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Research paper

Analytical and computer research of stability of three layer shells, supported by stiffness ribs

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Abstract

Development of calculation model and algorithm of research of stability of three-layer cylindrical shell with light-weight aggregate, sup-port-ed by regular transverse stiffness ribs is considered in the paper. In variational way, using the functional-action by Ostrograskiy-Hamilton, there were obtained differential equation of stability of shell's part, enclosed between ribs, and conditions along ribs lines and edges of three-layer sloping shell, supported by longitudinal stiffness ribs at simple support of edges. For external bearing layers of shell, the hy-potheses of Kirchhoff-Love were accepted. For aggregate was accepted the principle of linear change of tangential displacements along thickness. Transverse deformations were not considered. Bernoulli hypotheses were accepted for ribs. There was considered only the bent of ribs in vertical plane. Using the passage to the limit there were obtained conditions on the ribs lines without consideration of shear defor-mations in ribs. Using the theory of finite-difference equations there was obtained the stability equation for determining of parameter of critical stiffness of ribs and coefficient of critical load. Comparison of calculation results of stability of three-layer shell without ribs by au-thor's method with experimental data was performed for twenty variants of initial data.

Keywords: *Three-Layer Shell; Light-Weight Aggregate; Stability; Critical Load; Stiffness Ribs; Finite-Element Analysis; ANSYS.*

1. Introduction

Calculation of monolithic ribbed overlapping in traditional design is reduced to separate calculations of main and secondary beams and monolithic plate as beam structures without taking into account the effect of their interaction on stress-strain state.
The overall disadvantage of monolithic overlapping calculation methods at their traditional design is a lack of spatial work consideration and as a result underestimation or overestimations of their strength and crack resistance.
Meanwhile, reinforced concrete plate-beam systems are widely used in construction, in particular in bridge construction they are used in more than 90% of operated or spans or spans under construction.

2. Problem formulation

Plate-beam system is a connection of two different types of elements, which interacts at deformation – plates and ribs (one-dimensional rod). The stress-strain state of each of these elements, conditioned in the framework of known applied theories, has its own peculiarities. Concerning this at studying of such systems there is a need in a special theory construction, which consider main features inherent in separate elements and conditions of their common work. Analytical solutions of problem are not currently achieved. Calculations by numerical methods (mainly by the finite element method) require appropriate verification, so the development of new approaches seems very relevant.

2.1. Aim of paper

The aim of given paper is to research the stability of three-layer cylindrical shell at change of its geometric parameters and the number of supporting longitudinal ribs.

2.2. Materials and methods

The variation method, based on the principle of possible displacements, is used to derive the differential stability equations for the part of the shell enclosed between the ribs, as well as the conditions along the rib's lines and along the edges of the shell. For the numerical implementation of author's method there was developed the program "Three-layer-II", which was implemented in the Wolfram Mathematica 11 environment [14]. In order to verify the obtained results, there were prepared models of considered shells in the ANSYS [15] package. Corresponding calculations were made.

3. Research results

Let's consider the unclosed three-layer shell, supported by stiffness ribs. All ribs have the same stiffness and equidistant from each other (Figure 1).
For external and internal bearing layers of shell we will consider Kirchhoff-Love hypotheses to be valid, and for aggregate we accept

the principle of linear change of tangential displacements along length. We neglect the transverse deformations of aggregate. For ribs the Bernoulli hypothesis holds and it is accounted only the bending in vertical plane [16]. Due to the fact that external layers thickness δ is smaller in comparison with the shell thickness H, at considering of overall loss of stability we can neglect the flexural stiffness of external layers (we accept $D=0$) in comparison with the total flexural stiffness of shell. The limits of applicability of this assumption were investigated by A.P. Prusakov, L.M. Kurshin and other authors [16]. At considering of overall loss of stability the assumption is valid at shear parameter

$$k_0=\frac{\pi^2\cdot B\cdot h}{G_3\cdot b^2}<1{,}0;\ \frac{h}{\delta}\geq 3.$$

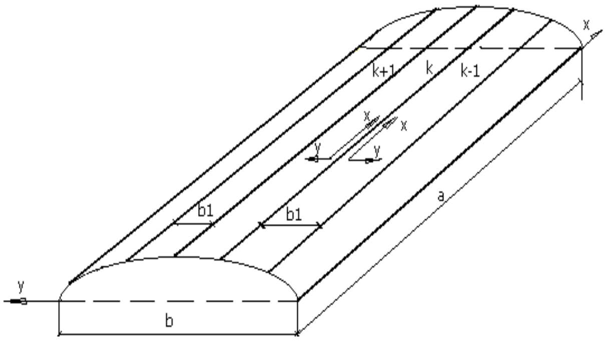

Fig. 1: Three-Layer Shell, Supported by Longitudinal Ribs.

