

Optimal Design of Heavy Vehicle Suspension System Subjected to Random Road Input

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Abstract

Vehicle suspension system plays a vital role in diminishing the vibration caused by the road roughness and prevent it from transmitting to the driver and the passenger. In this paper conventional type suspension is used for process of optimization. The computational model of suspension is created through bond graph modeling. Further this model is simulated subjected to h1 and h2 type random road condition as suggested by ISO 8608. So that the performance can be done as real time inputs and typical Indian scenario. Optimized parameters of leaf spring is obtained through L₉ modified orthogonal array generated by Taguchi method. These parameters are obtained for reducing the total stress by minimizing the signal to noise (Signal/Noise) ratio. Further, ANOVA is applied to determine the significant process parameters, which are mostly influencing the total stress and deflection of the spring.

Keywords: leaf spring; bond graph modeling; Taguchi method; stress analysis

1. Introduction

Leaf spring is the elastic member, which has the primary function to connect the wheel to the body. It is deflected or distorted under loading and return to its original shape when the load is removed. Suspensions of the vehicle mostly viewed as interconnections of rigid bodies with compliance and kinematic joints elements such as bushings and springs. Suspension design requires description of the interconnection points and the characteristics of compliance elements.

Tak et al. [1] suggested an algorithmic approach of concluding optimum design of suspension, where suspension parameters are expressed in terms of camber, toe, and compliance system. Koulocheris et al. [2] have combined stochastic and deterministic formulations for determining optimum vehicle suspension parameters. They have confirmed that these combination gives reliable and faster convergence to the optimum. Mitchell et al. [3] adopted a GA (Genetic Algorithm) for optimizing the geometries of vehicle suspension involving the description of a suspension model. Raghavan [4] developed an algorithm to find out the connection point locations of the tie-rod of a vehicle suspension, in order to achieve linear toe change characteristics with jounce and rebound of the wheel. Eskandari et al. [5] modified the front suspension parameters by optimize the steering behavior of passenger car. They utilized an objective function combination of eight criteria indicating steering characteristics and reduced the quantity of parameters, by implementing the design of experiments method capabilities.

In this work quarter car model of heavy vehicle suspension system is modelled through bond graph modeling. Simulation of this model is carried at average of 60 kmph in two different type random condition prescribed by ISO 8608. The other focus of this work is to optimize the various parameters of leaf of heavy vehicle

system using Taguchi method and suggest the best possible design of suspension spring in each road condition.

2. Stress Analysis of Heavy Road Truck Suspension Spring

The spring of suspension system is effected by various forces as shown in Fig 1. Dynamic force ($F_{z,dynamic}$) caused due to the road roughness and generated in Z-direction, whereas static force ($F_{z,stat}$) is influenced by nominal axle load of vehicle in Z-direction. Braking force (F_x) caused due to braking acceleration (b) of the vehicle, and acting to the X-direction. The relation between braking force and force acting to the Z-directions is presented in equations. (1) [6].

$$F_x = \mu(F_{z,stat} + F_{z,dyn}) \quad (1)$$

where $\mu = \frac{b}{g}$ and b is braking deceleration ($b = 2 \text{ m/sec}^2$), μ is

adhesive friction coefficient ($\mu = 0.2$). The total stress (τ_t) acting on spring can be calculated as the sum of static (τ_{static}), dynamic ($\tau_{dynamic}$) and braking stress (τ_{brake}).

$$\tau_{total} = \tau_{static} + \tau_{dynamic} + \tau_{brake} \quad (2)$$

Static stress of the suspension spring can be evaluated through following relation [8,9].

$$\tau_{static} = \frac{6 F_{z,stat} L_a L_b}{n w t^2 (L_a + L_b)} \quad (3)$$

Where, $L_a + L_b = 1700$ (length of the spring), t is the thickness of the spring and w is width of the spring. Static force (F_{stat}) of the spring can be assumed as total vertical act on a suspension spring.

