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Research paper

Analysis of gold precipitation based on point and block kriging

Nur Ali Amri [1] *, Winda [2], Gunawan Nusanto [2]

[1] *Magister of Mining Engineering, Faculty of Mineral Technology Universitas Pembangunan Nasional Veteran Yogyakarta 55283 SWK 104 Condong Catur, Yogyakarta INDONESIA*
[2] *Mining Engineering Department, Faculty of Mineral Technology Universitas Pembangunan Nasional Veteran Yogyakarta 55283 SWK 104 Condong Catur, Yogyakarta INDONESIA*
**Corresponding author E-mail: nuraliamri@upnyk.ac.id*

Abstract

This paper analyzes the prediction of gold distribution in veins using kriging on various block sizes. The empirical semivariogram parameters used are classical and robust, and the models used are weighted least squares and ordinary least squares based on exponential and spherical semivariogram theory. Fitting accuracy is based on the four smallest root mean square errors (RMSE), which are all obtained from the exponential base. An interesting phenomenon occurs in the theoretical exponential semivariogram-based predictions: the average value of block variance is directly proportional to the size of the widely used block. This relationship is also demonstrated by the inverse values of the validation index generated. While linked to the semivariogram parameters, the effectiveness relationship is that the length of the range of the fitting result is inversely related to the acquisition of the meanprediction.

Keywords: *Block Kriging; Gold Vein; Point Kriging; Semivariogram.*

1. Introduction

Gold is a mineral that forms through a mineralization process, that is, the inclusion of gold minerals in the rock (or earth) with the potential to form ore deposits. Through thermal energy boosting, minerals then move in and fill the spaces of open fractures in the rock, and when the rock settles, an ore body is formed [1]. Different media occur in the gold deposition process, one of which is the vein. The potential consequences arising from the formation processes include likeness properties (principally the value of the grade) between locations within the region, which in turn can allow the use of geostatistical concepts [2].
One of the crucial stages in mining exploration is to determine the average (and error rate) and value distribution in the area [3]. This study is performed to investigate one of the regions in the Pongkormount, namely, the gold mining region of UBPE Pongkor owned by PT. Aneka Tambang (Tbk). This region is administratively located in the district of Bayah, Lebak, Banten province, Indonesia and has an elevation range from 1.110 to 1.250 m above sea level. Geographically, the region's coordinates are 106°24'00"E – 106°26'00"E and 06°44'00"S – 06°46'00"S. The data used are 128 drilling samples representing an area around 1500×345 m^2, and the predictions use a block model kriging technique. The kriging selection is based on the nature of the gold mineralization, which is mainly gold distributed in veins [2]. There are six sizes of blocks, namely 15×15, 25×25, 35×35, 50×50, 75×75 and 100×100. The variations in this measure are intended to determine the relationship between the block area additions and the error rate of prediction.

2. Semivariogram

The semivariogram is a formula used as a basis in kriging calculation, based on fitted and obtained parameters particularly including the nugget, sill and range,. The semivariogram is defined as the variance of the {Z ($\mathbf{s}_i$)–Z($\mathbf{s}_j$)} increment, written as follow [4], [5]:

$$\begin{aligned} E\left[Z(\mathbf{s}_i)-Z(\mathbf{s}_j)\right]^2 &= E\left[\left(Z(\mathbf{s}_i)-\mu\right)-\left(Z(\mathbf{s}_i)-\mu\right)\right] \\ &= 2\sigma^2 - 2Cov\left(Z(\mathbf{s}_i),Z(\mathbf{s}_j)\right) \\ &= 2\sigma^2 - 2C(\mathbf{h});\ \text{with}\ \mathbf{h}=\mathbf{s}_i-\mathbf{s}_j \end{aligned} \quad (1)$$

Since $\gamma(\mathbf{s}_i,\mathbf{s}_j)=var\{Z(\mathbf{s}_i)-Z(\mathbf{s}_j)\}$, thus $(\mathbf{s}_i,\mathbf{s}_j)=\frac{1}{2}var\{Z(\mathbf{s}_i)-Z(\mathbf{s}_j)\}$. However, to compute the semivariogram, an empirical formula must be used, which is given as follows [6]:

$$\hat{\gamma}(\mathbf{h})=\frac{1}{2|N(\mathbf{h})|}\sum_{N(\mathbf{h})}\left(Z(\mathbf{s}_i)-Z(\mathbf{s}_j)\right)^2;\mathbf{h}\in \mathrm{R}^d \quad (2)$$