The differential equations of stability of the shell section enclosed between the ribs, as well as the conditions along the rib lines and along the edges of the three-layer shallow shell, which is supported by longitudinal stiffness ribs with the hinged support of the edges, are obtained in a variation way, using the Ostrogradskii-Hamilton functional-action [17].

$$\nabla^4\Phi-\frac{\overline{B}}{R}\frac{\partial^2}{\partial x^2}\left(\varphi-\frac{Bh}{G_3}\nabla^2\varphi\right)=0$$

(1)

$$\nabla^4\varphi-\frac{1}{RD^*}\frac{\partial^2\Phi}{\partial x^2}+\frac{2T_1}{D^*}\frac{\partial^2}{\partial x^2}\left(\varphi-\frac{Bh}{G_3}\nabla^2\varphi\right)=0$$

(2)

$$\psi-\frac{1-\mu}{2G_3}Bh\nabla^2\psi=0$$

(3)

There was performed the simplification of the system of differential equations, using the displacement function F [18]:

$$\varphi=\nabla^4F;\quad \Phi=-\frac{\overline{B}}{R}\frac{\partial^2}{\partial x^2}\left(1-\frac{Bh}{G_3}\nabla^2\right)F.\quad \nabla^4\nabla^4F+\frac{\overline{B}}{R^2D^*}\frac{\partial^4}{\partial x^4}\left(1-\frac{Bh}{G^3}\nabla^2\right)F+\frac{2T_1}{D^*}\frac{\partial^2}{\partial x^2}\left(1-\frac{Bh}{G^3}\nabla^2\right)\nabla^4F=0.$$

(4)

Solution of equations (3) and (4) for the shell section enclosed between the ribs we will found in form:

$$F=f_1(y)\sin\frac{n\pi}{a}x;\quad \Psi=f_2(y)\cos\frac{n\pi}{a}x.$$

(5)

Substituting expressions (5) into equations (3) and (4), solution of equations we will write in form:

$$F=\Big[\cos(y\varphi_1)C_1\rho_1^y+\sin(y\varphi_1)C_2\rho_1^y+\cos(y\varphi_1)C_3\rho_1^y-$$
$$-\sin(y\varphi_1)C_4\rho_1^y+\cos(y\varphi_2)C_5\rho_2^y+\sin(y\varphi_2)C_6\rho_2^y+$$
$$+\cos(y\varphi_2)C_7\rho_2^y-\sin(y\varphi_2)C_8\rho_2^y\Big]\sin(\frac{n\pi}{a}x).$$

(6)

$$\Psi=\left(C_9sh\beta y+C_{10}ch\beta y\right)\cos\frac{n\pi}{a}x.$$

(7)

Here

$$\beta=\frac{\pi}{b}\sqrt{\frac{2}{(1-\mu)k_0}+\alpha_n^2};$$

$$tg\varphi_1=\frac{r}{s},\ tg\varphi_2=\frac{d}{c},\ \rho_1=\left|\sqrt{s^2+r^2}\right|,\ \rho_2=\left|\sqrt{c^2+d^2}\right|,$$

where s, c — real, r, d — complex roots of characteristic equation

$$\lambda^8+\left(k_0m_t\alpha_n^2\right)\lambda^6+\alpha_n^2\left[\alpha_n^2\left(2-k_0m_t\right)+m_t\frac{\pi^2}{b^2}\right]\lambda^4+$$
$$+\left[k_0\left(\alpha^2\frac{\pi^2}{b^2}+m_t\right)\alpha_n^6\right]\lambda^2+$$
$$+\alpha_n^4\left[\alpha_n^4\left(1-k_0m_t\right)-\alpha_n^2\frac{\pi^2}{b^2}\left(m_t-\alpha^2k_0\right)+\alpha^2\frac{\pi^4}{b^4}\right]=0.$$