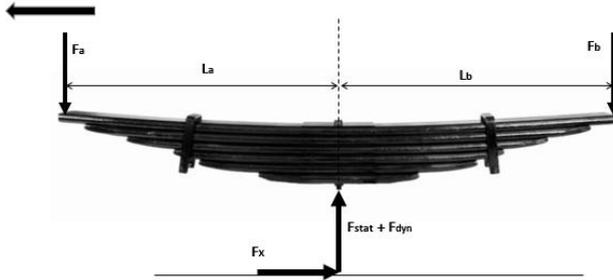


Fig 1: Forces acting on Leaf spring

Dynamic stress $\tau_{dynamic}$ occurs due to the road roughness and acting in the form of dynamic loads on the spring, which can be expressed as [2],

$$\tau_{z,dynamic} = \frac{F_{z,dyn} \tau_{stat}}{F_{stat}} \quad (4)$$

Similarly, the braking stress (τ_{brake}) can be evaluated through a relation;

$$\tau_{brake} = \frac{F_x \tau_{stat}}{F_{stat}}, F_x \text{ may be calculated from Eq. (1).}$$

The total stress can be calculated from Eq (2).

3. Model Formulation

The model considered in this work is a 2-degrees of freedom (DOF) quarter car model. A 2-DOF quarter car model consist with two masses as shown in Fig. 2. The top mass M_s , represents the vehicle body whereas the bottom mass, M_u , represents the tire. The parallel spring and damper combinations placed in between the vehicle body and the tire (k_s and c_d) represent the stiffness and damping of the suspension system. The tire stiffness is shown by the spring k_t . x_1 , x_2 and x_{in} are the vehicle displacement, wheel displacement and the road input to the quarter car model. According to Newton's law, the governing equation of the system can be represented as:

$$M_s \ddot{x}_1 + k_s(x_1 - x_2) + c_d(\dot{x}_1 - \dot{x}_2) = 0 \quad (5)$$

$$M_u \ddot{x}_2 - k_s(x_1 - x_2) - c_d(\dot{x}_1 - \dot{x}_2) + k_t(x_2 - x_{in}) = 0 \quad (6)$$

The same can be represented in the matrix form as:

$$\begin{bmatrix} M_s & 0 \\ 0 & M_u \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_d & -c_d \\ -c_d & c_d \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_s & -k_s & 0 \\ -k_s & k_s + k_t & -k_t \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_{in} \end{Bmatrix} = 0 \quad (7)$$

A bond graph model of the quarter car vehicle suspension system is developed in SYMBOLS Sonata® software [7]. The model is shown in Fig. 3.

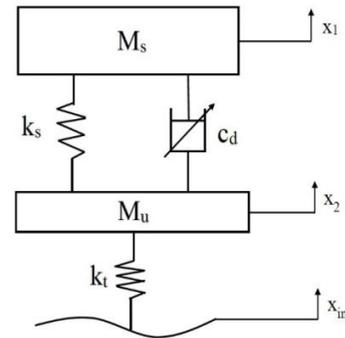


Fig. 2: 2-DOF quarter car model with controllable damper

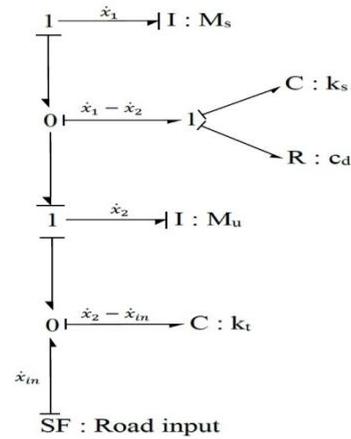


Fig. 3: BG model (Bond Graph) of 2-DOF quarter car suspension system

4. Numerical Simulation

Computer simulation can compress the performance of a system over years into a few minutes of computer running time. Simulation models are relatively flexible and can be modified to accommodate the changing environment to the real situation. At present, most of the simulation models are made by means of differential equations. In this research, quarter car suspension system are investigated at average speed of 60 kmph on four mentioned different road profile inputs using bond graphs technique and simulator of SYMBOLS-Sonata® and MATLAB/Simulink® software's are used.

The simulation of bond graph model is carried out for 10 sec to obtain different output responses. Total 1024 records are used in the simulation and error is kept in the order of 5.0×10^{-4} . Runge-Kutta Gill method of fifth order is used in present work to solve the differential equations generated through bond graph model. The parameters for the simulation of the 2-DOF quarter car model are shown in Table. The parameters used by Eryurek, [29] for a suspension model are used in this work.

