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Research paper

An Analytical Model for Dispersive Channel in Radio-over-Fiber Communication Systems Using Electro-Absorption Modulator with Chirp

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Abstract

Chirp effects induced by the electro-absorption modulator (EAM) and chromatic dispersion from the optical fiber in the radio-over-fiber (RoF) system are critical parameters of performance for the communication system design. In this paper, the fundamental and the inter-modulation component powers are expressed by closed-form equations with respect to the dispersion and chirp. And the optical double side-band (ODSB) signals are also investigated. Therefore the power fluctuation of the fundamental and the third harmonic components are presented with expressions for two special cases, ODSB and the optical single side-band signals.

Keywords: *chirp, electro-absorption modulator, nonlinear distortion, optical communication, optical fiber dispersion, optical link, and radio-over-fiber.*

1. Introduction

The high data rates are required by communication systems at any time and in any place cheaply in order to subscribers' demand such as full sound music, photograph data, and multimedia stream services [1]. For satisfying the needs, radio-over-fiber (RoF) systems have been a good solution, which has a broadband frequency range from several GHz to tens of GHz and is widely applied for various communication networks such as wireless local area networks, hybrid fiber coaxial systems and antenna remoting [2].

Normally the optically modulated RF signal of optical communication links is generated by electro-absorption modulator (EAM) or Mach-Zehnder electro-optical modulator (MZM) [3]. The EAM has been frequently used for optical communication systems including RoF systems since it has a compact shape, simple control circuit, large radio frequency (RF) bandwidth and potential for the ability of integration with other electro-optic elements like a semiconductor optical amplifier (SOA) or a laser diode (LD) [11].

The two nonlinearity phenomena, dispersion and chirp, are dominant factors to RoF links using EAM. The influence of the EAM chirp on optical communication systems has been measured and explained the reason of chirp and the range of chirp from EAM [3-6]. Most of the previous work studying the chirp and dispersion of RoF systems using EAM are limited to how to measure and what is the reason for chirp.

In this paper, we propose an analytical model to study the fiber chromatic dispersion and a chirping effect from EAM incorporating ODSB signals. These results can be applied to almost all types of operation of EAM. The calculated results are presented the power fading of the fundamental and third harmonic components.

2. Analytical Model

2.1. General Model

Figure 1 shows a general RoF link diagram with EAM. $E_{in}(t)$ is the optical carrier source from LD with angular frequencies ω_{LD}. V(t) is the electrical signal which composed with two RF signals, $v_{rf}(t)$, with angular frequencies ω_1 and ω_2 including reverse voltage, V_{dc}. The optical carrier $E_{in}(t)$ is modulated by two-tone $v_{rf}(t)$ in EAM.
The General Model presents the laser diode (LD) as the optical carrier with angular frequencies, linked to Electro-Absorption Module (EAM) to the Optical Filter as the optical signal sent to the Photodiode (PD). The optical signal and electrical signal could be defined as

$$E_{in}(t) = \sqrt{P_{in}}\exp(j\omega_{LD}t) \tag{1a}$$

$$v_{rf}(t) = \sqrt{2Z_{in}P_e}\,(\sin(\omega_1 t) + \sin(\omega_2 t)) \tag{1b}$$

$$V(t) = V_{dc} + v_{rf}(t) \tag{1c}$$

$$\therefore V(t) = V_{dc} + \sqrt{2Z_{in}P_e}\,(\sin(\omega_1 t) + \sin(\omega_2 t)) \tag{1d}$$

where P_{in} means the power of the input optical carrier, Z_{in} is the input RF electrode impedance, $v_{rf}(t)$ is the input two-tone signals, P_e is defined as the power of each tone, and V_{dc} is the reverse DC bias of the EAM. The EAM is modulated by two-tone signals.; they generate inter-modulation (IM) components which are primary problem of system performance since the IM products are within the pass-band.

Note that the defined equations (1) are acceptable only $P_e \ll P_{in}$. It is notable that the EAM chirp is difficult to handle for any communication system.

