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Research paper

Prime Graceful Labeling

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Abstract

A graph G with m vertices and n edges, is said to be prime graceful labeling, if there is an injection φ from the vertices of G to {1, 2, ..., k} where k = min {2m, 2n} such that gcd ($\varphi(v_i)$, $\varphi(v_j)$)=1 and the induced injective function φ^* from the edges of G to {1, 2, ..., k − 1} defined by $\varphi^*(v_iv_j) = |\varphi(v_i) - \varphi(v_j)|$, the resulting edge labels are distinct. In this paper path P_n , cycle C_n , star $K_{1,n}$, friendship graph F_n , bistar $B_{n,n}$, $C_4 \cup P_n$, $K_{m,2}$ and $K_{m,2} \cup P_n$ are shown to be Prime Graceful Labeling .

Keywords: *Prime labeling, graceful labeling and prime graceful labeling.*

1. Introduction

By a graph G = (V,E) , we mean a finite simple undirected graph. For standard terminology and notations related to graph theory we refer F. Harary [2] and J.A.Bondy and U.S.R.Murthy [3]. For various graph labeling problems, we refer to Gallian [4]. We provide here some definitions which are necessary for our present investigations. In this paper, the concept of Prime Graceful labeling is introduced and some results on these are established.

Definition 1.1

Let G = ((V (G), (E(G)) be a graph with m vertices. A bijection f : V → {1, 2, ...,m} is called a prime labeling if for each edge e= uv, gcd(f(u),f(v)) = 1. A graph which admits a prime labeling is called a Prime graph.

Definition 1.2

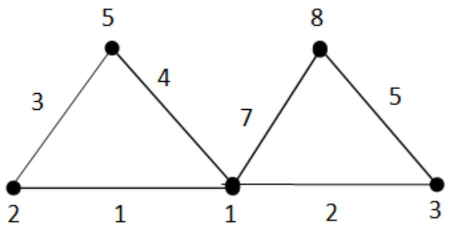
Let G = ((V (G), (E(G)) be a simple, finite and undirected graph with |V | = m and |E| = n. An injective function f : V → { 1, 2, ... , m} is called a graceful labeling of G if all the edge labels of G given by f(uv) = |f(u) − f(v)| for every uv ∈ E are distinct. A graph which admits a graceful labeling is called a graceful graph.

Definition 1.3

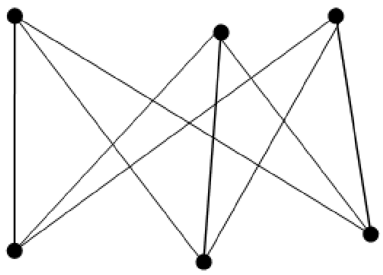
A graph G with m vertices and n edges, is said to be prime graceful labeling, if there is an injection φ from the vertices of G to {1, 2, ..., k} where k = min {2m, 2n} such that gcd ($\varphi(v_i)$, $\varphi(v_j)$)=1 and the induced injective function φ^* from the edges of G to {1, 2, ..., k − 1} defined by $\varphi^*(v_iv_j) = |\varphi(v_i) - \varphi(v_j)|$, the resulting edge labels are distinct.

Example 1.4

Triangular snake T2 is a prime graceful labeling



Example 1.5



In k3, 3, edges and vertices cannot be labeled, so that GCD of end vertices of each edge is one and edge labels are distinct. So k3, 3 is not a prime graceful labeling.

Theorem 1.6

The star graph k1, n admits prime graceful labeling.

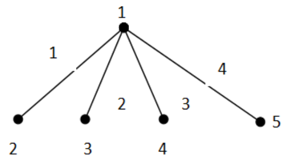
Proof

The star graph k1,n has n + 1 vertices and n edges.

k = min {2 (n + 1) , 2n}
= 2n

In k1, n, one vertex is adjacent with remaining n vertices.
Label the vertex of degree n with one remaining with 2, 3, 4, ... , n, n + 1.
The GCD of end vertices of each edge is 1
The edges labels 1, 2, 3, 4, ... are distinct.

Example 1.7



Theorem 1.8

The Bistar graph Bn, n admits prime graceful labeling.

