

An Efficient Hybrid Approach in Solving the Multiple Postman Problem

Asaad Shakir Hameed¹, Burhanuddin Mohd Aboobaid¹, Ngo Hea Choon¹,
Wassim Habib Bilal², Modhi Lafta Mutar¹

¹Faculty of Information and Communication Technology, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100, Durian Tunggal, Melaka, Malaysia.

²Faculty of Science, Tishreen University, Lattakia, Syria.

*Corresponding Author Email: asaadutem@yahoo.com

Abstract

In this study, the proposed algorithm contributed to the enhancing of solutions of the Multiple Postman Problem m-PP, which is one of the optimization problems that, has attracted a lot of attention at the present time. It is a problem of the NP-hard type. However, because of the complication of polynomial time, there is still no algorithm providing us with the optimal solution of this problem. All the used algorithms give solutions that are close to the optimal one. In this research, we will present the Hybrid Algorithm HA in two phases. In the first phase the Sweep Algorithm SW is applied, and in the second one, the Ant Colony Algorithm and the local search 3-opt are applied. we will then compare the quality of the solution resulted from this hybrid approach with the results of well-known standard tests to determine the effectiveness of the presented approach.

Keywords: Traveling Salesman Problem; Sweep Algorithm; 3-opt local search; Ant Colony Algorithm.

1.Introduction

The Multiple Postman Problem can be described as one of the most difficult optimization problems that were generalized to the TSP which has the same limitations and objective of the Traveling Salesman Problem TSP, thus it is NP-hard, [1] are the first to develop its mathematical model that can be defined as follows: we have m postman located in a single distribution center, and its mission is to serve a set of customers, and each customer is visited once per tour and by one postman, and each customer is a specific distance from the sole distribution center, with the aim of finding the lowest cost tours with observe for all the following restrictions: [2], [3].

- All tours must begin and end in the main mail center;
- Each customer serves once through a single postman.

The Multiple Postman Problem can be represented by a complete graph weighted directed or non-directed $G = (V, E)$ where $n = |V|$ The set of nodes and $V = \{0, 1, 2, \dots, n\}$ set the nodes and $E = \{(i, j) : i, j \in V, i \neq j\}$ the set of ribs that connect between nodes fully and $D = (d_{ij})$ the distance matrix among each the nodes and the only distribution center and the distance between every two nodes i and j is calculated in the Euclidean space as in the relationship $d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$, knowing that each $(i, j) \in E$ is associated with the cost of d_{ij} , and the distances between the nodes in the graph are symmetrical euclidean distances and $C = (c_{ij})$ the cost matrix and the two matrices are linked to E, assuming that the cost $c_{ij} = c_{ji}$ when the multiple postman problem is symmetrical and when it is asymmetrical $c_{ij} \neq c_{ji}$ and $c_{ij} = \infty$ and the cost function is $C: E \rightarrow Z^+$ and node 0 represents the main

center, and the requirement for main distribution center $d_0 = 0$, and the remaining nodes represent the requests of customers and each customer i has a request of a non-negative weight and is represented by the request function $d: V \rightarrow Z^+$ and m the set of symmetrical vehicles located in the distribution center, and the objective function is expressed in the following relationship [2], [3].

$$\min z = \sum_{v \in V} \sum_{(i,j) \in E} c_{ij} x_{ij}^v$$

Figure 1 shows the solution of the multiple postman problem for a small number of postal centers.

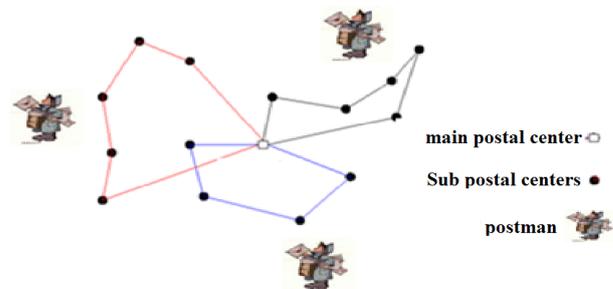


Figure 1: Solution of the multiple postman problem

The multiple postman problem that we address in this study is of the NP-hard category and all the algorithms used to provide solutions close to the optimal solution, and there is still no algorithm that provides the optimal solution for this problem [4].

