight. This weight will act as function to modify the curve. For generate cubic curve, the basis (6) to (9) will use with equation (2). The cubic rational Bezier curves with different value of weight are presented as follow. The curve shows the different curves can be plotted using different weight with same control polygon.



Curve with control polygon $w_0 = 3, w_1 = 0.4, w_2 = 0.5, w_3 = 3$

$w_0 = 4, w_1 = 2, w_2 = 0.2, w_3 = 7$ $w_0 = 0.7, w_1 = 0.4, w_2 = 2, w_3 = 3$

Fig 4 : The Cubic Bezier Curve

As discussed in Fig. 2, control point with higher weight values will pull the curve towards it. However, the effect of high weight value cannot be seen at the first and last control points, since these points are the first and last points of curve too. Fig. 2 and 4 show the curves were manipulated based on the value of weight in each curve. This property was applied in generating the spiral batik based on the reference figure and creating others design based on the manipulation of weight.

4. Result and Discussion

4.1. Spiral Curves

Spiral curve is one design that is used in designing Batik pattern. This curve was commercialized with many Batik designs [16]. In this paper, common spiral batik is selected as a reference figure and regenerate using Rational Bezier curve. From the reference figure, others design was created using the manipulation of weight in the curve. This curve helped us to design others shape of spiral easily and less cost and time [15].

Fig. 5 : Reference Figure of Spiral

Fig. 5 was used for determine the control points in order to sketch the control polygon. The control points were defining manually by using graph paper. From this figure, there are 10 curves was defined.

4.2. Quadratic Rational Bezier Spiral

The quadratic curve has three control points for each curve. Based on the figure 5, there have 10 curves defined and the results of spiral shape are as follows.

Table 1: The value of weight for each quadratic curve

Curve	Weights
Curve a	w0 = 2; w1 = 1.3; w2 = 2;
Curve b	w3 = 2; w4 = 0.55; w5 = 2;
Curve c	w6 = 2; w7 = 1.3; w8 = 2;
Curve d	w9 = 2; w10 = 0.9; w11 = 2;
Curve e	w12 = 2; w13 = 0.4; w14 = 2;
Curve f	w15 = 2; w16 = 0.6; w17 = 2;
Curve g	w18 = 2; w19 = 1.1; w20 = 2;
Curve h	w21 = 2; w22 = 0.8; w23 = 2;
Curve i	w24 = 2; w25 = 0.8; w26 = 2;
Curve j	w27 = 2; w28 = 1.02; w29 = 2;

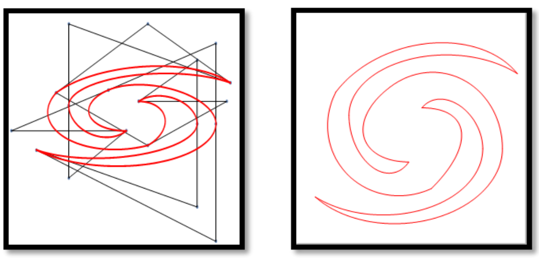


Fig 6 :Regenerating Reference Spiral Quadratic Curve

Fig. 6 is the design of quadratic spiral Bezier curve that regenerated from Fig. 5. This figure is produced by manipulating the value of weight that refer in the table 1.0 and it is closer to the real figure. From Fig 6, by using the same of control polygon and manipulating the value of weights that refer in the table 2, the new design of quadratic spiral can be created as show in Fig 7.

Table 2: The value of weight for new spiral quadratic curve

Curve	Weight			
	Fig 7 (a)	Fig 7 (b)	Fig 7 (c)	Fig 7 (d)

Curve a	w0 = 2;
	w1 = 0.3;
	w2 = 3;
		w0 = 19.3;
		w1 = 7.8;
		w2 = 7.1;
			w0 = 19.3;
			w1 = 7.8;
			w2 = 7.1;
				w0 = 2;
				w1 = 0.7;
				w2 = 2;

Curve b	w3 = 4;
	w4 = 1.1;
	w5 = 5;
		w3 = 4;
		w4 = 1.1;
		w5 = 5;
			w3 = 4;
			w4 = 1.5;
			w5 = 5;
				w3 = 2;
				w4 = 0.7;
				w5 = 2;

