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Research paper

Fuzzy $\mathscr{t}\mathscr{g}\mathscr{p}$-closed sets and fuzzy $\mathscr{t}^*\mathscr{g}\mathscr{p}$-closed sets

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Abstract

In this paper, we aim to address the idea of fuzzy $\mathscr{t}$-set and fuzzy $\mathscr{t}^*$-set in fuzzy topological space to present new types of the fuzzy closed set named fuzzy$\mathscr{t}\mathscr{g}\mathscr{p}$-closed set and fuzzy $\mathscr{t}^*\mathscr{g}\mathscr{p}$-closed set. We will study several examples and explain the relations of them with other classes of fuzzy closed sets. Moreover, in a fuzzy locally indiscrete space we can see that these two sets are the same.

Keywords: Fuzzy$\mathscr{t}$-set, fuzzy $\mathscr{t}^*$-set, fuzzy $\mathscr{t}\mathscr{g}$-closed set, fuzzy $\mathscr{t}\mathscr{g}\mathscr{p}$ –closed set, fuzzy $\mathscr{t}^*\mathscr{g}\mathscr{p}$-closed set.
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1. Introduction

Fuzzy sets were the expansion of the classical notion of sets which invented by Zadeh [9] in 1965. Fuzzy topology invented in 1968 by Chang in [4] when he put fuzzy set instead of classical set in the ordinary definition of the topology.
One of the most important concepts of fuzzy topology is fuzzy $\mathscr{g}$-closed set.The concept of $\mathscr{t}$-set was introduced by Tong [8] in 1989, the fuzzy per-closed set defined by Bin Shahna [3], in 1991. In 1997, Balasubramanian and Sundaram [2] introduced the notion of fuzzy generalized closed set. In 2012, thenotion of $\mathscr{t}^*$ -set was devised by Indira and Rekha [5].
In this article, we introduced two types of fuzzy closed set named fuzzy $\mathscr{t}$-generalized pre-closed sets (simply, fuzzy $\mathscr{t}\mathscr{g}\mathscr{p}$-closed sets), and fuzzy $\mathscr{t}^*$-generalized pre-closed sets (simply, fuzzy $\mathscr{t}^*\mathscr{g}\mathscr{p}$-closed sets) and the relations arestudy together beside numerous types of fuzzy closed set.

2. Preliminaries

First, we use the sample $(\mathcal{X},\mathcal{T})$ throughout this paper to represent fuzzytopological space (in short, fts). Any fuzzy set $\mathcal{N}$ in any fts $(\mathcal{X},\mathcal{T})$ in theChang's meaning, $\mathcal{N}^\circ$ and $\overline{\mathcal{N}}$ denote the fuzzy interior and fuzzy closure of $\mathcal{N}$respectively. Also, we use f.o. (resp., f.c., f.s.o., f.s.c., f.p.o.s., f.p.c., f.r.o. andf.r.c.) to denote fuzzy open (resp., fuzzy closed, fuzzy semi-open, fuzzy semi-closed, fuzzy pre-open, fuzzy pre-closed, fuzzy regular-open and fuzzy regular-closed)
We review several fundamental definitions and notations of most basic concepts
that are needed later in our paper.
Definition 2.1.A fuzzy subset $\mathcal{N}$ of any fts$(\mathcal{X},\mathcal{T})$so-called:

(1) f.p.o. if $\mathcal{N} \leq \overline{\mathcal{N}}^\circ$, [3] (resp. f.p.c. if $\overline{\mathcal{N}^\circ} \leq \mathcal{N}$).

(2) f.s.o. if $\mathcal{N} \leq \overline{\mathcal{N}^\circ}$, [1] (resp. f.s.c. if $\overline{\mathcal{N}}^\circ \leq \mathcal{N}$)

(3) f.r.o. if $\mathcal{N} = \overline{\mathcal{N}}^\circ$, [1] (resp. f.r.c. if $\mathcal{N} = \overline{\mathcal{N}^\circ}$).

Definition 2.2. [7] Given a fuzzy subset $\mathcal{N}$ in any fts$(\mathcal{X},\mathcal{T})$, we called,
(1) the union of whole f.p.o. subsets of $\mathcal{X}$ which are contained in $\mathcal{N}$ iscalled fuzzy pre-interior of $\mathcal{N}$ and it's symbolized by$\mathcal{N}^{\circ P}$.
(2) the intersection of whole f.p.c. subsets of $\mathcal{X}$ which are containing $\mathcal{N}$ iscalled fuzzy pre-closure of $\mathcal{N}$ and it's symbolized by$\overline{\mathcal{N}}^P$.
Proposition 2.3. [7] For fuzzy subsets $\mathcal{N}$ and $\mathcal{M}$ of any fts$(\mathcal{X},\mathcal{T})$, we have:

(1) If $\mathcal{N}$ is f.p.o. (resp., f.p.c.), then $\mathcal{N} = \mathcal{N}^{\circ P}$ (resp. $\mathcal{N} = \overline{\mathcal{N}}^P$).
(2) $(\mathcal{N}\wedge\mathcal{M})^{\circ P} = \mathcal{N}^{\circ P}\wedge\mathcal{M}^{\circ P}$, $\overline{(\mathcal{N}\wedge\mathcal{M})}^P \leq \overline{\mathcal{N}}^P\wedge\overline{\mathcal{M}}^P$.
(3) $(\mathcal{N}\vee\mathcal{M})^{\circ P} \geq \mathcal{N}^{\circ P}\vee\mathcal{M}^{\circ P}$, $\overline{(\mathcal{N}\vee\mathcal{M})}^P = \overline{\mathcal{N}}^P\vee\overline{\mathcal{M}}^P$.

Definition 2.4. A fuzzy subset $\mathcal{N}$ of an fts$(\mathcal{X},\mathcal{T})$ is called:
(1) fuzzy generalized closed set (in short, fuzzy $\mathscr{g}$-closed) [2] if $\overline{\mathcal{N}} \leq \mathcal{V}$whenever $\mathcal{N} \leq \mathcal{V}$and $\mathcal{V}$is an f.o. set.
(2) fuzzy generalized pre-closed set (in short, fuzzy $\mathscr{g}\mathscr{p}$-closed) [6] if $\overline{\mathcal{N}}^P \leq \mathcal{V}$whenever $\mathcal{N} \leq \mathcal{V}$and $\mathcal{V}$ is an f.o. set.

3. Fuzzy t and t*- Sets

In this part, the concepts of fuzzy $\mathscr{t}$-set and fuzzy $\mathscr{t}^*$-set are defined. Several interesting properties are investigated besides giving several examples.
Definition 3.1. If $(\mathcal{X},\mathcal{T})$) is any fts and $\mathcal{N}$ is a fuzzy subset of $(\mathcal{X},\mathcal{T})$, then$\mathcal{N}$ is called:
(1) fuzzy $\mathscr{t}$-set if $\mathcal{N}^\circ = \overline{\mathcal{N}}^\circ$.
(2) fuzzy $\mathscr{t}^*$-set if $\overline{\mathcal{N}} = \overline{\mathcal{N}^\circ}$.
Proposition 3.2. In any fts$(\mathcal{X},\mathcal{T})$, any fuzzy subset of it is a fuzzy $\mathscr{t}$-set iff it's complement is a fuzzy $\mathscr{t}^*$-set.
Definition 3.3. An fts$(\mathcal{X},\mathcal{T})$is said to be fuzzy locally indiscrete space iffevery fuzzy open set in $\mathcal{X}$ is fuzzy closed in $\mathcal{X}$.
Proposition 3.4. Any fuzzy subset $\mathcal{N}$ of a fuzzy locally indiscrete space is afuzzy $\mathscr{t}$-set iff it is a fuzzy $\mathscr{t}^*$-set.
Proof. Consider that $(\mathcal{X},\mathcal{T})$be a fuzzy locally indiscrete space, we must provethat the fuzzy $\mathscr{t}$-sets are the same as the fuzzy $\mathscr{t}^*$-sets.
First, consider that $\mathcal{N}$is a fuzzy $\mathscr{t}$-set, so $\mathcal{N}^\circ = \overline{\mathcal{N}}^\circ$, but since $(\mathcal{X},\mathcal{T})$is fuzzy locally indiscrete, thenany f.c. set is f.o., so $\overline{\mathcal{N}}$ is f.o. and $\overline{\mathcal{N}} = \overline{\mathcal{N}}^\circ = \mathcal{N}^\circ$, hence $\overline{\mathcal{N}} = \overline{\mathcal{N}^\circ}$ fuzzy $\mathscr{t}^*$-set.
Conversely, assume that $\mathcal{N}$ is a fuzzy $\mathscr{t}^*$-set, so $\overline{\mathcal{N}} = \overline{\mathcal{N}^\circ}$but $\mathcal{N}^\circ$ is f.o., so it'sf.c. (since $(\mathcal{X},\mathcal{T})$ is fuzzy locally indiscrete), hence

$\overline{\mathcal{N}} = \overline{\mathcal{N}^{\circ}} = \mathcal{N}^{\circ}$and this implies that $\mathcal{N}^{\circ} = \overline{\mathcal{N}}^{\circ}$, therefore $\mathcal{N}$ is a fuzzy t-set.

Proposition 3.5. Given a fuzzy subset $\mathcal{N}$ of any fts$(\mathcal{X},\mathcal{T})$. Then we have;

(1) $\mathcal{N}$ is a fuzzy$t$-set iff it's f.s.c.