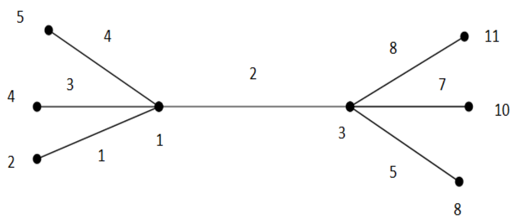
Proof

The Bistar graph has (2n + 2) vertices and (2n + 1) edges.
Bistar graph Bn, n has exactly two vertices of degree n, label these vertices by 1 and 3.

$$k = \min \{2(2n + 2), 2(2n + 1)\} = 2(2n + 1)$$

Label the vertices that are adjacent with vertex label 1 by 2, 4, 5, 6, ..., n + 2, so that gcd of end vertices of each edge is one.
Label the vertices that are adjacent with vertex label 3 from the set {n + 3, n + 4, ..., 2 (2n + 1)} and not a multiple of 3.
The gcd of end vertices of each edge is 1 and edge labels are distinct.

Example 1.9



Theorem 1.10

The path Pn admits prime graceful labeling.

Proof

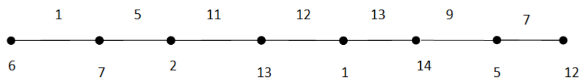
The path Pn has n and vertices (n − 1) edges.

$$k = \min \{2n, 2(n - 1)\} = 2n - 2$$

The vertices of Pn are labeled from the set S = {1, 2, 3, ..., 2n − 3, 2n − 2}
Choose $\left(\frac{n}{2}\right)^{th}$ vertex and label it with 1.
If n is odd, choose $\left(\frac{n+1}{2}\right)^{th}$ vertex and label it with 1.
Choose last two integers from the set S.
i.e., (2n – 3) and (2n – 2) and label it to the adjacent vertices of vertex label 1.
Choose two integers from the beginning of the set S and label with the vertex adjacent to the vertex label 2n−3 and 2n−2 , so that the gcd of two consecutive vertices is 1 . Alternatively choose the integers from the beginning the set and end of the set, label the vertices so that gcd of two consecutive vertices is 1. The resulting edge labels are distinct.

Example 1.11

P_8 is prime graceful labeling.



Theorem 1.12

The friendship graph F_n admits prime graceful labeling.

Proof

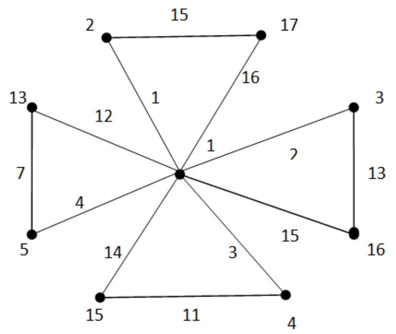
The friendship graph F_n has (2n + 1) vertices and 3n edges.

$$k = \min \{2(2n + 1), 6n\} = \min \{4n + 2, 6n\} = 4n + 2.$$

In friendship graph, one vertex of degree 2n is adjacent to the remaining 2n vertices, label the vertex of degree 2n with 1. Choose a vertex from each cycle C_3, label it with 2, 3, 4, 5, ..., (n + 1) and label the remaining vertices with (4n + 2) , (4n + 1), (4n), (4n − 1) ..., so that the gcd of end vertices of each edge is 1.
The resulting edge labels are distinct.

Example 1.13

Friendship graph F_4 is prime graceful labeling.



Theorem 1.14

The cycle graph Cn admits prime graceful labeling.

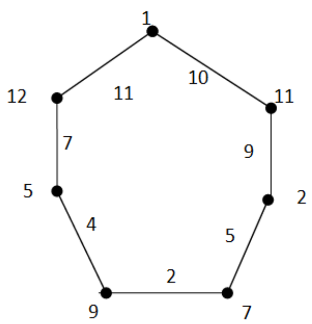
Proof

The cycle C_n has n edges and n vertices.

$$K = \min (2n, 2n) = 2n.$$

The vertices of C_n are labeled from the set S = {1, 2, 3, ..., 2n − 1, 2n}.
Choose an arbitrary vertex in C_n and label it with 1.
Choose last two integers from the set S.
i.e., 2n-1 and 2n label it to the adjacent vertices of vertex label 1.
Choose the integers from the beginning of the set S and label with the vertex adjacent to the vertex label 2n-1 or 2n, so that the gcd of two consecutive vertices is 1 and edge labels are distinct.

Example 1.15



Theorem 1.16

The graph $K_{m,2}$ admits prime graceful labeling for $m \geq 2$.