Curve c	w6 = 10;
	w7 = 5.2;
	w8 = 9;
		w6 = 10;
		w7 = 5.2;
		w8 = 9;
			w6 = 10;
			w7 = 4.8;
			w8 = 9;
				w6 = 2;
				w7 = 0.7;
				w8 = 2;

Curve d	w9 = 3;
	w10 = 3.4;
	w11 = 22;
		w9 = 20;
		w10 = 6.1;
		w11 = 22;
			w8 = 9;
			w10 = 7.9;
			w11 = 22;
				w9 = 2;
				w10 = 0.7;
				w11 = 2;

Curve e	w12 = 15;
	w13 = 3.3;
	w14 = 17;
		w12 = 15;
		w13 = 3.3;
		w14 = 17;
			w12 = 15;
			w13 = 5.4;
			w14 = 17;
				w12 = 2;
				w13 = 0.7;
				w14 = 2;

Curve f	w15 = 19;
	w16 = 5.5;
	w17 = 7;
		w15 = 19;
		w16 = 5.5;
		w17 = 7;
			w15 = 19;
			w16 = 5.5;
			w17 = 7;
				w15 = 2;
				w16 = 0.7;
				w17 = 2;

Curve g	w18 = 6;
	w19 = 0.8;
	w20 = 21;
		w18 = 6;
		w19 = 5.3;
		w20 = 21;
			w18 = 6;
			w19 = 5.3;
			w20 = 21;
				w18 = 2;
				w19 = 0.7;
				w20 = 2;

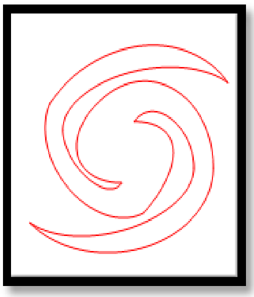
Curve h	w21 = 23;
	w22 = 9;
	w23 = 35;
		w21 = 23;
		w22 = 9;
		w23 = 35;
			w21 = 23;
			w22 = 9;
			w23 = 35;
				w21 = 2;
				w22 = 0.7;
				w23 = 2;

Curve i	w24 = 14;
	w25 = 4;
	w26 = 24;
		w24 = 14;
		w25 = 5.3;
		w26 = 24;
			w24 = 14;
			w25 = 7.3;
			w26 = 24;
				w24 = 2;
				w25 = 0.7;
				w26 = 2;

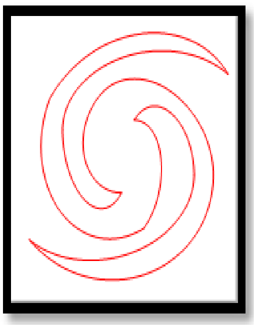
Curve j	w27 = 8;
	w28 = 0.42;
	w29 = 11;
		w27 = 8;
		w28 = 2.9;
		w29 = 11;
			w27 = 8;
			w28 = 2.9;
			w29 = 11;
				w27 = 2;
				w28 = 0.7;
				w29 = 2;

a)

b)



c)



d)

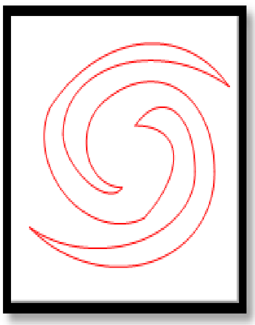


Fig. 7: New Design of Spiral Quadratic Curve

4.3. Cubic Rational Bezier Spiral

The cubic curve has four control points for each curve. Based on the figure 5, there have 10 curves defined and the results of spiral shapes are as follows.

Table 3: The value of weight for each cubic curve

Curve	Weights
Curve a	w0 = 2; w1 = 0.9; w2 = 0.9; w3 = 2;
Curve b	w4 = 2; w5 = 0.9; w6 = 0.9; w7 = 2;
Curve c	w8 = 2; w9 = 0.9; w10 = 0.9; w11 = 2;
Curve d	w12 = 2; w13 = 0.9; w14 = 0.9; w15 = 2;
Curve e	w16 = 2; w17 = 0.9; w18 = 0.9; w19 = 2;
Curve f	w20 = 2; w21 = 0.9; w22 = 0.9; w23 = 2;
Curve g	w24 = 2; w25 = 0.9; w26 = 0.9; w27 = 2;
Curve h	w28 = 2; w29 = 0.9; w30 = 0.9; w31 = 2;
Curve i	w32 = 2; w33 = 0.9; w34 = 0.9; w35 = 2;
Curve j	w36 = 2; w37 = 0.9; w38 = 0.9; w39 = 2;