4.1 Road Input

A random road profile is generated according to the International Organization for Standardization (ISO 8608). The ISO has suggested road roughness categorization using the power spectral density (PSD) values [8]. The artificial profile can be given as:

$$h(x) = \sum_{i=0}^N A_i \cos(2\pi \cdot n_i \cdot x + \phi_i) = \sum_{i=0}^N \sqrt{2 \cdot \Delta n \cdot G_d(i \cdot \Delta n)} \cdot A_i \cos(2\pi \cdot i \cdot x \cdot \Delta n + \phi) \quad (8)$$

where x is the abscissa variable from 0 to L ; Δn is $1/L$; n_{max} is taken as $1/B$; N is $\frac{n_{max}}{\Delta n}$ or L/B ; where $L=250$,

$N=100$; constant is denoted as k , depending from ISO road profile classification. In this work, four road conditions (H1-H4) are considered, which is shown in Figures.

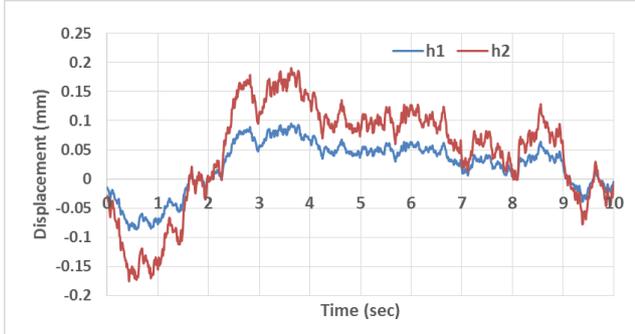


Fig 4: Road profiles at 60 kmph in four different road conditions as per ISO 8608

This road profiles are simulated for average speed of 60 kmph as shown in figure. It is evident from figure, H2 road condition is obtained maximum magnitude in an upward direction and minimum magnitude is identified at H1 road condition.

4.2 Simulation Results

The main objective of this work is to optimize all suitable parameters of leaf spring for heavy vehicle suspension system in order to improve the quality of the suspension can be enhanced and reduce the chances of failure. The simulation is carried out for two type road condition (h1 and h2) and the results are obtained in terms of total stress and spring deflection with respect to time.

5. Optimization

Present study used three significant parameters of suspension system. Entire parameter configuration is simulated for two types of road conditions with minimum count of simulation runs using Taguchi’s modified L9 orthogonal array. Table 1 summarizes the parameters and their range used for simulation runs. And Table 2 is formed as L9 orthogonal array (modified form) with seven columns and nine rows. Total degrees of freedom (DOF) is required to create a suitable orthogonal array and it must be higher than or equal to design parameters [9]. Therefore, total degrees of freedom for this study is 9. Total 9 parameter combinations are available whereas column of the orthogonal array assigned with each parameter of suspension. In this way, total 9 simulation runs are counted to investigate the whole parameter space by L9 improved orthogonal array.

A total of 18 simulation runs, two type of road conditions for every set of factors are simulated on a Matlab/Simulink/Symbols to determine their total stress and deflection. Table 3 & 4 summarized the spring deflection and total stress corresponding to each run for h1 and h2 road conditions respectively.

Table 1: Level of each of parameter

S. No	Factor	Levels of each parameter		
		Level 1	Level 2	Level 3
1	Thickness	35	40	45
2	No. of leaves	6	7	8
3	Width of spring	75	80	85

Table 2: Taguchi L9 modified orthogonal array

Experiment No	Columns		
	Factor 1	Factor 2	Factor 3
1	1	1	1
2	1	2	2
3	1	3	3
4	2	1	2
5	2	2	3
6	2	3	1
7	3	1	3
8	3	2	1
9	3	3	2

5.1 Analysis using Signal/Noise ratio

According to Taguchi method, Signal/Noise ratio is a deviation from the desired value and measure of quality characteristics. The ‘signal’ and noise shows the ‘mean desirable value’ and the ‘undesirable value’ (standard deviation from mean) respectively for the characteristic of output [10]. Further, signal/Noise ratio (μ) may be represented as

$$\mu = 10 \log (M.S.D) \tag{9}$$

Whereas, stress is undesired phenomenon in leaf spring so it has to be minimized and deflection of the leaf spring should also be minimized for good ride comfort. Thus, smaller the better value of Signal/Noise ratio is selected as a quality criterion in the analysis for reducing the total stress of the spring. This value can be obtained by:

$$M.S.D = \frac{1}{n} \sum_{i=1}^n S_i^2 \tag{10}$$

where, n =the total number of the simulation runs in the orthogonal array and S_i is the total stress and deflection for the i th simulation run.

