

# Numerical Solutions for Fed-Batch Fermentation Process by Using Leapfrog Method

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## Abstract

This article is in detail the dynamical models of two non-linear systems of fed-batch and group aging procedures about consequences of the investigation. With a specific end goal to comprehend the state conditions for the non-linear system and Leapfrog Method for the combination of the non-linear differential equations were produced. The stated algorithm has extended to incorporate with the different non-linearity problems.

**Keywords:** Non-Linear System, Feb-Batch Fermentation Process, Differential Equations, Non-Linear Differential Equations.

## 1. Introduction

The regularly numerous critical factors and parameters must be computed or assessed as a result of their unmeasurable, for example, cell mass and substrate fixations in a maturation procedure. Rather than the elements of the frameworks, the reproduction method can be connected to the frameworks for ideal control since loads of non-straight frameworks are by and large extremely perplexing and not totally known, similar to an aging procedure from [1, 2, 3, 4, 5, 6, 7, 8].

## 2. Structure for Non-Linear System

A maturation procedure is an established confused, non-linear, time-varying system. Its elements are not non-quantifiable, for example [8]. Consider

$$\text{Bio-Mass: } D[B(t)] = B(t)[\sigma - Dilu(t)]$$

$$\text{Substrate: } D[SE(t)] = Dilu(t)[SE_{gi} - SE(t)] - P_1\sigma B(t)$$

$$\text{Fermented Volume: } D(V) = G_{in} - G_{out}, \text{ where } G_{in} = D(V) \text{ and } G_{out} = \text{Zero}.$$

Oxygen Absorption:

$$D(O) = OX_{UR} + OX_{TR}$$

$$OX_{UR} = -D[B(t)] \frac{1}{Y_{X:O}}$$

$$OX_{TR} = P_{1a} [B_{OX_2} OX_1 - OX]$$

Absorption of Carbon-dioxide:

$$D(CE) = CP + CE$$

$$C_{PE} = P_{CO_2} \sigma B(t)$$

$$C_{TR} = P_d P_2 [B_{CO_2} CE_1 - CE]$$

Absorption of Ethanol:

$$D(EL) = PLE - OBE$$

$$PLE = \left( \frac{\lambda_{max}}{\frac{EL}{P_{ST}} + 1} \right) \left( \frac{SE}{SE + P_s} \right)$$

$$OBE = \left( \frac{P_{PO} OX}{OX + KOX_2} \right) \left( \frac{SE}{P_s + E} \right)$$

The stated notations as described as  $OX$  – oxygen,  $SE$  – Substrate,  $CE$  – Carbon dioxide,  $B$  – Biomass and  $EL$  – Ethanol. Here  $B$  and  $SE$  is none valuated and  $OX, CO$  and  $EL$  is possibly to measured.  $OX_{UR}$  be the uptake for oxygen rate,  $OX_{TR}$  be the transfer for oxygen rate,  $C_{PE}$  be the transfer for corbondioxide rate,  $PLE$  be the concentration for ethanol rate and  $Dilu$  is rate for the dilution.  $S_{gi}$  be the influent substrate.  $\lambda$  is the growth with  $\lambda_{max}$ .  $P_1$  be the yields coefficients,  $OX_1$  is the oxygen dissolved and  $CE_1$  be the concentrations of the corbondioxide.

## 3 Growth Rate for $\sigma$

The particular development rate  $\sigma$  is a key time-fluctuating parameter for description of biomass development, substrate utilization and items arrangement. For a process, the most regularly utilized models for  $\sigma$  are exhibited as takes after: [5, 6,8]

### 3.1 Contois Method:

The biomass development is frequently attempted to back off at high biomass focuses. A conceivable model for this situation is the accompanying structure:

$$\sigma(SE, OX) = \left[ \frac{SE}{[P_{ce}B + SE]} \right] \sigma_{Max}$$

### 3.2 Ollsons Method:

High-impact aging are forms where the miniaturized scale living being need oxygen to create legitimately. In such case, broke up oxygen (OX) in the way of life medium can be considered an extra substrate.