Assuming that diaphragms are installed on the edges of the shell, the boundary conditions for the case of free support will be written in the form:

at x = 0, a: $w=\frac{\partial u_\beta}{\partial x}=v_\beta=v_\alpha=\frac{\partial^2\Phi}{\partial y^2}=0;$

(8)

at y = 0, b: $w=\frac{\partial v_\beta}{\partial y}=u_\beta=u_\alpha=\frac{\partial^2\Phi}{\partial x^2}=0.$

(9)

Assuming for every section its own coordinate axes we place the origin of coordinates in the start of each section (it is shown for ($k+1$) section, which is located between k and $(k+1)$ rib) and designating $f_1(y)$ in the start and in the end of section (at $y=0$ and $y=b_1$, where $b_1=b/m$, m is the number of sections) as η_k and η_{k+1}, value $f_1''(y)$ as μ_k and μ_{k+1}, value $f_1^{IV}(y)$ as ζ_k and ζ_{k+1}, value $f_1^{VI}(y)$ as ξ_k and ξ_{k+1}, value $f_2'(y)$ as φ_k and φ_{k+1}.

Conditions along the lie of k rib account taking into account different direction of axes y for contiguous sections, that were obtained from variation equation will be written in form:

$$\frac{\partial^4}{\partial x^3\partial y}\left(F-\frac{Bh}{G_3}\nabla^2F\right)_{y=+0}+\frac{\partial^4}{\partial x^3\partial y}\left(F-\frac{Bh}{G_3}\nabla^2F\right)_{y=-0};$$

$$\left[\int\left(1-\frac{Bh}{G_3}\right)\left(\mu\frac{\partial^4}{\partial x^2\partial y^2}-\frac{\partial^4}{\partial x^4}+\nabla^4\right)F\right]_{y=+0}+$$
$$+\left[\int\left(1-\frac{Bh}{G_3}\right)\left(\mu\frac{\partial^4}{\partial x^2\partial y^2}-\frac{\partial^4}{\partial x^4}+\nabla^4\right)F\right]_{y=-0};$$

$$\left(\frac{\partial}{\partial y}\nabla^4F+\frac{\partial\psi}{\partial x}\right)_{y=+0}+\left(\frac{\partial}{\partial y}\nabla^4F+\frac{\partial\psi}{\partial x}\right)_{y=-0};$$

(10)

$$\left(\frac{\partial}{\partial y}\nabla^6 F\right)_{y=+0} + \left(\frac{\partial}{\partial y}\nabla^6 F\right)_{y=-0} + $$
$$+\left\{\left(1-\frac{Bh}{G_3}\nabla^2\right)\left(\frac{D_p}{D^*}\frac{\partial^4}{\partial x^4}\nabla^4 F - \frac{P_p}{D^*}\frac{\partial^2}{\partial x^2}\nabla^4 F\right)\right\}_{y=0}; \quad \left[\frac{\partial \psi}{\partial y} + \frac{Bh}{G_3}\frac{\partial}{\partial x}\nabla^6 F\right]_{y=0}.$$

It is denoted in equation:
$\alpha^2 = \frac{\bar{B}b^4}{R^2 D^* \pi^2}$ — shell curvature parameter; $m_t = \frac{2T_1 b^2}{\pi^2 D^*}$ — critical forces parameter; $k_0 = \frac{\pi^2 Bh}{G_3 b^2}$ — shear parameter; $\gamma = \frac{D_p}{D^* b}$ — stiffness parameter; D_p — flexural stiffness of rib; $H = h + 0{,}5\delta$; $\delta, 2h$ — thickness of external layer and aggregate; G_3 — shear modulus of aggregate; E, μ — modulus of elasticity and Poisson's ratio for external layers; u_1, v_1, u_2, v_2 — tangential displacements of middle surfaces of top and bottom layers; w — shell deflection.

$$D^* = 2BH^2;\ \bar{B} = 2\left(1-\mu^2\right)B;\ B = \frac{E\delta}{1-\mu^2};\ \alpha_n = \frac{n\pi}{a}.$$

Using the theory of finite-difference equations there was obtained the stability equation for determination of the parameter of critical stiffness of ribs γ and critical load coefficient m_t.