S/N ratio for each of the experiments is calculated and is presented in Table 3 and Table 4. The effect of a factor level is defined as the deviation it causes from the overall mean. The overall mean Signal/Noise ratio (β) of the runs is calculated by

$$\beta = \frac{1}{n} \sum_{i=1}^n \mu \tag{11}$$

where, β is the mean Signal/Noise ratio of the i th experiment. Every factor of all levels are similarly denoted in the 16 experiments. β can be presented as balanced overall mean for complete project. Since the simulation run array is orthogonal, it is viable to discretize the outcome of each aspect at every level. Output responses in the form of Signal/Noise ratio, total stress and deflection for each parameter at each level may be evaluated through mean Signal/Noise ratio, total stress and deflection of each of the experiment. Based on the Signal/Noise ratio analysis (Fig 5,6,7 and 8) and mean Signal/Noise ratio investigation, the optimum parameters for process to minimize total stress and deflection are obtained and presented in Table 5.

5.2 ANOVA (Analysis of Variance)

Analysis of variance is an extensively used statistical technique to interpret the simulation results. It is extensively used to identify the performance of group of parameters under investigation. The main purpose of this technique is to define the parameters, when combination of total variation is significant. Total sum of squares (SQ_T) can be calculated by [10].

Table 3: Signal/Noise ratio, total stress (MPa) and deflection for h1 road condition for each of the experiment

Exp no	Bidth of spring (w,mm)	Thickness of Leaf (mm)	No.of full leaves	DEFLECTION (mm)	Total stress (σ)	S/n ratio for deflection	S/n ratio for total stress
1	75	35	6	0.0087201	1124.77047	41.19	61.02
2	80	35	7	0.0071956	1082.81033	42.86	60.69
3	85	35	8	0.005185	891.726151	45.71	-
4	80	40	6	0.005741	967.197765	44.82	59.71
5	85	40	7	0.0036417	715.776053	48.77	57.10
6	75	40	8	0.0025906	581.917333	51.73	55.30
7	85	45	6	0.0025369	540.925335	51.91	54.66
8	75	45	7	0.0021124	525.470325	53.50	54.41
9	80	45	8	0.0016828	478.417995	55.48	53.60

Table 4: S/N ratio, total stress (Mpa) and deflection for h2 road condition for each of the experiment

Exp no	Bidth of spring (w,mm)	Thickness of Leaf (mm)	No.of full leaves	DEFLECTION (mm)	Total stress (σ)	S/n ratio for deflection	S/n ratio for total stress
1	75	35	6	0.011310287	1458.86071	38.93	-63.28
2	80	35	7	0.007195573	1082.81033	42.86	-60.69
3	85	35	8	0.005185045	891.726151	45.70	-59.00
4	80	40	6	0.005741048	967.197765	44.82	-59.71
5	85	40	7	0.003641718	715.776053	48.77	-57.10
6	75	40	8	0.002875269	645.864293	50.83	-56.20
7	85	45	6	0.002815712	600.367679	51.01	-55.57
8	75	45	7	0.002344511	583.214317	52.60	-55.32
9	80	45	8	0.001599519	454.733935	55.92	-53.16

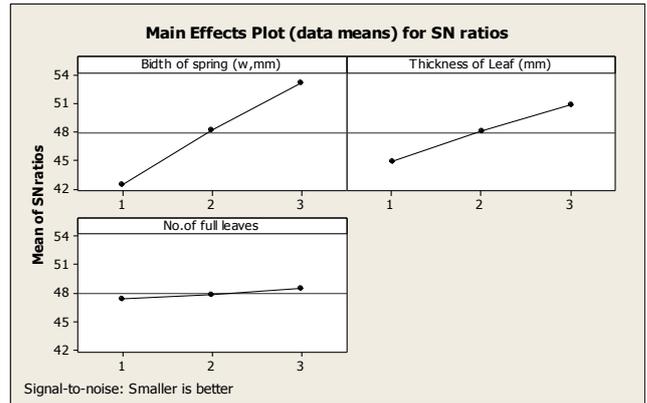


Fig 7: Signal/Noise ratio analysis of total stress of leaf spring at 60kmph in h2 road condition