$$\sigma(SE, OX) = \left[ \frac{OX}{[P_{O_2} + OX]} \right] \left[ \frac{SE}{[P_s + SE(t)]} \right] \sigma_{Max}$$

### 3.3 Monds Method:

A useful connection between the particular development  $\sigma$  and a fundamental compound focus was proposed by Monod technique.  $\sigma(SE) = \sigma_{Max} \frac{SE(t)}{[P_s + SE(t)]}$

### 3.4 Computer Simulation:

The principle of all numerical integration methods is to estimate the system states at time  $(\tau + \Delta\tau)$  given the state at time  $\tau$  where  $\Delta\tau$  is the sampling period [2]. For a general equation of the form  $g(\eta, \rho, \tau) = \dot{\eta}$

The standard of all numerical reconciliation strategies is to appraise the framework states at time  $(\tau + \Delta\tau)$  given the state at time  $\tau$  where  $\Delta\tau$  the examining time period [2] is. For a general condition of the system

$$g(\eta, \rho, \tau) = \dot{\eta}$$

At each progression calculation are finished by some equation typically in view of the Taylor's arrangement,

$$\eta(\tau) + \Delta\tau\dot{\eta}(\tau) + \frac{(\Delta\tau)^2}{2!}\ddot{\eta}(\tau) + \dots = \eta(\tau + h)$$

On the off chance that  $\Delta\tau$  is been adequately little and higher order subordinates of  $\eta$  and powers of  $\eta$  are taken, at that point  $\eta(\tau + \Delta\tau)$  the esteem can be precisely found. An established first order single-term Haar wavelet arrangement strategy and Leapfrog technique have been connected to the models for joining and are contrasted with the test information.

## 4. Conclusion

The results of discrete solutions by utilizing the Leapfrog strategy give more exact qualities when contrasted with the single-term Haar wavelet arrangement technique. From the Tables 1-8, we see that the arrangements acquired by the Leapfrog technique coordinate well with the exploratory information of the fed- batch fermentation process, however the single-term Haar wavelet arrangement method yields a mistake. It is to be noticed that from Tables 1-8, we can see that the Leapfrog technique yields less mistake when contrasted with single-term Haar wavelet arrangement strategy in Contois Method, Ollsons Method Monds method. Thus, the Leapfrog strategy is more appropriate for concentrate the reenactment of the fed-batch fermentation process.

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**Table 1:** Solution Values for Monds Method

Time $\tau$	Bio-Mass	Substrate	Fermentation Volume	Oxygen	Carbon-dioxide	Ethanol	Solution by Monds Method
0	0	0	0	0	0	0	0
0.5	1.6179	0.6395	1.6567	0.0496	0.0249	0.0154	0.0132
1	2.6504	0.9814	2.7446	0.0536	0.0970	0.0374	0.0255
1.5	4.3963	1.4563	4.5470	-0.0929	0.2672	0.0517	0.0257
2	7.3839	2.0895	7.5329	-0.6511	0.6312	0.0141	0.0140

**Table 2:** Solution Values for single-term Haar wavelet series method and Monds Method

Time $\tau$	Bio-Mass	Substrate	Fermentation Volume	Oxygen	Carbon-dioxide	Ethanol	Solution by Monds Method
0	0	0	0	0	0	0	0
0.5	1.6181	0.6395	1.6570	0.0496	0.0179	0.0086	0.0093
1	2.6512	0.9813	2.7456	0.0536	0.0862	0.0228	0.0190

1.5	4.3984	1.4561	4.5494	-0.0934	0.2756	0.0391	0.0239
2	7.3891	2.0887	7.5383	-0.6530	0.7302	0.0413	0.0108

**Table 3:** Solution Values for Leapfrog method and Monds Method

Time $\tau$	Bio-Mass	Substrate	Fermentation Volume	Oxygen	Carbon-dioxide	Ethanol	Solution by Monds Method
0	0	0	0	0	0	0	0
0.5	1.6179	0.6395	1.6567	0.0496	0.0249	0.0154	0.0132
1	2.6504	0.9814	2.7446	0.0536	0.0970	0.0374	0.0255
1.5	4.3963	1.4563	4.5470	-0.0929	0.2672	0.0517	0.0257
2	7.3839	2.0895	7.5329	-0.6511	0.6312	0.0141	0.0140