(2) $\mathcal{N}$ is a fuzzy t^*-set iff it's f.s.o.

Proof.

(1) Assume that $\mathcal{N}$ is a fuzzy t-set, therefore $\mathcal{N}^{\circ} = \overline{\mathcal{N}}^{\circ}$, then $\overline{\mathcal{N}}^{\circ} \le \mathcal{N}$, then $\mathcal{N}$ is a f.s.c. set.

Conversely, consider that $\mathcal{N}$ is a f.s.c. set, we have$\overline{\mathcal{N}}^{\circ} \le \mathcal{N}$, so $\overline{\mathcal{N}}^{\circ} \le \mathcal{N}^{\circ}$. Since, we also have $\mathcal{N}^{\circ} \le \overline{\mathcal{N}}^{\circ}$. hence $\mathcal{N}^{\circ} = \overline{\mathcal{N}}^{\circ}$. Then $\mathcal{N}$ is afuzzy t-set.

(2) Similar to (1).

Proposition 3.6.

(1) Every f.o. subset of any fts$(\mathcal{X},\mathcal{T})$is a fuzzy t^*-set.

(2) Every f.c. subset of any fts$(\mathcal{X},\mathcal{T})$is a fuzzy t-set.

Proof. Obvious.

Remark 3.7. The reverse implication of (1) and (2) of the above-mentioned proposition may be not true as illustrated in the example below.

Example 3.8. Define fuzzy sets $\mathcal{G}$, $\mathcal{H}$ and $\mathcal{K}$ on a set $\mathcal{X} = [0,1]$, as follows:

$$\mathcal{G} = \begin{cases} 1 & 0 \le a \le \frac{1}{2} \\ 2-2a & \frac{1}{2} \le a \le 1 \end{cases}$$

$$\mathcal{H} = \begin{cases} 0 & 0 \le a \le \frac{1}{4} \\ 4a-1 & \frac{1}{4} \le a \le \frac{1}{2} \\ 1 & \frac{1}{2} \le a \le 1 \end{cases}$$

$$\mathcal{K} = \begin{cases} 1-4a & 0 \le a \le \frac{1}{4} \\ 0 & \frac{1}{4} \le a \le 1 \end{cases}$$

Consider the fuzzy topology $\mathcal{T} = \{0_{\mathcal{X}}, \mathcal{K}, 1_{\mathcal{X}}\}$on a set $\mathcal{X}$. Therefore $\mathcal{G}$ is afuzzyt^*-set, however it's not f.o. set. As well, $\mathcal{H}$ is a fuzzy t-set, while it isn'tf.c. set.

Proposition 3.9.

(1) Every f.r.o. subset of any fts$(\mathcal{X},\mathcal{T})$is a fuzzy$t$-set.

(2) Every f.r.c. subset of any fts$(\mathcal{X},\mathcal{T})$is a fuzzy t^*-set.

Proof.

(1) Consider that $\mathcal{N}$ is a f.r.o. set, then $\mathcal{N} = \overline{\mathcal{N}}^{\circ}$. Since every f.r.o.set is f.o., therefore $\mathcal{N}^{\circ} = \mathcal{N} = \overline{\mathcal{N}}^{\circ}$, hence $\mathcal{N}$ is a fuzzy t-set.

(2) Similar to (1).

Remark 3.10. We can see in the example below that the converse of (1) and(2) of the above Proposition isn't true in general:

Example 3.11. Define fuzzy sets $\mathcal{G}$, $\mathcal{H}$ and $\mathcal{K}$ on a set $\mathcal{X} = [0,1]$, as below:

$$\mathcal{G} = \begin{cases} 0 & 0 \le a \le \frac{1}{2} \\ 2a-1 & \frac{1}{2} \le a \le 1 \end{cases}$$

$$\mathcal{H} = \begin{cases} 1 & 0 \le a \le \frac{1}{4} \\ \frac{4}{3}(1-a) & \frac{1}{4} \le a \le 1 \end{cases}$$

$$\mathcal{K} = \left\{(1-a)^2 \quad 0 \le a \le 1\right.$$

Consider the fuzzy topology $\mathcal{T} = \{0_{\mathcal{X}}, \mathcal{K}, 1_{\mathcal{X}}\}$ on a set $\mathcal{X}$. Then $\mathcal{G}$ is a fuzzy$t$-set, however it's not f.r.o.. As well, H is a fuzzy t^*-set, whereas it's not f.r.c.set.

Proposition 3.12. If $\mathcal{N}$ is a fuzzy subset of any fts$(\mathcal{X},\mathcal{T})$, then we have.

(1) $\mathcal{N}$ is f.r.o. set iff it's fuzzy t-set and f.p.o. set.

(2) $\mathcal{N}$ is f.r.c. set iff it's fuzzy t^*-set and f.p.c. set.