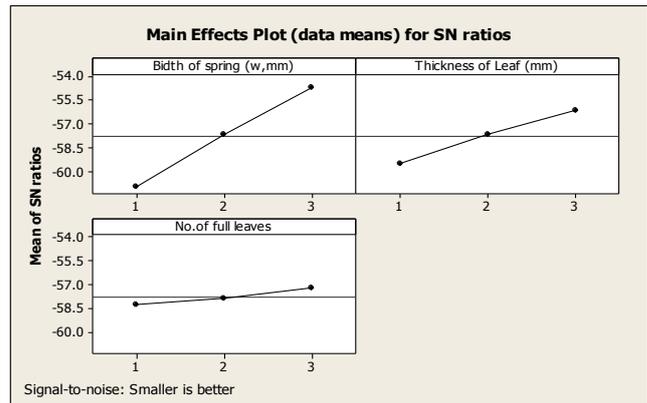


Fig 8: Signal/Noise ratio analysis of deflection of leaf spring at 60kmph in h2 road condition

Table 5: Optimum values of parameters for minimizing the total stress and spring deflection in both type road conditions

S. No.	Parameter	H1-road condition		H2-road condition	
		Magnitude	Unit	Magnitude	Unit
1	Width of spring	75	mm	75	mm
2	Thickness	35	mm	35	mm
3	No. of leaves	7	---	6	---

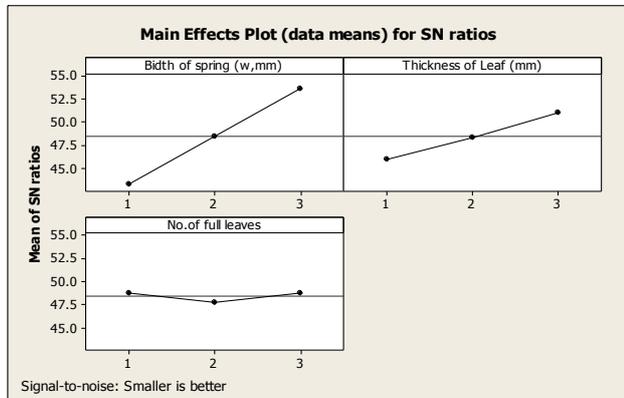


Fig 5: Signal/Noise ratio analysis of total stress of leaf spring at 60kmph in h1 road condition

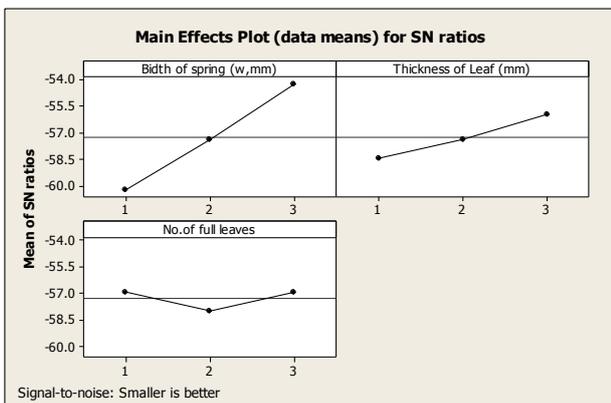


Fig 6: S/N ratio analysis of deflection of leaf spring at 60kmph in h1 road condition

$$SQ_T = \sum_{i=1}^n (\mu - M)^2 \tag{12}$$

Where, M is the overall mean of Signal/Noise ratio.

$$SQ_r = SQ_e + \sum_{j=1}^{n_p} SQ_j \tag{13}$$

$$SQ_j = \sum_{i=1}^l (\mu_{ji} - M)^2 \tag{14}$$

In eq (14), μ_{ji} = number of important factors and

l = number of levels

SQ_e = sum of squared error without or with pooled factor,

The process response majorly influenced by the factor then F-value is high. In the present work F-values obtained for the total stress and deflection. The F-value for h1 road condition is presented in Table (7 and 9) and for h2 road condition is presented in Table (11 and 13).