There are three types of algorithms were used to solve the multiple postman problem:

a) Exact algorithms

They are suitable algorithms for cases of relatively small measurement, and cases of problems cannot be resolved for more than 50 sub-postal centers in a reasonable period of time because they are of the NP-hard type difficult to solve even using computers and examples of exact algorithms [4], such as: Branch & Cut Algorithm and Branch, Cut & Price algorithm.

b) Approximate algorithms

Are algorithms that use heuristic and metaheuristic with repetitive improvements for large-scale problems and are divided into two main groups [5], [6], as follows:

• **Heuristic Algorithms**

Are algorithms that use heuristic methods to accelerate the finding of a convincing solution to an issue because comprehensive research is impractical, but it is not effective to escape local optimization and there is a large gap in the resulting solutions compared to the best known solutions, such as the local search algorithm [5], [6], [7].

• **Metaheuristic Algorithms**

Is a class of algorithms and techniques that use a degree of randomization to find the best solutions to difficult problems, and is more general than the other types of algorithms, it is a calculation method improve a candidate solution to the issue repetitively the solution is related to a certain measure of quality optimization as one of the appropriate methods and is applied to a very wide set of problems thus, such algorithms do not guarantee the optimal solution. This approach is very effective to escape local optimization. It is one of the best set of algorithms to solve optimization problems, such as ACO colony algorithm and genetic algorithm (GA) [8], [9], [7].

c) Hybrid Algorithms

Many researchers have recently found that the use of hybridization in optimization problems can improve the quality of solutions that can be found in comparison to the heuristic and metaheuristic approaches [8], [10], [11], [9], [7]Despite the large and significant differences between these algorithms, they share some elements that have been ignored and left untapped. We will use hybrid algorithms to focus on strengths and to compensate for weaknesses in previous approximation algorithms. The goal is to cleverly combine the main elements of competing methodologies to find a superior solution [8], [9], [7].

The aim of this study, a hybrid algorithm that integrates the Sweep algorithm and the ACO colony algorithm with the 3-opt local search algorithm has been proposed to enhance the Multiple Postman Problem solutions, the proposed algorithm has been compared with standard results. The rest of the paper is organized as follows: section 2 has been presented with the classification of the Multiple Postman Problem, then, the Formulation of the Multiple Postman Problem has been introduced in section 3, next, Methods have been presented in section 4, on the other hand, we discuss the experiment result in section 5, finally, the Conclusion and future studies have been given in section 6.

2. Classification of the Multiple Postman Problem

The multiple postman problem is of the NP-hard type, and the complexity of vehicle routing problem has been studied by the world [12]. [13] have been proved multiple postman problem is unsolvable in polynomial time and this means that the solution time for the multiple postman problem is steadily growing with the number of nodes increasing [5], [6]. Thus, there are no efficient algorithms to solve the multiple postman problem so far, and any

increase in the number of nodes will exponentially increase their calculation time.

Assumptions of the Problem

- a) The messages are separate and ready within sets and the postman moves it to the centers and distributed by the centers;
- b) Each postman starts from all sub-centers and each center will be visited only once and then the postman will return to the main postal center from which he started;
- c) Each postman uses different means of transportation to perform his mission (private car, public transport, bicycle ...);
- d) The cost of moving back and forth between center and other is the same;
- e) The multiple postman problem has the same limitations and objective of the traveling salesman problem.