Proof. Obvious.

Proposition 3.13. If $\mathcal{N}$ and $\mathcal{M}$ are fuzzy subsets of any fts$(\mathcal{X},\mathcal{T})$. Then:

(1) $\mathcal{N} \wedge \mathcal{M}$ is a fuzzy t-set, when $\mathcal{N}$ and $\mathcal{M}$ are fuzzy t-sets.

(2) $\mathcal{N} \vee \mathcal{M}$ is a fuzzy t^*-set, when $\mathcal{N}$ and $\mathcal{M}$ are fuzzy t^*-sets.

Proof.

(1) Assume that $\mathcal{N}$ and $\mathcal{M}$ be fuzzy t-sets, therefore $\mathcal{N}^{\circ} = \overline{\mathcal{N}}^{\circ}$ and$\mathcal{M}^{\circ} = \overline{\mathcal{M}}^{\circ}$. Since $(\mathcal{N}\wedge\mathcal{M})^{\circ} \le \overline{(\mathcal{N}\wedge\mathcal{M})}^{\circ} \le (\overline{\mathcal{N}}\wedge\overline{\mathcal{M}})^{\circ} = \overline{\mathcal{N}}^{\circ}\wedge\overline{\mathcal{M}}^{\circ} = \mathcal{N}^{\circ}\wedge\mathcal{M}^{\circ} = (\mathcal{N}\wedge\mathcal{M})^{\circ}$. Then $(\mathcal{N}\wedge\mathcal{M})^{\circ} = \overline{(\mathcal{N}\wedge\mathcal{M})}^{\circ}$, so $\mathcal{N}\wedge\mathcal{M}$ is afuzzy t-set.

(2) Consider that $\mathcal{N}$ and $\mathcal{M}$ be fuzzy t^*-sets, therefore $\overline{\mathcal{N}} = \overline{\mathcal{N}^{\circ}}$and $\overline{\mathcal{M}} = \overline{\mathcal{M}^{\circ}}$. Since$\overline{(\mathcal{N}\vee\mathcal{M})} = (\overline{\mathcal{N}}\vee\overline{\mathcal{M}}) = (\overline{\mathcal{N}^{\circ}}\vee\overline{\mathcal{M}^{\circ}}) = \overline{(\mathcal{N}^{\circ}\vee\mathcal{M}^{\circ})} \le \overline{(\mathcal{N}\vee\mathcal{M})^{\circ}}$. Since $\overline{(\mathcal{N}\vee\mathcal{M})^{\circ}} \le \overline{(\mathcal{N}\vee\mathcal{M})}$, so $\overline{(\mathcal{N}\vee\mathcal{M})^{\circ}} \le \overline{(\mathcal{N}\vee\mathcal{M})}$. Then $\overline{(\mathcal{N}\vee\mathcal{M})} = \overline{(\mathcal{N}\vee\mathcal{M})^{\circ}}$. Hence $\mathcal{N}\vee\mathcal{M}$ is a fuzzy t^*-set.

Remark 3.14. From the following example we can see that the converse of(1) and (2) of the above Proposition need not be true in general.

Example 3.15. Define fuzzy sets $\mathcal{G}$, $\mathcal{H}$ and $\mathcal{K}$ on a set $\mathcal{X} = [0,1]$, as below:

$$\mathcal{G} = \begin{cases} 1-\frac{4}{3}a & 0 \le a \le \frac{3}{4} \\ 0 & \frac{3}{4} \le a \le 1 \end{cases}$$

$$\mathcal{H} = \begin{cases} 0 & 0 \le a \le \frac{3}{4} \\ 4a-3 & \frac{3}{4} \le a \le 1 \end{cases}$$

$$\mathcal{K} = \left\{1-a \quad 0 \le a \le 1\right.$$

Consider the fuzzy topology $\mathcal{T} = \{0_{\mathcal{X}}, \mathcal{G}, \mathcal{H}, \mathcal{G}\vee\mathcal{H}, 1_{\mathcal{X}}\}$ on a set $\mathcal{X}$. Then $\mathcal{H}$and $\mathcal{K}$ are fuzzy t^*-sets, however $\mathcal{H}\wedge\mathcal{K}$isn't a fuzzy t^*-set. Also, $\mathcal{H}^c$ and $\mathcal{K}^c$ are fuzzy$t$-sets, but $\mathcal{H}^c\vee\mathcal{K}^c$ isn't a fuzzy t-set.

4. Fuzzy tgp-closed set and fuzzy t*gp-closed set

A new type of fuzzy closed sets named fuzzy tgp-closed set and fuzzy t^*gp-closed set have been provided in this section and also we explain their relationswith other types of fuzzy closed sets.