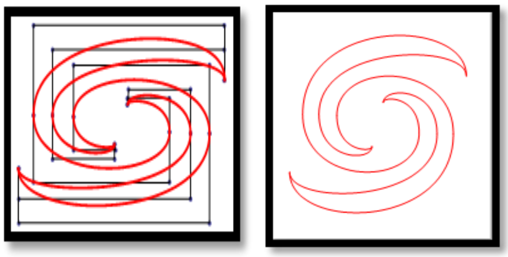


Fig. 8: Regenerating Reference Spiral Cubic Curve

Fig 8 is the design of cubic spiral Bezier curve that regenerated from the reference figure. This figure is produced by manipulating the value of weight that refer in table 3.0 and it is closer to the real figure. From Fig 8, by using the same of control polygon and manipulating the value of weight in the table 4.0, the new design of cubic spiral can be created as shown in Fig 9.

Table 4: The value of weight for new spiral quadratic curve

Curve	Weight			
	Fig 9 (a)	Fig 9 (b)	Fig 9 (c)	Fig 9 (d)

Curve a	w0 = 1;
	w1 = 0.9;
	w2 = 2.1;
	w3 = 23;
		w0 = 1;
		w1 = 0.9;
		w2 = 2.1;
		w3 = 4;
			w0 = 2;
			w1 = 4;
			w2 = 0.4;
			w3 = 2;
				w0 = 2;
				w1 = 4;
				w2 = 0.4;
				w3 = 2;

Curve b	w4 = 5;
	w5 = 4.1;
	w6 = 0.19;
	w7 = 8;
		w4 = 5;
		w5 = 4.1;
		w6 = 0.19;
		w7 = 8;
			w4 = 2;
			w5 = 4;
			w6 = 0.4;
			w7 = 2;
				w4 = 2;
				w5 = 4;
				w6 = 0.4;
				w7 = 2;

Curve c	w8 = 9;
	w9 = 1.11;
	w10 = 10.83;
	w11 = 12;
		w8 = 9;
		w9 = 1.11;
		w10 = 1.1;
		w11 = 12;
			w8 = 2;
			w9 = 4;
			w10 = 0.4;
			w11 = 2;
				w8 = 2;
				w9 = 4;
				w10 = 0.4;
				w11 = 2;

Curve d	w12 = 13;
	w13 = 6;
	w14 = 9;
	w15 = 16;
		w12 = 13;
		w13 = 6;
		w14 = 9;
		w15 = 16;
			w12 = 2;
			w13 = 4;
			w14 = 0.4;
			w15 = 2;
				w12 = 2;
				w13 = 4;
				w14 = 0.4;
				w15 = 2;

Curve e	w16 = 17;
	w17 = 1.8;
	w18 = 0.39;
	w19 = 20;
		w16 = 17;
		w17 = 1.8;
		w18 = 1.9;
		w19 = 20;
			w16 = 2;
			w17 = 4;
			w18 = 0.4;
			w19 = 2;
				w16 = 2;
				w17 = 4;
				w18 = 0.4;
				w19 = 2;

Curve f	w20 = 21;
	w21 = 9.9;
	w22 = 18.3;
	w23 = 24;
		w20 = 21;
		w21 = 9.9;
		w22 = 18.3;
		w23 = 24;
			w20 = 2;
			w21 = 4;
			w22 = 0.4;
			w23 = 2;
				w20 = 2;
				w21 = 5;
				w22 = 5;
				w23 = 2;

Curve g	w24 = 25;
	w25 = 14.1;
	w26 = 3.9;
	w27 = 28;
		w24 = 25;
		w25 = 14.1;
		w26 = 17.2;
		w27 = 28;
			w24 = 2;
			w25 = 4;
			w26 = 0.4;
			w27 = 2;
				w24 = 2;
				w25 = 5;
				w26 = 5;
				w27 = 2;

Curve h	w28 = 29;
	w29 = 3;
	w30 = 3.1;
	w31 = 32;
		w28 = 29;
		w29 = 3;
		w30 = 3.1;
		w31 = 32;
			w28 = 2;
			w29 = 4;
			w30 = 0.4;
			w31 = 2;
				w28 = 2;
				w29 = 5;
				w30 = 5;
				w31 = 2;