Where $N(\mathbf{h})=\{(\mathbf{s}_i,\mathbf{s}_j):\mathbf{s}_i-\mathbf{s}_j=\mathbf{h};i,j=1,\dots,n\}$, and $|N(\mathbf{h})|$ is the number of point pairs within h.
The development of the classical semivariogram is known as a robust formula. The concept of robustness originated from the idea that semivariograms are affected by atypical observations. To improve this situation, made some modifications were made to obtain the following formula [7]:

$$\bar{\gamma}(\mathbf{h}) = \frac{\left(\frac{1}{2|N(\mathbf{h})|}\sum_{N(\mathbf{h})}(z(\mathbf{s}_i)-z(\mathbf{s}_j))^{1/2}\right)^4}{\left(0.457+\frac{0.494}{|N(\mathbf{h})|}\right)} \quad (3)$$

Table 1: List of Semivariogram Models [8]

Model	Semivariogram
Spherical	$\gamma(\mathbf{h}) = \begin{cases} 0, & \mathbf{h}=0 \\ C_0 + C\left[\frac{3}{2}\left(\frac{\lvert\mathbf{h}\rvert}{a}\right)-\frac{1}{2}\left(\frac{\lvert\mathbf{h}\rvert}{a}\right)^3\right], & 0<\lvert\mathbf{h}\rvert\le a \\ C_0 + C, & \lvert\mathbf{h}\rvert \ge a \end{cases}$
Exponential	$\gamma(\mathbf{h}) = \begin{cases} 0, & \mathbf{h}\neq 0 \\ C_0 + C\left[1-\exp\left(-\frac{\lvert\mathbf{h}\rvert}{a}\right)\right], & \mathbf{h}\neq 0 \end{cases}$

Fitting refers to obtaining the parameters that are used as the basis of the kriging calculation (i.e., nugget, sill and range), using two theoretical semivariogram model approaches, both spherical and exponential as in Table 1. The nugget parameter, geologically, is especially likely to allow analysis of the existence of an ore-body [9]. The two model approaches involve ordinary least squares (OLS) and weighted least squares (WLS) [10].

3. Kriging

Kriging is an interpolation technique for spatially sampled data. This technique uses the stationary concept which chooses optimal weights by minimizing the estimation of variance error [11]. The assumptions and simplifications in this method include that the observation data can be seen as a realization of a random variable, which is formally presented as $Z(s); s \in \mathcal{D}$ in Cressie [4] and Sarma [12].In general, if $z(s_i)$ (i=1, …, n) aren observations of data residing in many locations of s_i ($s_i \in R^d$, d in 2 dimension), and s_0 ($s_0 \in R^d$) is a position of the predicted point, the prediction value, $\hat{Z}(s_0)$, can be written as follows [13]:

$$\hat{Z}(s_0) = \sum_{i=1}^{n} w_i z(s_i) \quad (4)$$

Which is a weighted average of the grade sample. The weights, w_i, are such that $\sum_{i=1}^{n} w_i = 1$, and the estimation is assumed to be unbiased.The prediction value $\hat{Z}(s)$ is a realization of a random function Z(s) that in practice is never really known and is determined by the distribution value qualified by the cut-off grade [14]. This function minimizes the mean squared error of prediction, given by

$$\sigma_e^2 = E[Z(s_0)-\hat{Z}(s_0)]^2 \quad (5)$$

In addition, the variance of the intrinsically stationary process can be written as follows:

$$\sigma_e^2 = 2\sum_{i=1}^{n} w_i \gamma(s_0 - s_i) - \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \gamma(s_i - s_j) \quad (6)$$

4. Analysis

Semivariogram construction began by setting the initial lag distance h = 17.5, which applies periodically, where the maximum distance lag used is 500 m. The maximum distance is one-third of the regional landscape. This lag stipulation applies to both principles in the preparation of either a classical empirical semivariogram, $\hat{\gamma}(\mathbf{h})$, or a robust one, $\bar{\gamma}(\mathbf{h})$. Beginning from the first lag, by Equation(1) and Equation(2)the classical and robust semivariograms produce different values. The robust semivariogram value was almost half the classical value.