Definition 4.1. A fuzzy subset $\mathcal{N}$ of an fts $(\mathcal{X},\mathcal{T})$is named fuzzy tg-closed set (resp., fuzzy t^*g-closed set) if $\overline{\mathcal{N}} \le \mathcal{V}$ whenever $\mathcal{N} \le \mathcal{V}$ and $\mathcal{V}$ is a fuzzy$t$-set (reps., fuzzy t^*-set). A fuzzy tg-open set (resp., fuzzy t^*g-open set)is thecomplement of a fuzzy tg-closed set (resp., fuzzy t^*g-closed set).

Example 4.2. Any fuzzy subset of a fuzzy discrete topological space is a fuzzytg-closed set.

Solution: Assume that $(\mathcal{X},\mathcal{T})$is the fuzzy discrete topology, and let $\mathcal{N}$ beany non-empty fuzzy subset of $(\mathcal{X},\mathcal{T})$, so that $\mathcal{N}$ is an f.c. set, hence $\mathcal{N} = \overline{\mathcal{N}}$,this implies that $\mathcal{N}$ is a fuzzy tg-closed set.

Example 4.3. Any fuzzy subset of a fuzzy indiscrete topological space is afuzzy tgp-closed set.

Solution: Consider that $(\mathcal{X},\mathcal{T})$is the fuzzy indiscrete topology. Let us takeany non-empty fuzzy set $\mathcal{N}$in $(\mathcal{X},\mathcal{T})$, therefore $\mathcal{N} = 1_{\mathcal{X}}$, since the only f.c.sets are $0_{\mathcal{X}}$ and $1_{\mathcal{X}}$, also $1_{\mathcal{X}}$ is the only fuzzy t-set which is contains $\mathcal{N}$, then$\mathcal{N}$ is a fuzzy tg-closed set.

Remark 4.4. We can also obtain a fuzzy t^*gp-closed set from Example 4.2and Example 4.3, because of both these examples are fuzzy locally indiscrete spaces see Proposition 3.4.

Proposition 4.5. For any fts$(\mathcal{X},\mathcal{T})$,

(1) Every f.c. subset of any fts$(\mathcal{X},\mathcal{T})$is fuzzy tg-closed.

(2) Every f.c. subset of any fts$(\mathcal{X},\mathcal{T})$ is fuzzy t^*g-closed.

Proof.

(1) Assume that $\mathcal{N}$ is an f.c. subset of an fts$(\mathcal{X},\mathcal{T})$, this mean$\overline{\mathcal{N}} = \mathcal{N}$, hence this implies that $\mathcal{N}$ is a fuzzy tg-closed set.

(2) Consider that $\mathcal{N}$ is anf.c. subset of an fts$(\mathcal{X},\mathcal{T})$and let $\mathcal{V}$ be any fuzzyt^*-set such that $\mathcal{N} \leq \mathcal{V}$, so $\overline{\mathcal{N}} = \mathcal{N} \leq \mathcal{V}$, then $\mathcal{N}$is a fuzzy t^*g-closedset.

Remark 4.6. The reverse implication of (1) and (2) of the above-mentioned proposition may be not true as illustrated in the examples below.

Example 4.7. Define fuzzy sets $\mathcal{G}$ and $\mathcal{H}$ on a set $\mathcal{X} = [0,1]$, as below:

$$\mathcal{G} = \begin{cases} 0 & 0 \leq a \leq \frac{2}{3} \\ 3a - 2 & \frac{2}{3} \leq a \leq 1 \end{cases}$$

$$\mathcal{H} = \begin{cases} 1 - \frac{3}{2}a & 0 \leq a \leq \frac{2}{3} \\ 0 & \frac{2}{3} \leq a \leq 1 \end{cases}$$

Consider the fuzzy topology $\mathcal{T} = \{0_{\mathcal{X}}, \mathcal{H}, 1_{\mathcal{X}}\}$on a set $\mathcal{X}$. Therefore $\mathcal{H}^c$ is afuzzy t-set and $\mathcal{G} \leq \mathcal{H}^c$, also, $\overline{\mathcal{G}} \leq \mathcal{H}^c$, hence $\mathcal{G}$ is fuzzy tg-closed, while it's notan f.c. set.

Example 4.8. Define a fuzzy subset $\mathcal{G}$ on a set $\mathcal{X} = [0,1]$, as below:

$$\mathcal{G} = \begin{cases} 1 & 0 \leq a \leq \frac{1}{4} \\ 2 - 4a & \frac{1}{4} \leq a \leq \frac{1}{2} \\ 0 & \frac{1}{2} \leq a \leq 1 \end{cases}$$

Consider that $\mathcal{T} = \{0_{\mathcal{X}}, 1_{\mathcal{X}}\}$ is a fuzzy topology on $\mathcal{X}$. Therefore $1_{\mathcal{X}}$ is the only fuzzy t^*-set which is containing $\mathcal{G}$ is $1_{\mathcal{X}}$ and $\overline{\mathcal{G}} = 1_{\mathcal{X}}$, therefore $\mathcal{G}$ is a fuzzyt^*g-closed set while it isn't an f.c. set.