Unknown $\eta_k, \mu_{kr}, \zeta_k, \xi_k, \varphi_k$, which this system includes, should satisfy conditions at $k=0$ and $k=m$ on the edges of shell $y=0, b$.

Conditions on the edges of shell at $k=0$ и $k=m$ for simply supported shell according to expressions (9) condition should be written in form:

$$\eta_0 = \mu_0 = \zeta_0 = \xi_0 = \varphi_0 = \eta_m = \mu_m = \zeta_m = \xi_m = \varphi_m = 0$$
(11)

To evaluate the reliability of the computational model of a three-layer shell and the algorithm for studying its stability by the author's method, the results of the calculation are compared with known experimental data and with the results of finite element analysis in the ANSYS program.

For evaluation of reliability of computational model of three-layer shell and algorithm of its stability research by the author's method, the results of calculations were compared with known experimental data and with the results of finite element analysis in the ANSYS program. To implement the algorithm of the author's method, the "Three-layer shell-II" program was compiled, and a series of calculations was performed in the Wolfram Mathematica 11 environment.

On the first stage the three-layer shell without ribs with simple support of longitudinal edges and fixed support of transverse edges was considered. Material of bearing layers – duralumin D-16T; thickness of layers – δ = 0.001 m; modulus of elasticity – E = 6.9 $\cdot 10^7$ kN/m^2; cylindrical stiffness at tension – B = 15800 N/m; Poisson's ratio – μ = 0.33. Light-weight aggregate – FK-20 type styrofoam; Poisson's ratio – μ = 0.4; shear modulus G_s and thickness 2h – variable values that are given for every of researched variant of shell in Table 1 and Table 2. Shell parameters (Figure 1) are: a = 0.6 m; b = 0.4 m; R = 1.0 m. Stiffness rib has rectangular form (height 2h and width b_r are given in Table 1 and Table 2; material – duralumin D-16T.

A comparison of results, obtained by author's method for twenty variants of initial data, with experimental data of A.Y. Aleksandrov, L.E. Brukker and other [19] are given in table 1.

Note that, as it is shown by S.N. Kahn and other [10], the conditions of support of cylindrical shell does not affect the value of critical load at its axial compression. Consequently, results, obtained for simple support of edges are also valid for other boundary conditions.

The same shell but supported with one and three longitudinal ribs with dimensions that are given in the Table 2 was considered on the second stage. For ten variants of initial data there was obtained the critical load by author's method. Then all three shells (without ribs, with one and three longitudinal ribs) were modelled in the program ANSYS 17 with following stability calculation and critical load determination.

On the Figure 2 as an example there are shown third forms of loss of stability (shells with one and three ribs) at initial data, stated in row 6 of Table 1. All other results of calculations by author's method and finite element method are shown in Table 2.

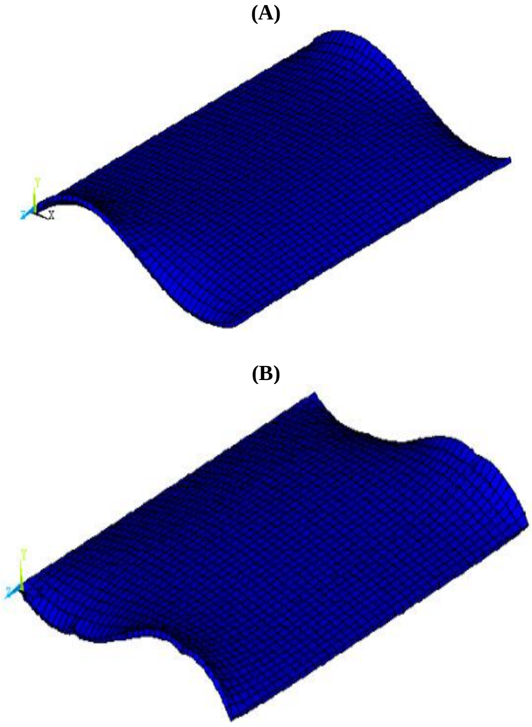

Fig. 2: Forms of Stability Loss.