Proof

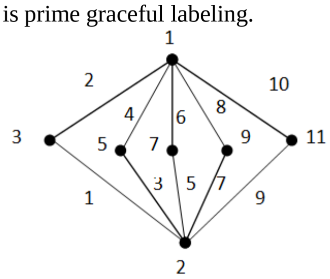
The graph $K_{m,2}$ contains (m + 2) vertices and 2m edges.

$$K = \min\{2(m+2), 2(2m)\}$$
$$= \min\{2m+4, 4m\}$$
$$= 2m+4.$$

Label the vertices having degree m of $K_{m,2}$ with 1 and 2 remaining vertices with odd numbers 3, 5, ..., [(2m + 4) − 1] . Hence the gcd of two consecutive vertices of each edge is 1 and resulting edge labels are distinct.

Example 1.17

The graph $K_{5,2}$ is prime graceful labeling.



Theorem 1.18

The Comb graph Pn ⊙ K1 admits prime graceful labeling.

Proof

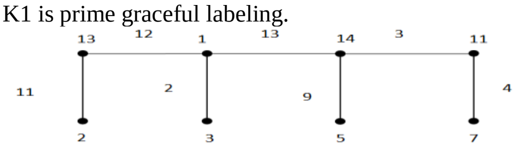
The Comb graph Pn ⊙ K1 has 2n vertices and (2n − 1) edges.

$$k = \min\{4n, 4(n-1)\}$$
$$= 4(2n-1)$$
$$= 8n-4$$

The vertices of Pn are labeled from the set S = {1, 2, 3, ..., 8n − 4}as in **Theorem1.10**. Label the remaining n vertices, so that gcd of two consecutive vertices of each edge is 1 and resulting edge labels are distinct.

Example 1.19

P4 ⊙ K1 is prime graceful labeling.



Theorem 1.20

The graph Pn ∪ C4 admits prime graceful labeling.

Proof

The graph Pn ∪ C4 contains (n + 4) vertices and (n + 3) edges.

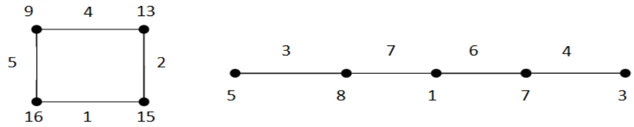
$$K = \min\{2(n+4), 2(n+3)\}$$
$$= \min\{2n+8, 2n+6\}$$
$$= 2n+6.$$

Label the vertices of Pn with $v_1, v_2, ..., v_n$, the vertices are labeled from the set S = {1, 2, ..., 2n + 6} as in **Theorem1.10** .

Label the vertices of C4 from the set {2n, 2n + 1, ..., 2n + 6} , so that the gcd of two consecutive vertices is 1 and the edge labels are distinct.

Example 1.21

The graph C4 ∪ P5 is prime graceful labeling.



Theorem 1.22

The graph Pn ∪ Km,2 (m ≥ 1) admits prime graceful labeling.

Proof

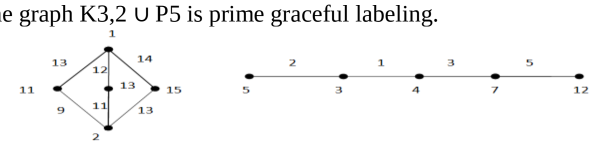
The graph Km,2 ∪ Pn contains (m + n + 2) vertices and (2m + n − 1) edges.

$$K = \min\{2(m+n+2), 2(2m+n-1)\}$$
$$= \min\{2m+2n+4, 4m+2n-2\}$$
$$= 2m+2n+4.$$

Label the vertices of Pn with v1, v2, ..., vn , the vertices are labeled from the set
S = {1, 2, ..., 2m + 2n + 4} as in **Theorem1.10** . Label the vertices having degree
2m of Km,2 with 1 and 2 and remaining vertices with odd numbers 3, 5, 7, ...
Hence the gcd of two adjacent vertices is 1 and edge labels are distinct.

Example 1.23

The graph K3,2 ∪ P5 is prime graceful labeling.



2. Conclusion

In this paper the concept of prime graceful labeling is introduced and prove the existence of prime graceful labeling for graphs such as path Pn, cycle C_n, star $K_{1,n}$, friendship graph F_n, bistar $B_{n,n}$, $C_4 \cup P_n$,$K_{m,2}$ and $K_{m,2} \cup P_n$.

It would be interesting to do further research on this topic and to find more graphs that satisfy the prime graceful labeling.

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