3-Formulation of the Multiple Postman Problem

We have a $G = (V; E)$ directed or non-directed graph where V is the set of nodes, E the set of ribs Which connects between nodes in full and $c = (c_{ij})$ is a cost matrix and both are linked to E . And x_{ij} is a variable corresponds to the rib that connects node i with node j and c_{ij} , the weight that represents li the length of that rib, and the mathematical model of the multiple postman problem was developed by [1], [2], [3] As follows:

$$x_{ij}^k = \begin{cases} 1 & \text{if vehilek goes from i to j} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$\min z = \sum_{i=0}^n \sum_{j=0}^n \sum_{k=1}^m c_{ij} x_{ij}^k \quad (2)$$

Subject to the following restrictions:

$$\sum_{i=0}^n \sum_{k=1}^m x_{ij}^k = 1 \quad \forall j \in \{1, \dots, n\} \quad (3)$$

$$\sum_{j=0}^n \sum_{k=1}^m x_{ij}^k = 1 \quad \forall i \in \{1, \dots, n\} \quad (4)$$

$$\sum_{i=0}^n x_{ip}^k - \sum_{j=0}^n x_{pj}^k = 0 \quad \forall p \in \{1, \dots, n\}, k \in \{1, \dots, K\} \quad (5)$$

$$\sum_{j=0}^n q_j (\sum_{i=0}^n x_{ij}^k) \leq Q \quad \forall k \in \{1, \dots, K\} \quad (6)$$

$$\sum_{i=0}^n \sum_{j=0}^n t_{ij} x_{ij}^k \leq L \quad \forall k \in \{1, \dots, K\} \quad (7)$$

$$\sum_{j=1}^n x_{0j}^k \leq 1 \quad \forall k \in \{1, \dots, K\} \quad (8)$$

$$\sum_{i=1}^n x_{i0}^k \leq 1 \quad \forall k \in \{1, \dots, K\} \quad (9)$$

$$x_{ij}^k \in \{0,1\} \quad \forall i, j \in \{1, \dots, n\}, k \in \{1, \dots, K\} \quad (10)$$

Where:

Equation (2) represents the target trajectory that reduces the total distance traveled.

Equations (3) and (4) represent the degree of constraints.

While equation (5) expresses continuity to direct the postman to the roads imposed on each postman to visit and leave each node once. Constraint (6) indicates that customers' requests do not exceed the capacity of the postman and constraint (7) on the maximum length

of the road. Constraints (8) and (9) guarantee each customer service once and through one postman departing and returning to the main distribution center. The constraint (10) is a binary constraint so that $x_{ij} = 1$ and $x_{ij} = 0$ otherwise.

4- Method

In this section, a hybrid algorithm has been proposed to solution the multiple postman problem as follows:

4.1 Sweep Algorithm (SW)

The Sweep Algorithm (SW) has been suggested by [14] for Euclidean networks that classify and bind the demand contract at their polar angle as shown in Figure 2.

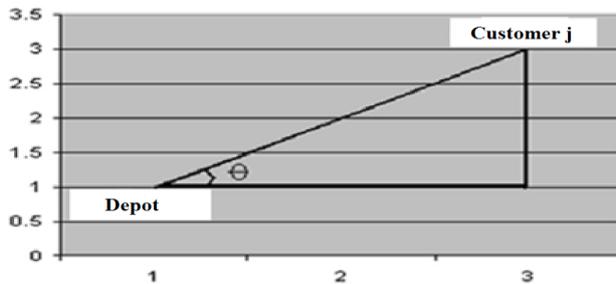


Figure 2: Binding of the demand nodes to the polar angle

The polar angle is calculated as follows:

$$An(i) = \arctan \left\{ \frac{(y(i) - y(0))}{(x(i) - x(0))} \right\}$$

Notes that if it is $y(i) - y(0) < 0$ and $y(i) - y(0) \geq 0$

$$\Rightarrow -\pi < An(i) < 0 \text{ and } 0 \leq An(i) \leq \pi \text{ respectively,}$$

this heuristic algorithm is one of the most famous strong heuristic algorithms [5], [6]. At first, the polar coordinates are calculated for all customers, and the depot is the node so that $An(1) = 0$, then moves (clockwise or counterclockwise) and starts from customer i who has not visited from the smaller angle to the larger corner until All customers are included in the tour.