Definition 4.9. If $\overline{\mathcal{N}}^P \leq \mathcal{V}$ whenever $\mathcal{N} \leq \mathcal{V}$and $\mathcal{V}$ is a fuzzy t-set, then thefuzzy set $\mathcal{N}$ in an fts $(\mathcal{X},\mathcal{T})$is said to be fuzzy tgp-closed set. A fuzzy tgp-openset is the complement of a fuzzy tgp-closed set.

Example 4.10. If $\mathcal{G}$ and $\mathcal{H}$ are fuzzy sets in a set $\mathcal{X} = [0,1]$, defined as:

$$\mathcal{G} = \begin{cases} 0 & 0 \leq a \leq \frac{2}{3} \\ 2 - 2a & \frac{2}{3} \leq a \leq \frac{4}{5} \\ \frac{a}{2} & \frac{4}{5} \leq a \leq 1 \end{cases}$$

$$\mathcal{H} = \begin{cases} 0 & 0 \leq a < \frac{2}{3} \\ 2 - 2a & \frac{2}{3} \leq a \leq 1 \end{cases}$$

Consider that $\mathcal{T} = \{0_{\mathcal{X}}, \mathcal{H}, 1_{\mathcal{X}}\}$ is the fuzzy topology on $\mathcal{X}$. Therefore $1_{\mathcal{X}}$ is a fuzzy t-set and $\mathcal{G} \leq 1_{\mathcal{X}}$, also, $\overline{\mathcal{G}} \leq \mathcal{H}^c$, thus $\mathcal{G}$ is a fuzzy tgp-closed set.

Definition 4.11. If $\overline{\mathcal{N}}^P \leq \mathcal{V}$ whenever $\mathcal{N} \leq \mathcal{V}$and $\mathcal{V}$ is a fuzzyt^*-set, thenthe fuzzy set $\mathcal{N}$ in an fts $(\mathcal{X},\mathcal{T})$is said to be fuzzy t^*gp-closed set. A fuzzyt^*gp-open set is the complement of a fuzzy t^*gp-closed set.

Example 4.12. Define fuzzy sets $\mathcal{G}$ and $\mathcal{H}$on a set $\mathcal{X} = [0,1]$, as below:

$$\mathcal{G} = \begin{cases} \frac{5}{4}a & 0 \leq a \leq \frac{4}{5} \\ 1 & \frac{4}{5} \leq a \leq 1 \end{cases}$$

$$\mathcal{H} = \begin{cases} 0 & 0 \leq a \leq \frac{3}{4} \\ 4a - 3 & \frac{3}{4} \leq a \leq \frac{4}{5} \\ 1 - a & \frac{4}{5} \leq a \leq 1 \end{cases}$$

Consider that $\mathcal{T} = \{0_{\mathcal{X}}, \mathcal{H}, 1_{\mathcal{X}}\}$ is a fuzzy topology on a set $\mathcal{X}$. Thus $\mathcal{G}$ is afuzzy t^*gp-closed set.

Proposition 4.13.

(1) Every fuzzy tg-closed subset of an fts$(\mathcal{X},\mathcal{T})$is afuzzy tgp-closed set.

(2) Every fuzzy t^*g-closed subset of an fts $(\mathcal{X},\mathcal{T})$is a fuzzy t^*gp-closed set.

Proof.

(1) Suppose that $\mathcal{N}$ be any fuzzy tg-closed subset of an fts$(\mathcal{X},\mathcal{T})$, and let $\mathcal{V}$ be a fuzzy t-set when $\mathcal{N} \leq \mathcal{V}$, so $\overline{\mathcal{N}} \leq \mathcal{V}$, so $\overline{\mathcal{N}}^P \leq \overline{\mathcal{N}} \leq \mathcal{V}$,this implies that $\mathcal{N}$ is a fuzzy tgp-closed set.

(2) Similar to part (1).

Proposition 4.14. For any fts$(\mathcal{X},\mathcal{T})$,

(1) Every f.p.c. subset of an fts$(\mathcal{X},\mathcal{T})$is a fuzzy tgp-closed set.

(2) Every f.p.c. subset of an fts$(\mathcal{X},\mathcal{T})$is a fuzzy t^*gp-closed set.

Proof. It is clear.

Remark 4.15. The converse of the above Proposition is not true in general, the fuzzy set $\mathcal{G}$ in Example 4.10 is fuzzy tgp-closed, while it couldn't be an f.p.c.set, as well the fuzzy set $\mathcal{G}$ in Example 4.12 is a fuzzy t^*gp-closed set, howeverit couldn't an f.p.c. set.