The empirical semivariogram fitting is based on eight combinations, four types for the classical and the other four for the robust. The classical semivariogram fitting begins with an initial C code, while the robust semivariogram uses the R code, followed by the OLS or WLS models. The semivariogram theories are encoded as in Table 1, with the letter S for spherical and E for exponential. Concretely, CWLSS is the fitting based on the classical semivariogram approach with WLS model and the spherical semivariogram theoretical models. RWLSE is a fitting based on the robust semivariogram approach with the WLS model and the spherical theoretical model. COLSE is a fitting based on the classical empirical OLS model and the exponential theory. The various options result in a total of eight fitting combinations, as shown in Figure 1.

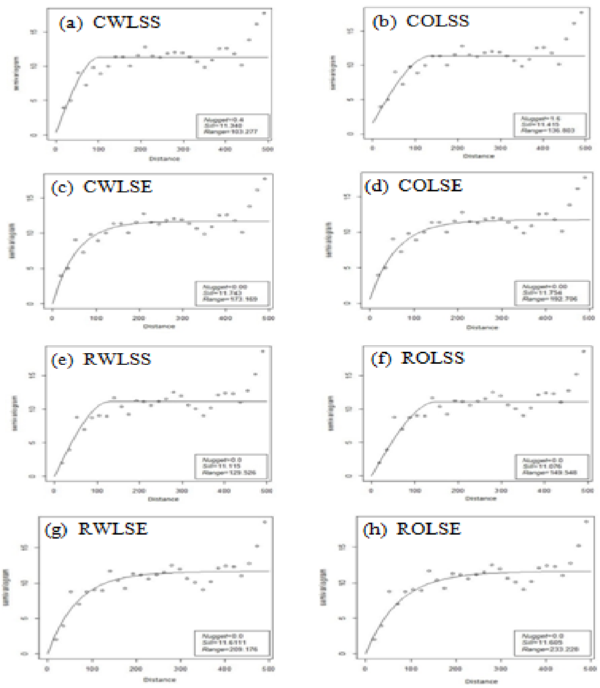

Fig. 1: Fitted Spherical and Exponential Models.

The empirical semivariogram parameters fitted against the semivariogram theory, i.e., nugget, sill and range, are presented in Table 2. It appears that the nugget effect only occurs in the CWLSS and COLSS combinations, with values of 0.4 and 1.6, respectively. The sill value is almost uniform in the range of 11.076 to 11.754. In general, however, it can be said that both exponential bases produce larger sill values than the spherical. The range value, as described in Figure 2, varies from a minimum of 103.28 m (CWLSS) to a maximum of 233.23 (ROLSE). The range produced by exponential fitting (CWLSE, COLSE, RWLSE and ROLSE) is in general longer than for the spherical fitting models.

Table 2: Semivariogram Parameters and Fitting with Root Mean Square Error (RMSE)

Category	Nugget	Sill	Range	RMSE
CWLSS	0.40	11.340	103.277	1.268
COLSS	1.60	11.415	136.803	1.224
CWLSE	0.00	11.743	173.169	1.124
COLSE	0.00	11.754	192.706	1.103
RWLSS	0.00	11.115	129.526	1.313
ROLSS	0.00	11.076	149.548	1.594
RWLSE	0.00	11.611	209.176	1.198
ROLSE	0.00	11.605	233.228	1.179

Table 2 shows that the four acquisition fittings with the lowest error (RMSE) are the ones with an exponential basis. Of these four, two are from the semivariogram classical basis, i.e., COLSE is 1.103 and CWLSE is 1.198. The other two are based on the robust semivariogram, i.e., ROLSE is 1.224 and RWLSE is 1.594. The semivariogram parameter is used as a primary basis for block kriging prediction.

Table 3 gives a complete summary of the block kriging prediction for various sizes, based on an exponential fitting semivariogram. Itdescribes the point and block kriging prediction on various sizes for a spherical base. In both of these tables, the first column shows the size of the block, while the second column is the prediction value (Pred. Value), which consists of the mean, variance and validation index. The validation index (Valid. Index) is a value derived from the ratio between the differences in mean prediction and the variance of the mean prediction values. This index describes the interval between the mean prediction and the variance results, where a higher value indicates greater reliability.