4. Conclusion

Analysis of Table 1 shows that the results of calculation of a three-layer shell without ribs are in general satisfactorily consistent with the experimental data of A.Y. Aleksandrov, L.E. Brukker and other [19]. In 12 of 20 variants considered the discrepancy between results does not exceed 20%. In some cases, the critical load obtained by the author's method is greater than the experimentally obtained value, in some cases it is less. Unfortunately, authors do not have another experimental data for comparison.

As for the comparison of the results by the author's method and the finite element method (Table 2), the discrepancy here is insignificant (no more than 5.4%). Two regularities can be clearly seen: the finite element calculation in all cases gives higher values of the critical load; the discrepancy between the results of the two methods increases as the number of stiffeners increases.

This allows us to recommend the proposed method for calculating of three-layer sloping shells and supported shells for engineering calculations, since its numerical implementation does not require the use of complex and expensive finite-element packages.

Table 1: Comparison of Results Obtained by Author's Method and Experimental Data

№	2h, mm	b_r, mm	G, kN/m^2	k_0	α^2	m_t	P_{author}, kN	P_{exp}, kN	Error %
1	15.0		8130	4.500	2.751	0.354	86.57	82.00	5.3
2	15.0	16.6	12400	2.950	2.751	0.499	112.00	92.00	17.9
3	15.0		8400	4.360	2.751	0.319	78.01	78.00	0.01

4	9.5	14.6	15000	1.540	6.388	1.323	119.00	89.00	25.2
5	10.0	14.8	9330	2.610	5.821	0.633	73.00	84.50	13.6
6	10.0		7950	3.070	5.821	0.525	61.00	86.00	29.1
7	4.0	11.9	12700	0.768	28.173	2.795	66.70	62.80	5.8
8	4.0		12100	0.807	28.173	2.356	56.50	69.60	18.8
9	4.0		56630	0.172	28.173	6.756	161.40	146.00	9.5
10	4.5	12.2	9200	1.193	23.284	2.196	63.00	50.00	20.6
11	4.0	11.9	17800	0.518	28.173	2.431	63.10	65.00	2.9
12	5.0	12.5	30400	0.401	19.565	3.220	111.00	102.00	8.1
13	15.0	16.6	15900	2.300	2.751	0.419	102.50	84.00	18.0
14	14.5	16.4	8900	3.970	2.932	0.510	118.00	115.00	2.5
15	13.0	15.9	1240	2.560	3.594	0.612	114.60	116.00	1.2
16	14.5	16.4	42900	0.825	2.932	0.629	144.40	160.00	9.8
17	15.0	16.6	28200	1.300	2.751	0.912	173.00	157.00	9.2
18	15.0		31500	1.160	2.751	1.267	229.00	207.00	9.6
19	10.0	14.8	32060	7.480	5.821	0.322	37.30	44.00	15.2
20	9.5	14.6	4000	5.790	6.388	0.415	43.70	46.20	5.4

Table 2: Calculations Results at Three Ribs in Every Direction

№	2h, mm	br, mm	P_{cr}, kN without ribs			Pcr, kN 1 rib			P_{cr}, kN 3 ribs		
			Author's method	FEM	%	Author's method	FEM	%	Author's method	FEM	%
1	15,0	16,6	86,57	89,12	2,9	107,0	112,1	4,5	149,0	157,1	5,2
2	15,0		122,00	126,0	3,2	134,0	139,1	3,7	186,6	195,7	4,7
3	9,5	14,6	139,00	145,1	4,2	198,0	207,3	4,5	229,6	240,8	4,7
4	4,0	11,9	66,70	69,92	4,6	67,0	70,3	4,7	68,0	71,9	5,4
5	4,0		161,40	167,0	3,4	164,0	171,2	4,2	171,0	180,3	5,2
6	10,0	14,8	61,00	63,76	4,3	91,0	95,7	4,9	102,5	108,3	5,4
7	10,0		37,30	39,00	4,4	39,0	40,9	4,6	76,0	80,2	5,2
8	14,5	16,4	118,00	122,2	3,4	172,0	180,4	4,7	227,0	238,5	4,8
9	13,0	15,9	114,60	119,1	3,8	164,0	172,0	4,7	181,7	190,6	4,7
10	20,0	18,1	75,41	78,18	3,5	117,0	122,8	4,7	168,1	177,0	5,0

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