Though the EAM has nonlinear characteristics, its transmission function could be expressed by a simple exponential term for RoF system analysis [7-8].

$$E_{out}(t) = P_{in}^{\frac{1}{2}}\exp(-\frac{1}{2}\gamma\alpha_{loss}(V(t))L)\exp(j\omega_{LD}t) \tag{2a}$$

$$E_{out}(t) \approx P_{in}^{\frac{1}{2}}\exp(-\frac{1}{2}\frac{V_{dc} + \sqrt{2Z_{in}P_e}\sin(\omega_e t)}{V_0})\exp(j\omega_{LD}t) \tag{2b}$$

where γ means the confinement value, $\alpha_{loss}(V(t))$ is the variation of absorption coefficient induced by the modulated voltage V(t), and L is the electrode length of the EAM. Assume $\alpha_{loss}(V(t)) \approx \alpha_0 V(t)$ for small signal efficiency; α_0 is the 1st order coefficient of $\alpha_{loss}(V(t))$ expansion. The unit of the 1st order coefficient, α_0, is cm^{-1} and the electrode length, L, is cm. Therefore, the three factors could be simplified as a constant V_0. The model constant could be defined by

$$V_0 \square (\alpha_0\gamma L)^{-1} \tag{3}$$

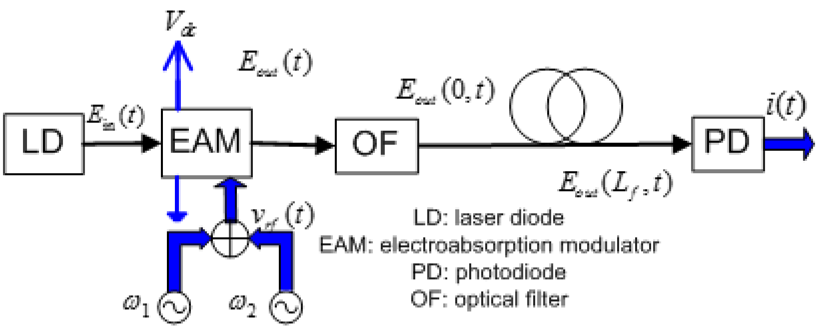


Fig. 1: The block diagram of radio-over-fiber link using EAM

The V_0 can lead the EAM transmission function comparing with equations (2) and (3) using a nonlinear curve fitting. EAM transmission function, which is defined by the optical transfer function for a EAM, is shown in Figure 2. The effective well width of a n EAM makes the transmission function shape and the shape makes the optical confinement factor and the modulation length of EAM [7, 8]. The two factors are modeled as a model constant, V_0. The relation is (3). To get the model constant, V_0, EAM transmission function is needed like Figure 2 and fitting is necessary to get the model constant, V_0. Most EAM characteristic function can be expressed as exponential form. Therefore the model constant can be used by one of link model parameters.

In Figure 2, the EAM transmission function is modeled as an exponential function as follows:

$$T = A_1 \exp(\frac{x}{t_1}) + y_0 \tag{4}$$

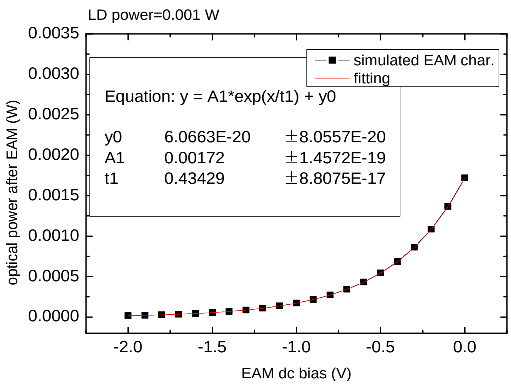
where A_1 is modeled as an input optical power, P_{in}, of EAM and y_0 is a DC term. Therefore t_1 is V_0 in (2b) and x is input electrical power, V(t), in (2b). In Figure (2), the experimental characteristic function of EAM has the model constant 1.63 and the simulation characteristic function of EAM has 0.43.