$$0 = An(1) \leq An(2) \leq \dots \leq An(n)$$

4.1.1 Sweep Algorithm Steps (SW)

The sweep algorithm is a way of grouping sub-centers into sets so that sub-centers in one set that are geographically close together can be served by the same postman [8], [9], [7], and uses for that the following steps:

Pseudo Code the Sweep Algorithm for CVRP

1. Locate the depot as the center of the two-dimensional plane.
2. Compute the polar coordinates of each customer with respect to the depot.
3. Start sweeping all customers by increasing polar angle.
4. Assign each customer encompassed by the sweep to the current cluster.
5. Stop the sweep when adding the next customer would violate the maximum vehicle capacity.
6. Create a new cluster by resuming the sweep where the last one left off.
7. Repeat steps 4 – 6, until all customers have been included in a cluster.

4.2 The Ant Colony Optimization Algorithm (ACO)

The ant colony optimization algorithm is a random search algorithm, based on the behavior of natural ants and search the behavior of ants. This is part of the so-called swarm intelligence, which was proposed by Italian scientists [15], [10], [11]. Each ant

is placed in the randomly selected nodes. The ant K in node i is chosen to move to node j according to the following rule:

$$j = \begin{cases} \arg \max_{j \in U} \{ [\tau_{iu}]^\alpha [\eta_{iu}]^\beta \} & \text{if } q < q_0 \\ J & \text{otherwise} \end{cases} \quad (1)$$

Where $\tau_{iu}(t)$ is amount pheromone On the rib (i, u) , η_{iu} a variable denotes the value chosen proportionally in inverse to the distance between nodes i, u and α, β is an adjustable Positive parameters and controls the relative weights of the effect of the pheromone and heuristic information, respectively representing the density of the pheromone that leads the ant The choice of its way and the relationship of choosing the road in its length where $\alpha, \beta > 0$. If $\alpha = 0$, the nearest contract is likely to be selected, either if $\beta = 0$, the amplification of the pheromone will lead to a rapid emergence of the recession that makes the ants follow the same path and thus build the same solution, and generate of ants tours is not optimal, and q_0 parameter where $0 \leq q_0 \leq 1$, and q random number achieves $0 \leq q \leq 1$.

$u \in allowed_k(t)$ set of nodes that not visited by the ant K and J The random variable that was determined according to the probability transition rule:

$$J: p_{ij}^k(t) = \begin{cases} \frac{[\tau_{ij}(t)]^\alpha [\eta_{ij}]^\beta}{\sum_{i \in J_k(i)} [\tau_{il}(t)]^\alpha [\eta_{il}]^\beta} & \text{if } j \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Clearly, this strategy increases the potential for diversity of research and thus avoids the incidence of local optimization [16], [10], [11]

• Local update law

The pheromone is updated after the ants complete their tour according to the law:

$$\tau_{ij}(t+1) = (1 - \rho) \cdot \tau_{ij}(t) + \rho \tau_0 \quad (3)$$

- Where ρ is the evaporation parameter and $0 < \rho < 1$

$$\tau_0 = \frac{1}{nl_{mn}}$$

And $\tau_0 > 0$ is parameter represents the initial value of the effect of the pheromone, n the number of nodes, and l_{mn} is the cost produced by the nearest heuristic neighbor [10], [8], [11], [9]. The local pheromone update base helps to explore unused routes in order to produce different solutions and avoid local optimal. This is the main objective of the local pheromone update that makes the road less desirable for the next ants.

