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Research paper

Performance measure of satellite flying in coplanar and non-coplanar formation

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Abstract

In order to fulfil specific mission objective demand, spacecraft performance can be further optimized by means of various methods or configurations. Like for instance, selection of orbit type and inclination with a periodically repeated ground track will ensure the high efficiency of ground target coverage be accomplished throughout the whole duration of mission. Unfortunately, a single monolithic satellite most often unable to accommodate the requirement solicitated by many multi background users. So, to deal with the issue, an alternative solution would be to operate a swarm of satellites flying in synchronized formation. In this paper, three satellites flying in co-planar and non-coplanar formation were simulated. Here, the resulting model of two deputy satellites operating in the same orbital plane but different phase angle moved along the orbit path while both still maintaining constant relative distance with the non-coplanar chief spacecraft throughout the whole orbit period were presented. The use of unique projected circular orbit (PCO) formation arrangement allows the assessment of some important performance measure parameters like average overlapping coverage area and optimum swath width coverage distance. For the determination of area on the surface of the Earth overlapped by three satellites, the analysis was done using the multiple boundary overlap condition. Parametric studies were conducted involving different formation distance and formation height to observe pattern variation of average total overlapping area and maximum coverage distance. Preliminary result showed that at a specific Earth central angle, the total overlapped area decreased substantially with the increased distance in formation. Height factor does not have significant influence in the total overlapped area variation due to constraint imposed on satellites operating in Low Earth Orbit (LEO) altitude regime. Results were tabulated using 3-dimensional graphs to study the relationships exist between multiple variables. Finally, conclusions were made based on our findings with regards to the performance of positioning satellites in such configuration.

Keywords: *coverage overlap area; formation flying; multiple boundary overlap; projected circular orbit; swath width.*

1. Introduction

Historically, the subject of satellite formation flying evolved from study conducted for the purpose of docking and rendezvous mission like the GEMINI manned space mission. The key solution to this problem is the ability to maneuver and control close distance between spacecraft and later assemble them while in orbit. From there, the concept of close formation flying emerged whereby the satellites should maintain a specific distance among themselves without collision or any rendezvous act. Research related to satellite formation flying literature have gained a lot of attention these days. The reason is due to its effectiveness in solving many problems associated with single multi-mission spacecraft operation. In the case of Earth observation, satellite flying in formation greatly enhanced the overall performances by capable of providing wider ground coverage region, extending the longevity of mission and reduce the risk of failure factor [1-4]. Through formation flying also, multiple objectives can be achieved simultaneously by flexibly pointing some instruments on different spacecraft at differ location at the very same time [5, 6]. Thus, having few other identical satellites to 'assist' during critical space operation, would be an indigenous solution.

This paper elaborates on the performance measure of three satellites flying in projected circular orbit (PCO) formation configuration about the low Earth orbit (LEO). The popularly known equations of relative motion developed by Clohessy and Wiltshire [7] were applied. Section 2 describes the relevant theoretical modelling involved in our analysis whereas Section 3 presents the results of simulation done and parametric study analysis carried out. Based on the results, conclusions are made in Section 4.

2. Satellite formation flying theoretical modeling

2.1. Clohessy-Wiltshire equation

Mathematically, satellite flying in formation can be modelled using the well-known Clohessy-Wiltshire (CW) equations of relative motion [7, 8]. These fundamental equations sufficiently described the dynamic behavior of satellite motion moving relative with respect to a reference satellite in a specific frame particularly for the case of close formation flying. The CW equations are also used to model spacecraft formation flying under the conditions of which the reference (chief) satellite orbiting the Earth in circular or near-circular type orbit with no sort of perturbation involved. The governing equations are given by Eqn. 1, where $\delta r = (\delta x, \delta y, \delta z)$ = the relative distance between chief and deputy satellite in the local-vertical local-horizontal (LVLH) rotating frame, $n = \sqrt{\frac{\mu}{a^3}}$ = the chief satellite mean motion (rad/s), a = chief satellite semi-major axis (km), μ = 398,600 km^3/s^2 = Earth gravitational constant parameter and $\rho = \sqrt{\delta x^2 + \delta y^2 + \delta z^2}$ = magnitude of relative distance of deputy in LVLH frame.

$$\begin{aligned} &\delta\ddot{x} - 3n^2\delta x - 2n\delta\dot{y} = 0 \\ &\delta\ddot{y} + 2n\delta\dot{x} = 0 \\ &\delta\ddot{z} + n^2\delta z = 0 \end{aligned} \tag{1}$$

This linearized model of relative motion equations is specified in time domain frame with the assumption that the ratio between the magnitude of relative position in LVLH frame, ρ to the magnitude of chief radius, r_c is very small and can be neglected $\left(\frac{\rho}{r_c} \leq 0\right)$.