For the purpose of the harmonic analysis, (2a) should be expanded by Bessel series. An exponential function could be separated by n-th harmonic expressions using Bessel function definition. For easy calculation, the chirp factor assumed constant. Therefore the chirped wave equation is employed as follows:

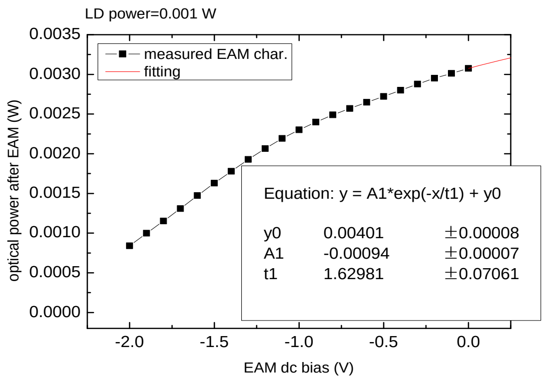
In order to analyze an optical fiber which is dispersive medium, a wave equation could be assumed as follows [9]:

$$E_{out}(L_f, t) = \exp(L_f \overline{D}) E_{out}(0, t)$$

$$\overline{D}(j\omega) = \frac{j}{2}\beta_2\omega^2 - \frac{\alpha_f}{2} \tag{5}$$



(a) simulated EAM function



(b) measured EAM function

Fig. 2: EAM transmission function

where L_f is fiber length, $\overline{D}$ is defined as the dispersion, β_2 means the second order dispersion parameter and αf is attenuation per a unit length of fiber (km). A photocurrent at photodiode (PD) could be calculate by being employed the square law model. The photocurrent i(t) could be derived by Eq. (6) as

2.2. ODSB signal

$$
\begin{aligned}
E_{out}(0,t) &= (P_{in})^{\frac{1}{2}} e^{j\omega_{LD}t} \\
&\times\Big(\sum_{n=-\infty}^{\infty} (\sqrt{1+C^2})^n e^{-\frac{V_{dc}}{2V_0}} J_n\Big(j\frac{\sqrt{2Z_{in}P_e}}{2V_0}\Big) e^{jn\omega_1 t} e^{-jn\arctan(C)}\Big) \\
&\times\Big(\sum_{m=-\infty}^{\infty} (\sqrt{1+C^2})^m e^{-\frac{V_{dc}}{2V_0}} J_m\Big(j\frac{\sqrt{2Z_{in}P_e}}{2V_0}\Big) e^{jm\omega_2 t} e^{-jm\arctan(C)}\Big)
\end{aligned} \tag{6}
$$

where C denotes the chirp factor of EAM.

$$
\begin{aligned}
i(t) &= R\left|E_{out}(L_f,t)\right|^2 + n(t) \\
&= i_{\omega_1}(t) + i_{\omega_2}(t) + i_{2\omega_2-\omega_1}(t) + i_{2\omega_1-\omega_2}(t) + i_o(t) + n(t)
\end{aligned} \tag{7}
$$

where R is PD responsivity, $i_{\omega 1}$(t) and $i_{\omega 2}$(t) means the two-tone RF components of the photocurrent, $i_{2\omega 1-\omega 2}$(t) and $i_{2\omega 2-\omega 1}$(t) denote the 3rd inter-modulation (IM3), i_o(t) defines other harmonic terms including the 2nd inter-modulation (IM2) components, and n(t) is noise. IM2 components could be easily eliminated by using a band-pass filter, since the components RF frequency are far from the pass-band. Hence, we could focus on the two-tone frequency and its IM3 terms.

ODSB modulation is one of the most popular formats in RoF links. ODSB signals composed of the upper (left) and lower (right) side optical bands. For ODSB signals, we substitute (7) with (5), (6) and expand the terms. After that, the mainly remained ω_1 terms are selected. Then the small signal power of a tone frequency component, $P_{\omega 1,ODSB}$, could be calculated and reduced by several terms as