• Global update law

The level of the pheromone is updated after all the ants build their tour, and choose the best roads that belong to the best global tour of the ant according to the following rule:

$$\tau_{ij}(t+1) = (1 - \rho) \cdot \tau_{ij}(t) + \rho \Delta \tau_{ij}(t) \quad (4)$$

$\Delta \tau_{ij}(t)$ is given in the following relationship:

$$\Delta \tau_{ij}(t) = \sum_{k=1}^m \Delta \tau_{ij}^k(t) \quad (5)$$

Where m is the number of ants, $\Delta \tau_{ij}^k(t)$ the sum of the pheromone left by the ant K between node i and node j , at time t .

$$\Delta \tau_{ij}(t) = \begin{cases} \frac{1}{L_{gb}} & \text{if } (i, j) \in \text{best global tour of the ant} \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

$\frac{1}{L_{gb}}$: The best global tour to be found until now [16], [10], [8], [17], [11], [9]

4.2.1 Steps of Ant Colony Optimization Algorithm (ACO)

Pseudo Code Canonical ACO for CVRP

1. initialize pheromones
2. repeat
3. for $k = 1$ to number of ants m do
4. start tour of ant k
5. for $i = 1$ to number of reachable nodes n do
6. choose next node of tour
7. end for
8. local search method (optional)
9. local pheromone update (optional)
10. end for
11. evaluate tours
12. update best solution found so far
13. global pheromone update
14. until stop condition
15. return best solution found

4.3 Local Search Algorithm 3-opt

Is a simple local search algorithm, it was used to solve TSP and network problems, including the deletion of three ribs of a network or a tour, and reconnecting them to three alternative ribs to form a network of all other possible ribs in order to find optimal tours, then repetition this process to a different set of three other ribs, and there can be referred to the existence of many ways to tie the nodes and produce the tour again while respecting the limitations of the problem [18], [19], [7].

4.3.1 Steps of the local search algorithm 3-opt for CVRP

Pseudo Code 3-opt for CVRP

Input : unimproved Tour

Output : Tour improvement

1. def three opt(tour):
- "Iterative improvement based on 3 exchange."
2. $\delta = 0$
3. for (a, b, c) in all segments($len(tour)$)
4. $\delta = reverse_segment_if_better(tour, a, b, c)$
5. if $\delta < 0$:
6. return three opt(tour)
7. return tour

4-4 Proposed a Hybrid Algorithm

Building algorithms produce a practical initial solution on their own, and optimization algorithms can improve solutions by having a workable solution, knowing that the building algorithm is the Sweep algorithm (SW), and the optimization algorithms are the ant colony optimization algorithm ACO and the local search algorithm 3-opt. In this hybrid algorithm, we will apply the ant colony optimization algorithm ACO to improve the route for each postman, where the nodes of each postman remain unchanged. On the other hand, the 3-opt local search algorithm is used to change the nodes and improve the route for each postman. The hybrid algorithm is divided into two:

Section I: To improve the postman's road without changing the nodes.

Section II: The process of improving the postman by changing the nodes.

Where an ant colony algorithm works to improve the postman's road without changing the nodes and the local search algorithm 3-

opt to improve the postman's road by changing the nodes through which it passes as in figure 3.

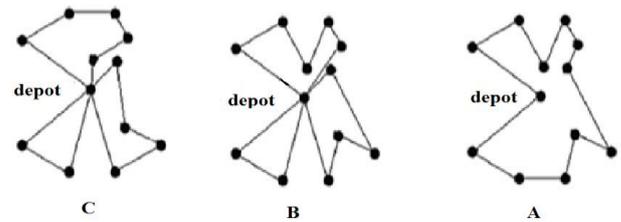


Figure 3: (A, B) Represent the application of the sweep algorithm (SW) and (C) represents the application of the ant colony algorithm for postman constraints

Through this approach

First: the nodes that should be visited from the postmen in the distribution center are arranged by the Sweep algorithm (SW) and then assigned to a destination.