**Table 4:** Values for Experimental Data Ollsons Model

Time $\tau$	Bio-Mass	Substrate	Fermentation Volume	Oxygen	Carbon-dioxide	Ethanol	Solution by Ollsons Method
0	0	0	0	0	0	0	0
0.5	1.6179	0.6395	1.6567	0.0495	0.0249	0.0156	0.0084
1	2.6504	0.9814	2.7446	0.0536	0.0970	0.0374	0.0179
1.5	4.3963	1.4563	4.5470	-0.0929	0.2673	0.0517	0.0232
2	7.3839	2.0895	7.5329	-0.6511	0.6312	0.0141	0.0089

**Table 5:** Solution Values for Single term-Haar wavelet series and Ollson Method

Time $\tau$	Bio-Mass	Substrate	Fermentation Volume	Oxygen	Carbon-dioxide	Ethanol	Solution by Ollson's Method
0	0	0	0	0	0	0	0
0.5	1.6181	0.6395	1.6570	0.0496	0.0179	0.0086	0.0083
1	2.6512	0.9813	2.7456	0.0536	0.0862	0.0228	0.0174
1.5	4.3984	1.4561	4.5494	-0.0934	0.2746	0.0392	0.0222
2	7.3891	2.0887	7.5383	-0.6530	0.7302	0.0413	0.0093

**Table 6:** Solution Values for Single Leapfrog Method and Ollson Method

Time $\tau$	Bio-Mass	Substrate	Fermentation Volume	Oxygen	Carbon-dioxide	Ethanol	Solution by Ollson Method
0	0	0	0	0	0	0	0
0.5	1.6179	0.6395	1.6567	0.0495	0.0249	0.0156	0.0084
1	2.6504	0.9814	2.7446	0.0536	0.0970	0.0374	0.0179
1.5	4.3963	1.4563	4.5470	-0.0929	0.2673	0.0517	0.0232
2	7.3839	2.0895	7.5329	-0.6511	0.6312	0.0141	0.0089

**Table 7:** Values for Experimental Data Contois Model

Time $\tau$	Bio-Mass	Substrate	Fermentation Volume	Oxygen	Carbon-dioxide	Ethanol	Solution by Contois Method
0	0	0	0	0	0	0	0
0.5	1.6178	0.6395	1.6567	0.0496	0.0249	0.0156	0.0098
1	2.6503	0.9813	2.7446	0.0536	0.0970	0.0374	0.0199
1.5	4.3963	1.4563	4.5470	-0.0929	0.2672	0.0517	0.0228
2	7.3839	2.0895	7.5328	-0.6511	0.6312	0.0141	0.0036

**Table 8:** Solution Values for Single-term Haar wavelet Method and Contois Method

Time $\tau$	Bio-Mass	Substrate	Fermentation Volume	Oxygen	Carbon-dioxide	Ethanol	Solution by Contois Method
0	0	0	0	0	0	0	0
0.5	1.4750	0.5980	1.5050	0.0533	0.0000	0.0013	0.0067
1	2.1940	0.8703	2.2650	0.0927	0.0212	0.0042	0.0134
1.5	3.2911	1.2413	3.4089	0.0841	0.0945	0.0106	0.0200
2	4.9778	1.7412	5.1303	-0.0545	0.2818	0.0234	0.0265

**Table 9:** Solution Values for Leapfrog method and Contois Method

Time $\tau$	Bio-Mass	Substrate	Fermentation Volume	Oxygen	Carbon-dioxide	Ethanol	Solution by Contois Method
0	0	0	0	0	0	0	0
0.5	1.6178	0.6395	1.6567	0.0496	0.0249	0.0156	0.0098
1	2.6503	0.9813	2.7446	0.0536	0.0970	0.0374	0.0199
1.5	4.3963	1.4563	4.5470	-0.0929	0.2672	0.0517	0.0228
2	7.3839	2.0895	7.5328	-0.6511	0.6312	0.0141	0.0036