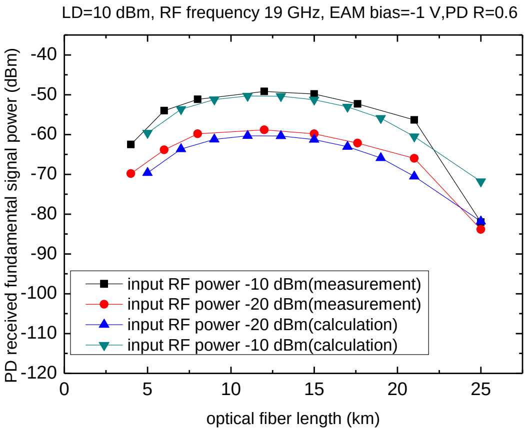
$$
\begin{aligned}
P_{\omega_1,ODSB} \approx 2P_{in}{}^2\, exp\Big(\frac{2V_{dc}}{V_0}\Big)10^{\frac{-\alpha_f L_f}{5}} \{2R\sqrt{1+C^2} \\
\mathrm{Im}[&\left|J_{-2}\right|^2 J_{-2}J_{-1}{}^* + J_{-2}\left|J_{-1}\right|^2 J_{-1}{}^* + J_{-2}\left|J_0\right|^2 J_{-1}{}^* \\
&+ J_{-2}\left|J_1\right|^2 J_{-1}{}^* + J_{-2}\left|J_2\right|^2 J_{-1}{}^* + J_{-1}\left|J_{-2}\right|^2 J_0{}^* \\
&+ \left|J_{-1}\right|^2 J_{-1}J_0{}^* + J_{-1}\left|J_0\right|^2 J_0{}^* + J_{-1}\left|J_1\right|^2 J_0{}^* \\
&+ J_{-1}\left|J_2\right|^2 J_0{}^*] \times \cos\Big(\frac{\beta_2}{2}\omega_1^2 L_f + \arctan(C)\Big)\}^2
\end{aligned} \tag{8}
$$

Be aware that the another tone component, $P_{\omega 2,ODSB}$, is identical result as $P_{\omega 1,ODSB}$. As the same process, the small signal power of the IM3, $P_{2\omega 1-\omega 2,ODSB}$, could be calculated and reduced by several terms as

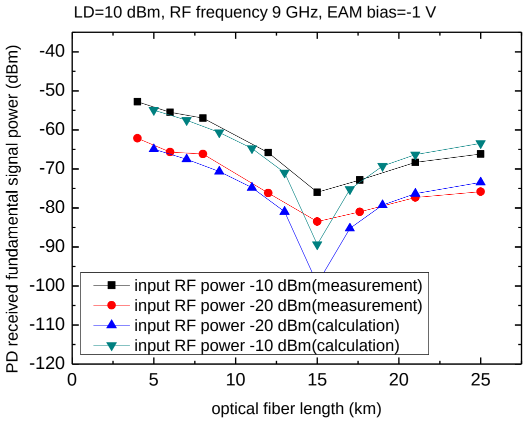
$$
\begin{aligned}
P_{2\omega_1-\omega_2,ODSB} \approx 2P_{in}{}^2\, exp\Big(\frac{2V_{dc}}{V_0}\Big)10^{\frac{-\alpha_f L_f}{5}} \{2R(\sqrt{1+C^2})^3 \\
\mathrm{Im}[&J_0\left|J_{-2}\right|^2 J_{-1}{}^* + \left|J_0\right|^2 J_{-1}J_{-2}{}^* + J_0{}^2 J_{-2}{}^* J_1{}^* \\
&+ J_0 J_1 J_{-2}{}^* J_2{}^* + J_1 J_{-2}(J_{-1}^2)^* + J_{-1}\left|J_{-1}\right|^2 J_0{}^*] \\
\times \cos\Big(\frac{\beta_2}{2}\omega_1^2 L_f &+ \arctan(C)\Big)^2 \cos\Big(\frac{\beta_2}{2}\omega_2^2 L_f - \arctan(C)\Big)\}^2
\end{aligned} \tag{9}
$$

where the variable of each Bessel function is constant, $j\sqrt{2Z_{in}P_e}/2V_0$.