Corollary 4.16. Every f.c. subset of an fts$(\mathcal{X},\mathcal{T})$is a fuzzy tgp-closed set(resp., fuzzy t^*gp)-closed set.

Proof. The result follows from the fact that every f.c. set is an f.p.c. set and the Proposition 4.14.

Proposition 4.17. Any fuzzy set in a fuzzy locally indiscrete space is fuzzytgp-closed iff it is fuzzy t^*gp-closed.

Proof. Since $(\mathcal{X},\mathcal{T})$is a fuzzy locally indiscrete space, then by Proposition 3.4the fuzzy t-set and fuzzy t^*-set are same. So, we get the following, that anyfuzzy tgp-closed set is a fuzzy t^*gp-closed set, also, we get the following, thatany fuzzy t^*gp-closed set is a fuzzy tgp-closed set.

Remark 4.18. It is clear that fuzzytgp-closed and fuzzy t^*gp-closed sets areindependent notions of each other and the following examples are illustratethis.

Example 4.19. Define fuzzy sets $\mathcal{G}$ and $\mathcal{H}$ on a set $\mathcal{X} = [0,1]$, as below:

$$\mathcal{G} = \begin{cases} 1 - \frac{5}{4}a & 0 \leq a \leq \frac{4}{5} \\ 0 & \frac{4}{5} \leq a \leq 1 \end{cases}$$

$$\mathcal{H} = \begin{cases} 1 & 0 \leq a \leq \frac{3}{4} \\ 4 - 4a & \frac{3}{4} \leq a \leq \frac{4}{5} \\ a & \frac{4}{5} \leq a \leq 1 \end{cases}$$

Consider the fuzzy topology $\mathcal{T} = \{0_{\mathcal{X}}, \mathcal{H}, 1_{\mathcal{X}}\}$ on a set $\mathcal{X}$. Thus $\mathcal{G}$ is a fuzzytgp-closed set, while it isn't fuzzy t^*gp-closed.

Example 4.20. Define fuzzy sets $\mathcal{G}$, $\mathcal{H}$and $\mathcal{K}$ on a set $\mathcal{X} = [0,1]$, as below:

$$\mathcal{G} = \begin{cases} 1 - 4a & 0 \leq a \leq \frac{1}{4} \\ 0 & \frac{4}{5} \leq a \leq 1 \end{cases}$$

$$\mathcal{H} = \begin{cases} 0 & 0 \leq a \leq \frac{1}{2} \\ 2a - 1 & \frac{1}{2} \leq a \leq 1 \end{cases}$$

$$\mathcal{K} = \begin{cases} 1 & 0 \leq a \leq \frac{1}{4} \\ \frac{4}{3}(1 - a) & \frac{1}{4} \leq a \leq 1 \end{cases}$$

Consider the fuzzy topology $\mathcal{T} = \{0_{\mathcal{X}}, \mathcal{H}, 1_{\mathcal{X}}\}$ on a set $\mathcal{X}$. Thus $\mathcal{G}$ is a fuzzyt^*gp-closed set, while it isn't fuzzy tgp-closed, when $\mathcal{K}$ is a fuzzy t-set.

Proposition 4.21. If $\mathcal{N}$ is a fuzzy tgp-closed set and fuzzy t-set, then it is anf.p.c. set.

Proof. Assume that $\mathcal{N}$ is both fuzzy t-set and fuzzy tgp-closed set, therefore$\mathcal{N}$ is a fuzzy t-set which contains itself, so it should be contains $\overline{\mathcal{N}}^P$, therefore we have $\mathcal{N} \leq \overline{\mathcal{N}}^P \leq \mathcal{N}$, then $\mathcal{N} = \overline{\mathcal{N}}^P$, hence $\mathcal{N}$ is an f.p.c. set.

Proposition 4.22. If $\mathcal{N}$ is a fuzzy subset of any fts$(\mathcal{X}, \mathcal{T})$, then we have.

(1) $\mathcal{N}$ is fuzzy tgp-open set iff$\mathcal{V} \leq \mathcal{N}^{\circ P}$where $\mathcal{V}$ is a fuzzy t^*set and$\mathcal{V} \leq \mathcal{N}$.

(2) $\mathcal{N}$ is fuzzy t^*gp-open set iff $\mathcal{V} \leq \mathcal{N}^{\circ P}$ where $\mathcal{V}$ is a fuzzy t-set and$\mathcal{V} \leq \mathcal{N}$.

Proof.