Table 6: Analysis of Variance for DEFLECTION (mm) at 60 kmph in h1 road condition

Source	DF	Seq SS	Adj SS	Adj MS	F	P	Percentage contribution
Bidh of spring (w,mm)	2	0.0000018	0.0000018	0.0000009	3.63	0.216	3.68852459
Thickness of Leaf (mm)	2	0.0000370	0.0000370	0.0000185	74.26	0.013	75.81967213
No.of full leaves	2	0.0000095	0.0000095	0.0000047	19.04	0.050	19.46721311
Error	2	0.0000005	0.0000005	0.0000002			1.024590164
Total	8	0.0000488					

Table 7: ANOVA table for regression model for deflection in h1 road condition

Source	DF	SS	MS	F	P
Regression	3	0.000046533	0.000015511	33.83	0.001
Residual Error	5	0.00002292	0.00000458		
Total	8	0.000048825			

Table 8: Analysis of Variance for total stress at 60 kmph in h1 road condition

Source	DF	Seq SS	Adj SS	Adj MS	F	P	Percentage contribution
Bidh of spring (w,mm)	2	26576	26576	13288	4.87	0.170	5.18
Thickness of Leaf (mm)	2	403468	403468	201734	74.00	0.013	78.65
No.of full leaves	2	77477	77477	38738	14.21	0.066	15.10
Error	2	5452	5452	2726			1.06
Total	8	512973					

Table 9: ANOVA table for regression model for total stress in h1 road condition

Source	DF	SS	MS	F	P
Regression	3	481165	160388	25.21	0.002
Residual Error	5	31808	6362		
Total	8	512973			

$S = 79.7591$ $R-Sq = 93.8\%$ $R-Sq(adj) = 90.1\%$

Table 10: Analysis of Variance for DEFLECTION (mm) at 60 kmph in h2 road condition

Source	DF	Seq SS	Adj SS	Adj MS	F	P	Percentage contribution
Bidh of spring (w,mm)	2	0.0000040	0.0000040	0.0000020	1.49	0.402	5.38
Thickness of Leaf (mm)	2	0.0000497	0.0000497	0.0000249	18.39	0.052	66.80
No.of full leaves	2	0.0000179	0.0000179	0.0000090	6.63	0.131	24.06
Error	2	0.0000027	0.0000027	0.0000014			3.63
Total	8	0.0000744					

Table 11: ANOVA table for regression model for deflection in h2 road condition

Source	DF	SS	MS	F	P
Regression	3	753692	251231	43.02	0.001
Residual Error	5	29197	5839		
Total	8	782888			

$S = 76.4153$ $R-Sq = 96.3\%$ $R-Sq(adj) = 94.0\%$

Table 12: Analysis of Variance for total stress at 60 kmph in h2 road condition

Source	DF	Seq SS	Adj SS	Adj MS	F	P	Percentage contribution
Bidh of spring (w,mm)	2	39129	39129	19564	2.55	0.282	5.00
Thickness of Leaf (mm)	2	546576	546576	273288	35.63	0.027	69.82
No.of full leaves	2	181844	181844	90922	11.86	0.078	23.23
Error	2	15338	15338	7669			1.96
Total	8	782888					

Table 13: ANOVA table for regression model for total stress in h2 road condition

Source	DF	SS	MS	F	P
Regression	3	753692	251231	43.02	0.001
Residual Error	5	29197	5839		
Total	8	782888			

$S = 76.4153$ $R-Sq = 96.3\%$ $R-Sq(adj) = 94.0\%$

5.3 Empirical Model Derivation

Empirical model for this study is derived through linear regression formulation. The obtained regression expressions is presented below.

For h1 road condition

The regression equation is

Deflection (mm) = 0.0384 - 0.000069 Bidh of spring (w,mm) - 0.000492 Thickness of Leaf (mm) - 0.00126 No.of full leaves

The regression equation is

Total stress (σ) = 3858 - 2.79 Bidh of spring (w,mm) - 51.8 Thickness of Leaf (mm) - 113 No.of full leaves

For h2 road condition

The regression equation is

Total stress (σ) = 5702 - 16.0 Bidh of spring (w,mm) - 59.8

Thickness of Leaf (mm) - 172 No.of full leaves

6. Conclusions

In the current work, correlation in the middle of the total stress and a most significant factors namely width, thickness, and no of leaves of the spring is investigated. The outcome of present study will support to recognize concerns by changing these factors in a ride comfort level of the driver and also road holding capability of wheels. The computational model of quarter car model was run for real parameters of leaf spring of truck and obtained 9 responses for different configuration of parameters. So that this study can be extended to implement a Taguchi methodology for obtaining an optimal setting parameters for quarter car suspension model.

Optimal suspension parameters for two different road conditions as per ISO 8608 obtained through Taguchi methodology is presented in Table 5.

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