Since the absolute value of J_3 is about 20 dB smaller than J_2, the higher order terms than J_2 are removed in (8) and (9). All parameters in (8) and (9) could be get using datasheets of devices (EAM, LD, the optical fiber and PD) and set by a system designer such as the input optical power, EAM reverse DC bias, the optical fiber length, PD responsivity, and the 2nd order dispersion parameter. The chirp parameter and model constant could be obtained by measuring. (8) and (9) represents that the RoF communication system affects the nonlinearity of EAM chirp and optical fiber dispersion.



(a) RF frequency 19 GHz



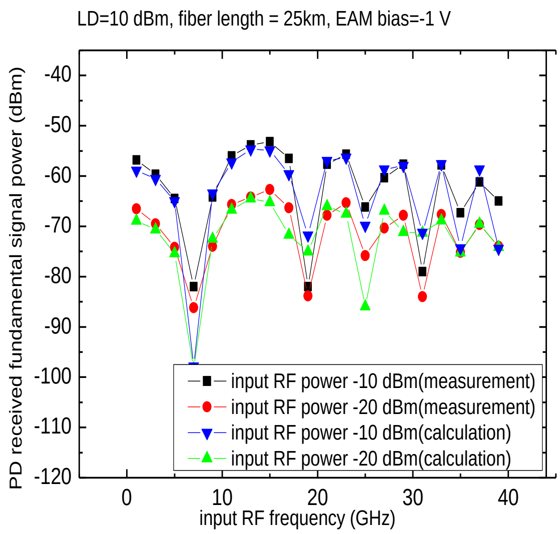
(b) RF frequency 9 GHz

Fig. 3: Received RF power of the fundamental, with different input RF power at chirp 1.732 as a function of the fiber span.

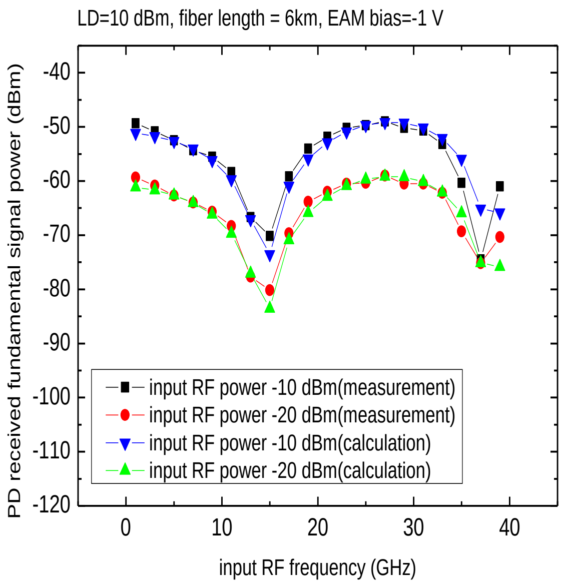
In Figure 3 the experimental and model calculation results are plotted following optical fiber length and RF frequency. In Figure 3 (b), the experimental limitation of mismatch by RF source phase noise and LD source linewidth and so on makes the difference of the valley depth. But the other data is well matched about 2 dB difference. In this experiment, laser diode wavelength is 1550 nm, EAM reverse bias is -1 V, photodetector responsivity is 0.6, chirp is 1.731 and the second order fiber dispersion parameter is 20.473 ps^2/km. The simulation parameters are appeared in Table 1. And experimental parameters are same as Table 1 except chirp factor, bias point and the model constant.

Table 1: The simulation and experiment parameters in a RoF system with EAM

Parameter	Value
Frequency of LD	193.1 X 10^{12} Hz (1550 nm)
Power of LD	10 dBm
EAM bias	-1 V
Input RF power	-20 and -10 dBm
Chirp factor	1.732
PD responsivity	0.6
Fiber dispersion parameter	20.473 ps^2/km



(a) fiber length 25 km



(b) fiber length 6 km

Fig. 4: Received RF power of the fundamental with chirp 1.732 as a function of the RF frequency.

And the dispersion and RF frequency are compared. In Figure 4, the measured results are compared with calculated results as the RF frequency. The optical fiber chirp is 1.732 and fiber lengths are 25 and 6 km. The valley is well matched the calculated and measured cases. But the PD's responsivity is linearly lower from 0.98 to 0.46 in the range of RF frequency from 0 to 40 GHz. Therefore, the PD responsivity is altered in Figure 4. The experimental parameters are also same as Table 1 except chirp factor, bias point and the model constant.