(1) Let $\mathcal{N}$ be a fuzzy tgp-open set and $\mathcal{V}$ is a fuzzy t^*-set and $\mathcal{V} \leq \mathcal{N}$. Then$\mathcal{N}^c \leq \mathcal{V}^c$. Since $\mathcal{N}^c$ is a fuzzy tgp-closed and $\mathcal{V}^c$ is a fuzzy t-set, we have$\overline{\mathcal{N}^c}^P \leq \mathcal{V}^c$. Then $\leq \mathcal{N}^{\circ P}$.

Conversely, Let $\mathcal{V} \leq \mathcal{N}^{\circ P}$where $\mathcal{V}$ is a fuzzy t^*-set and $\mathcal{V} \leq \mathcal{N}$. Then$\mathcal{N}^c \leq \mathcal{V}^c$and $\mathcal{V}^c$ is a fuzzy t-set. Now $\overline{\mathcal{N}^c}^P \leq \mathcal{V}^c$, by hypothesis. Then$\mathcal{N}^c$ is a fuzzy tgp-closed set. Hence $\mathcal{N}$ is a fuzzy tgp-open.

(2) Similar part (1).

Proposition 4.23. If $\mathcal{N}$ and $\mathcal{M}$ are fuzzy subsets of any fts$(\mathcal{X}, \mathcal{T})$, we have:

(1) If $\mathcal{N} \leq \mathcal{M} \leq \overline{\mathcal{N}}^P$, and $\mathcal{N}$ is a fuzzy tgp-closed set, then $\mathcal{M}$ is also a fuzzytgp-closed set.

(2) If $\mathcal{N} \leq \mathcal{M} \leq \overline{\mathcal{N}}^P$, and $\mathcal{N}$ is a fuzzy t^*gp-closed set, then $\mathcal{M}$ is also a fuzzy t^*gp-closed set.

Proof.

(1) Suppose that$\mathcal{N} \leq \mathcal{M} \leq \overline{\mathcal{N}}^P$, so$\overline{\mathcal{N}}^P \leq \overline{\mathcal{M}}^P \leq \overline{\mathcal{N}}^P$, therefore $\overline{\mathcal{N}}^P = \overline{\mathcal{M}}^P$. Now, if $\mathcal{V}$ is a fuzzy t-set, such that $\mathcal{M} \leq \mathcal{V}$, so $\mathcal{N} \leq \mathcal{V}$ itfollows that $\overline{\mathcal{N}}^P = \overline{\mathcal{M}}^P \leq \mathcal{V}$, then $\mathcal{M}$ is a fuzzy tgp-closed set.

(2) Similar part (1).

Proposition 4.24. If $\mathcal{N}$ and $\mathcal{M}$ are fuzzy subsets of any fts $(\mathcal{X}, \mathcal{T})$, we have:

(1) If $\mathcal{N}$ is a fuzzy tgp-open set and $\mathcal{N}^{\circ P} \leq \mathcal{M} \leq \mathcal{N}$, then $\mathcal{M}$ is a fuzzytgp-open set.

(2) If $\mathcal{N}$ is a fuzzy t^*gp-open set and $\mathcal{N}^{\circ P} \leq \mathcal{M} \leq \mathcal{N}$, then $\mathcal{M}$ is a fuzzyt^*gp-open set.

Proof. The proof is straight from Proposition 4.23.

Proposition 4.25.

(1) The union of two fuzzy tgp-closed subsets of any fts$(\mathcal{X}, \mathcal{T})$is a fuzzy tgp-closed set.

(2) The union of two fuzzy t^*gp-closed subsets of any fts $(\mathcal{X}, \mathcal{T})$is a fuzzy$t^*gp$-closed set.

Proof.

(1) Consider that $\mathcal{N}$ and$\mathcal{M}$both of them are fuzzy tgp-closed sets. If$\mathcal{V}$ is any fuzzy t-set, such that $\mathcal{N} \vee \mathcal{M} \leq \mathcal{V}$, therefore $\mathcal{N} \leq \mathcal{V}and\mathcal{M} \leq \mathcal{V}$,so $\overline{\mathcal{N}}^P \leq \mathcal{V}$ and $\overline{\mathcal{M}}^P \leq \mathcal{V}$, we have $\overline{\mathcal{N} \vee \mathcal{M}}^P = \overline{\mathcal{N}}^P \vee \overline{\mathcal{M}}^P \leq \mathcal{V}$, hence$\mathcal{N} \vee \mathcal{M}$is a fuzzy tgp-closed set.

(2) Similar part (1).

Proposition 4.26. In any fts$(\mathcal{X}, \mathcal{T})$,

(1) The intersection of two fuzzy tgp-open subsets of any fts $(\mathcal{X}, \mathcal{T})$is fuzzy$tgp$-open.

(2) The intersection of two fuzzy t^*gp-open subsets of any fts $(\mathcal{X}, \mathcal{T})$is fuzzy$t^*gp$-open.

Proof. A proof is straight from Proposition 4.25.

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