Second: Cases which was done obtained be preserved despite changing their acquired nodes, and for each destination, the postman begins to move from the distribution center and visits the nodes in the other matrices. This means that if the postman's load is greater than his capacity, return to the warehouse and repeat the steps until there are no nodes did not be visited. The ant colony algorithm is applied to each road until we get the best road. When applying the 3-opt local search algorithm 3-opt, we get further optimization of the algorithm's performance. Figure 4 shows how to obtain an improved solution by applying the local search algorithm 3-opt.

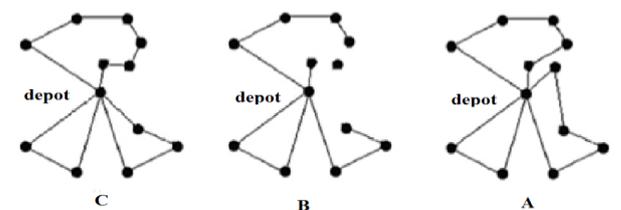


Figure 4: (A) Represents the solution unimproved. (B) represents the deletion of three ribs. (C) represents the solution improved.

4.4.1 Steps of Proposed a Hybrid Algorithm

The Hybrid Algorithm for multiple Postman Problem (m-PP)

Pseudo Code.

1. Let m number of Postman.
2. Generate random solution s^* // Represents the best solution that exists so far.
3. $f^* = \infty$ // f^* Represents the best value for a solution to be obtained.
4. Set parameters and initialize the pheromone trails.
5. Use the SW algorithm for the node requisition.
- Which must be visited by Postman from the main center called R.
6. put cases for R obtained in matrix T.
7. For $i=1$ TO length (T).
8. Use the SW algorithm for $T(i, :)$ to produce a solution called $T^*(i, :)$
9. For each Postman route we apply the ACS colony algorithm
10. If the new solution is better than $T^*(i, :)$, replace with the new best solution.
11. Apply the 3-opt local search algorithm for $T^*(i, :)$.
12. if $f(T^*(i, :)) < f^*$ then
13. $f^* = f(T^*(i, :))$;

14. $s^* = T^*(i, :);$
15. Save the best solutions until the end of the conditional loop.
16. End for i .
17. Return s^* & f^* as the best Solution.
18. End.

5. The Experimental Results

The experimental results of the proposed hybrid algorithm have been carried out by using C ++, and we performed computer experiments using a corei7 processor and 4 GB of RAM. Our results were compared with the standard inputs and results known in scientific researches VRPLIB [21, 22, 23,24,25]. The accuracy of the algorithm has been calculated by using the value of the gap, the gap calculated as follows:

$$\text{Gap} = \frac{\text{ProposedAlgorithm} - \text{Best Known Standard Results}}{\text{Best Known Standard Results}} \times 100$$

The smaller the gap, the more robust the evaluated algorithm. Table 1 below show the results obtained by the proposed algorithm as follows:

Table 1 Experimental results of the proposed algorithm

Number of nodes	Results Known Standard [20]	Proposed algorithm	Gap%
75	1,618	1,618	0
75	1,344	1,344	0
75	1,291	1,295	0.31
75	1,365	1,380	1.10
100	2,041	2,050	0.44
100	1,939	1,940	0.052
100	1,406	1,411	0.36
100	1,581	1,585	0.25
150	3,055	3,055	0
150	2,656	2,732	2.86
150	2,341	2,364	0.98

The results in above Table show that the proposed algorithm finds optimal solutions for three cases out of 11 known cases, and their results represent a competitive approach compared to the known standard results, and in this case, the performance of the high-accuracy algorithm has a gap 0%, on the other hand, in the remaining eight cases out of 11 cases, the proposed algorithm performed was good and the gap was limited to more than 0% and less than 3%. The algorithm was able to escape the local optimum points, and it improved the solutions and succeeded in reducing the total distances as shown in the experimental results in Table 1. Figure 5 shows a comparison the results between our proposed with known standard results as follows:

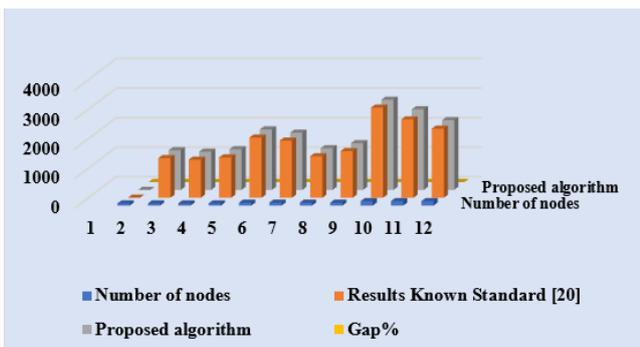


Figure 5: Comparison of results from Table 1

The experimental results in Table 2 below show that the proposed hybrid algorithm that integrates the sweep algorithm (SW) and the ant colony optimization algorithm ACO with the local search

algorithm 3-opt is more efficient than the ant colony algorithm alone, especially for large-scale problems and give better results within a reasonable time, and the results showed that the efficiency of the proposed hybrid algorithm is better than the algorithms that were compared with it where the performance increased and reduced the cost to solve. The gap was small between this algorithm and record results are known. On the other hand, the limitation of this study, the convergence of the proposed algorithm was guaranteed, but the time to convergence uncertain, the coding was not straightforward and needs more parameters for the setting.

Table 2 Summary compared the proposed algorithm with Ant colony algorithm

A standard example	Proposed algorithm	Ant colony algorithm (ACO) [20]
E-n51-k5	0	0
E-n76-k10	0.72	5.41
E-n101-k8	2.97	3.68
E-n101-k10	0.46	2.20
E-n121-k7	0.76	14.10
M-n151-k12	0.20	7.45
M-n200-k17	2.65	24.36

Figure 6 below show the compare between the proposed algorithm and the ant colony algorithm [20] from Table 2.

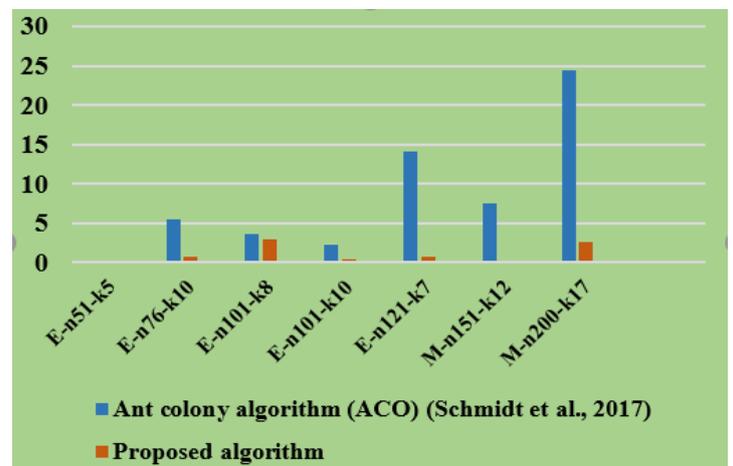


Figure 6: Comparison of results from Table 2

6. Conclusion

In this work, the multiple postman problems have been addressed, depending on the hybrid algorithms and their impact on the quality of the solution, and the experimental results confirmed that the proposed algorithm is an effective tool to improve the solution and is able to perform better in terms of speed of convergence and the ability to find better solutions than the heuristic and metaheuristic algorithm, however, the development of hybrid algorithms offers an area that was not well exploited by experimental research and in order to exploit it well, the following can be recommended:

1. Apply the proposed hybrid algorithm to other types of routing problems;
2. Proposing methods and other hybrid techniques and employing them for optimization problems;
3. Expand the field of research and studies to determine the most effective approach to integrate these methods.

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