~~Actually~~ General experiment is hard to operate for more specific conditions such as appointed chirp value with predetermined bias point, and EAMs are practically difficult to make a same transmission function and chirp characteristic. Therefore, simulation is reasonable for confirmation of the model.

Table 2: The simulator and model parameters in a RoF system with EAM

Parameter	Value
Frequency of LD	193.1 X 10^{12} Hz (1550 nm)
Power of LD	10 dBm
EAM bias	-1.5 V
Fiber length	0 ~ 62 km

Input RF power	-7 dBm
Chirp factor	0 ~ 2
PD responsivity	1 A/W
Fiber dispersion parameter	20.473 ps^2/km
Optical fiber loss	0.2 dB/km

For the purpose of simulation, the power of LD is 10 dBm and the wavelength sets 1550 nm. The power for RF input is -7 dBm. Chirp factors have been set up from 0 to 2 since ~~actually~~ general modulator has between 0 to 2 chirp factors [3]. Optical fiber loss, 0.2 dB/km and dispersion parameters, 20.473 ps/km·nm, are commercially normal values by [10]. The reverse bias of EAM is 1.5 V and PD responsivity R is 1 A/W. The simulation parameters are represented in Table 2.

From (8) and (9) at ODSB, the simulation and calculation results of the two-tone and 3rd inter-modulation (IM3) signals are shown in Figure 5.

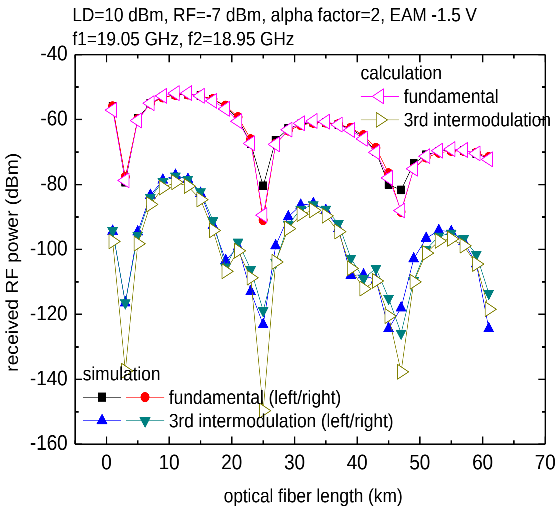
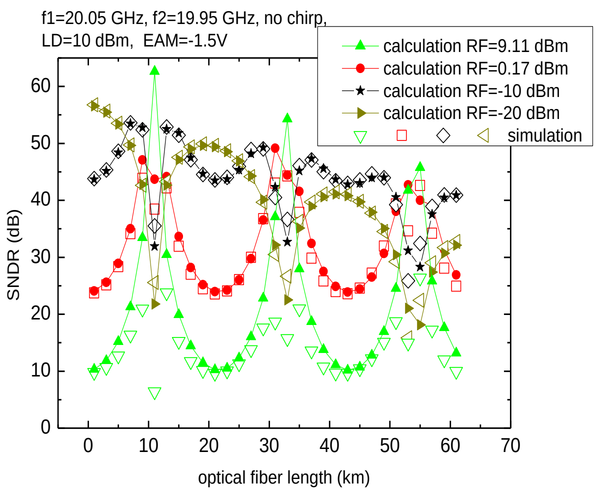