Curve i	w32 = 27.02;
	w33 = 28.7;
	w34 = 0.77;
	w35 = 24.16;
		w32 = 27.02;
		w33 = 28.7;
		w34 = 0.77;
		w35 = 24.16;
			w32 = 2;
			w33 = 4;
			w34 = 0.4;
			w35 = 2;
				w32 = 2;
				w33 = 5;
				w34 = 5;
				w35 = 2;

Curve j	w36 = 22;
	w37 = 0.8;
	w38 = 3.9;
	w39 = 21;
		w36 = 22;
		w37 = 0.8;
		w38 = 3.9;
		w39 = 21;
			w36 = 2;
			w37 = 4;
			w38 = 0.4;
			w39 = 2;
				w36 = 2;
				w37 = 5;
				w38 = 5;
				w39 = 2;

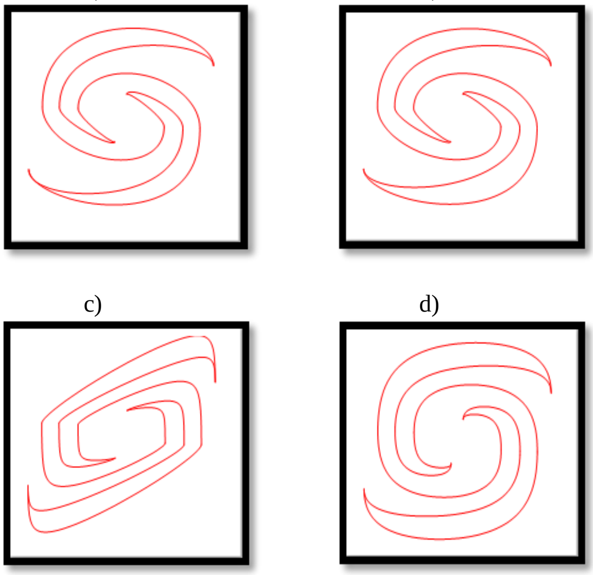


Fig. 9: New Design of Spiral Cubic Curve

From the result in Fig 6 and Fig 8, they show the cubic curve can regenerate closer to the reference figure compared to quadratic curve. The results show the control points and the control polygon also affected the shape of curve. Besides that, the advantage of weight in the curve can be manipulating the shape of the curve and it also helped us to generate the spiral shape easily close to the real figure.

Using the property in this curve and the existence of control polygon, the new designs of spiral was generated. Fig 7 and 9 show the new designs that were produced by manipulating the value of weight. Here, the weight can help the designer to generate the new design easily and can produce a variety of designs. Hence, the design can be used to produce a variety of spiral Batik patterns.

4.4. Example of Spiral Batik Patterns

Fig.10 are the examples of Spiral Batik patterns that can be designed using the results that were generated using Rational Bezier curves. This shows that the properties of the curves can generate new and variety designs by using a reference figure only.

Fig. 10: Example of Spiral Batik Patterns

5. Conclusion and Future Work

From the result, the spiral cubic rational Bezier curve has a better result compared to spiral quadratic rational Bezier curve. It can be said that the higher the degree, the better the curve is generated. Quadratic have 3 control points and cubic have 4 control points. This make cubic curve easier to be manipulated since it has more control on the curve shape.

The rational Bezier curve is discussed from the Bernstein polynomial. The basis functions for each degree are obtained. Control points are plotted and curves for each segment formed and the designs are then generated. There are a few methods to manipulate the curve such as by manipulating the control points and increasing the degree. For this paper, we focus on manipulation of weight magnitude. Each control point has its own weights. The higher the magnitude of the weight, the closer the curve will move to the control point. The weights are manipulated in many ways and many designs are formed [5]. It can be concluded that higher degree will produce a better curve as cubic curve that will provide a better result compared to quadratic curve. The shape also can be manipulated by changing the magnitude of weight.

By using the property in this curve, the spiral curve can be regenerated and the new and variety designs also can be created. This method is an alternative method to digital batik design and it will help the designer to generate new and variety designs easily and low cost of time [15]. The results of the spiral Batik can produce a variety pattern of Batik motif. This property also can be applied in many designs of Batik motifs such as flower, butterfly and others. This method will improvise the digital Batik design industry.

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