All of the predictions derive from exponential fitting yield positive values.A representative example of the entire block kriging prediction is the distribution produced by the block kriging prediction size 15×15 based on ROLSE fitting, which has the highest validation index. In general, this block kriging produces 58.40% of an area containing a distribution of values greater or equal to 3.00 (g/t Au), 5.88% of an area containing values of at least 10.00, and 0.96% of an area containing a distribution of values of at least12.00 (a rich zone). The rich zone in Figure 2 (shown as a yellow contour image), here represented by 24 prediction blocks located around the abscissa coordinates 7486-7891 and the ordinate coordinates 464-659.

Table 3: List of Best Predicted Values of Mean, Variance and Validation Index Based on Exponential Fitting

Point(*) /block	Pred. Value	CWLSE	Fitting COLSE	base RWLSE	ROLSE
4×16*)	Mean	4.552	4.493	4.472	4.448
	Variance	4.407	4.087	3.800	3.497
	Valid. Index	0.025	0.090	0.150	0.214
8×16*)	Mean	4.585	4.559	4.541	4.519
	Variance	4.085	3.776	3.502	3.211
	Valid. Index	0.109	0.172	0.229	0.289
53×75*)	Mean	4.625	4.600	4.546	4.525
	Variance	3.887	3.583	3.458	3.169
	Valid. Index	0.160	0.211	0.239	0.300
15×15	Mean	4.611	4.586	4.568	4.548
	Variance	3.763	3.446	3.359	3.075
	Valid. Index	0.190	0.249	0.265	0.324
25×25	Mean	4.618	4.592	4.574	4.554
	Variance	3.774	3.482	3.396	3.110
	Valid. Index	0.183	0.242	0.257	0.317
35×35	Mean	4.590	4.564	4.546	4.525
	Variance	3.838	3.544	3.458	3.169
	Valid. Index	0.164	0.233	0.239	0.300
50×50	Mean	4.580	4.553	4.534	4.513
	Variance	3.955	3.653	3.565	3.268
	Valid. Index	0.136	0.198	0.214	0.276
75×75	Mean	4.673	4.645	4.627	4.605
	Variance	4.030	3.730	3.645	3.347
	Valid. Index	0.138	0.197	0.212	0.273
100×100	Mean	4.570	4.543	4.524	4.502
	Variance	4.233	3.919	3.820	3.520
	Valid. Index	0.074	0.137	0.156	0.218

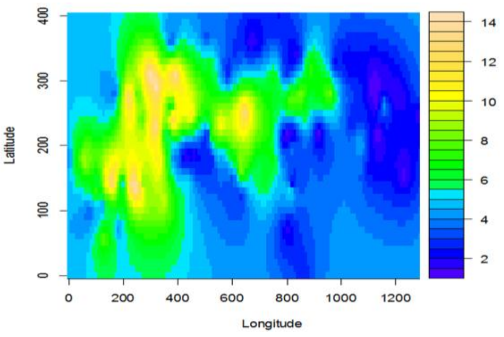

Fig. 2: The Distribution Values of 15×15block Kriging Based on ROLSE Fitting.

5. Conclusion

Based on this analysis done in section 4, several conclusions can be drawn. We found that the mean prediction produced by block kriging based on exponential fitting (i.e., KWLSE, KOLSE, RWLSE and ROLSE) produces a value greater than the mean variance. We also found that the block size is inversely related to the validation index value (Valid. Index). When using a larger block size for prediction, the value of the validation index will be smaller, i.e., there is a negative correlation. In fact, the block size is proportional to the mean variance prediction. If the block size is smaller, the variance produced will also be smaller.If the judgment is based on the greatest value generated by the prediction of the exponential equivalent, the mean prediction then occurs for a kriging block size of 75×75. Apart from that, the range length is in inverse proportion to the mean prediction generated.In considering this provision, the most realistic prediction is produced using a 15×15 block of ROLSE. This prediction-base produces a mean of 4.548 (g/t Au) and a mean variance of 3.075 (g^2/t^2 Au).

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