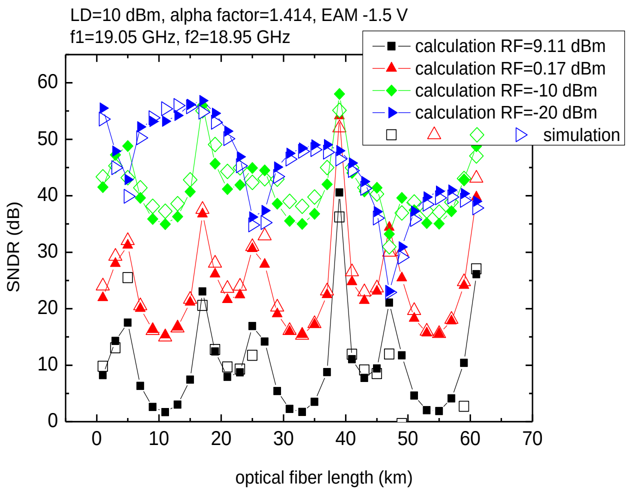
Fig. 5: A simulation result of the two-tone and 3rd inter-modulation (IM3) power as a function the optical fiber length at ODSB signal.

The power of two-tone and IM3 signals increase and decrease as the fiber length. Effects of the chirp and dispersion are also representing as phase change at the tone and IM3 signals in Figure 5. Results of calculation and simulation are well matched. As these results shown, the simple expressions in from (8) and (9) make the power fading prediction quite easy and enable us to optimize setting of RoF systems considering the chirp and dispersion.

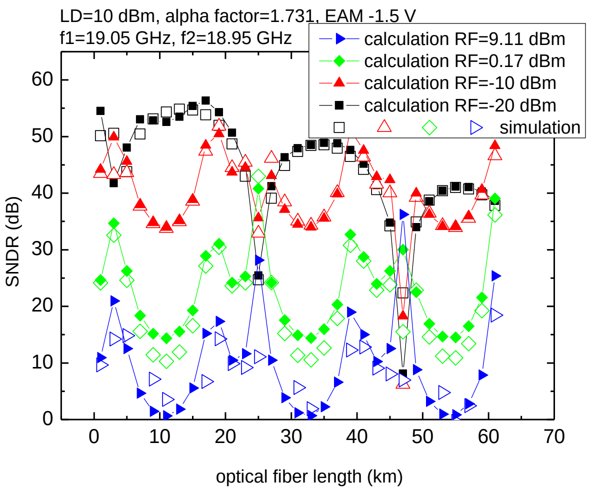
Signal-to-noise distortion ratio (SNDR) is drawn in related with the optical fiber length to optimize the communication system performance using closed form expressions in Figure 6.



(a) Chirp 0



(b) Chirp 1.414



(c) Chirp 1.731

Fig. 6: The signal-to-noise distortion ratio (SNDR) as a function of the optical fiber span at ODSB signals. The empty figures denote simulation points.

The thermal noise and IM3 are considerate to calculate the SNDR. Since the two-tone gap is 100 MHz, the frequency bandwidth is assumed as adding the IM3 and noise power. In Figure 6, the optimum SNDR could be between the input RF power -20 and -10 dBm. If the input RF power is larger than 0 dBm, the SNDR is smaller. Since the IM3 power is less than the thermal noise (-124 dBm) at the case of that the input RF power is -20 dBm, the SNDR shape is similar to the fundamental power shape. The SNDR shape of the input RF power -10 dBm is squeezed because the IM3 power fluctuating around the thermal noise. Otherwise the IM3 power is larger than the thermal noise at the input RF power 0 and 9 dBm.

3. Conclusion

The proposed analytical model for RoF links with EAM is presented to study the optical fiber dispersion and EAM chirp. This model could be applied to several operating conditions of EAM, such as input LD power, EAM bias point, input RF frequency and input RF power. This model results in closed-form expressions which can be applied to the operating conditions of EAM. This model is also presented without iterative numerical simulation and iterative matrix calculation. The case of ODSB signals is demonstrated. The SNDR of RoF links using EAM could be also calculated and it is helpful to design new RoF links.

These expressions are used to calculate the power fluctuation of the two-tone and third harmonic components.

4. Discussion

The proposed model analysed the case of ODSB signals which have the fiber dispersion. Normally the optical single sideband (OSSB) signal is using for reducing the fluctuation from the optical fiber nonlinearity, dispersion. Therefore, the chirp effect should be analysed when the OSSB signal is used. Furthermore, the optimal or near-optimal parameters should also have analysed such as the EAM bias point (chirp factor), input RF power, SNDR, and so on.

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