Consider three spacecrafts in which one of them is the chief while the other two are called the deputies. Both deputy satellites orbiting the Earth in the same orbital plane but with different phase angle whereas the chief cruise around Earth at specific inclination in a non-coplanar orbit different from the two deputy satellites. The configuration of formation for these satellites is the projected circular orbit (PCO) type with the phase angle difference between the first deputy and the second deputy satellite is set at 45°. In all cases, the deputy spacecrafts while orbiting the Earth at constant phase angle separation, they also consistently moving and maintaining same relative distance with the chief satellite in different orbital plane. Let us denote the chief satellite as S_c at any given point of time, t. The deputy satellites are represented by S_{d1} and S_{d2}, respectively. The desired formation flying configuration can be illustrated by the following Figure 1. The red axes represent the LVLH reference frame used to describe spacecraft relative motion with respect to the chief satellite. δ_{rd1} and δ_{rd2} denoted relative distance between the chief and deputy satellite 1 and deputy satellite 2, respectively.

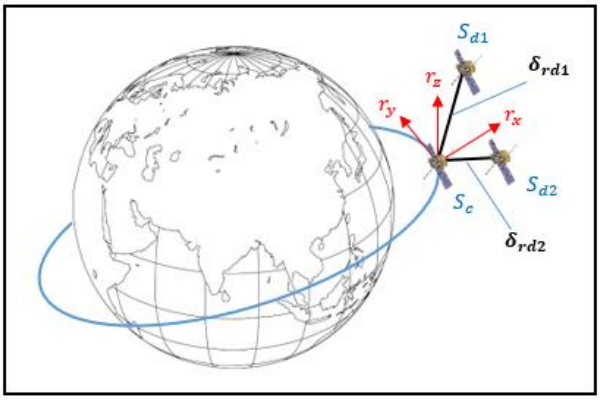


Fig. 1: Satellite relative motion in LVLH frame

2.2. Projected circular orbit formation configuration

The CW equations would be useful to analyze satellite formation flying application if certain conditions hold or satisfied. In our case, bounded relative motion is desired. Applying CW equations, the *locally bounded criteria* for satellite relative motion (one chief and one deputy) requires the following initial condition to be fulfilled where, for relative velocity component in y-direction at time $t = 0$ [8],

$$v_y = -2n\delta x \tag{2}$$

As for the *global bounded criteria*, the corresponding condition is simply given by the energy matching condition, or tentatively presented by these relationships:

$$\frac{T_c}{T_d} = \left(\frac{\varepsilon_d}{\varepsilon_c}\right)^{\frac{3}{2}} = \left(\frac{a_c}{a_d}\right)^{\frac{3}{2}} \tag{3}$$

T_c and T_d stand for the chief and deputy satellite orbital period, respectively. ε_d is the deputy orbital energy while ε_c is the chief satellite orbital energy. Parameters a_c and a_d represent chief and deputy orbit semi-major axis, correspondingly. In energy matching condition, beside ensuring orbital period commensurability, the ratio between orbital energies, ε_d and ε_c, and semi-major axes, a_c and a_d should always satisfies Eqn. 3. Alternatively, when using orbital elements for describing satellite relative motion, the *globally bounded criteria* is simply reduced to

$$\delta a = 0 \tag{4}$$

Once bounded motion has been determined, the desired formation configuration for the mission is selected. Among all configurations available for Earth monitoring mission purposes, the projected circular orbit configuration is the most desirable one. An example of PCO formation configuration is illustrated by Figure 2. PCO is formed when motion of deputy satellite is projected onto the chief's satellite reference frame. Again, to achieve PCO setting, certain conditions comply. If α stands for deputy phase angle with respect to chief and ρ is as defined above, then the following requirements hold for PCO which

$$\alpha_x = \alpha_z \quad \& \quad \rho_z = 2\rho_x \tag{5}$$

$$\delta\dot{x}(t=0) = \delta\dot{z}(t=0) = 0\,;\; \delta z(t=0) = 2\delta x(t=0) \tag{6}$$

Such conditions were used when the CW equations have been solved yet simplified and written in the magnitude-phase form of

$$\begin{aligned} x(t) &= \rho_x \sin(nt + \alpha_x) \\ y(t) &= \rho_y + 2\rho_x \cos(nt + \alpha_x) \\ z(t) &= \rho_z \sin(nt + \alpha_z) \end{aligned} \tag{7}$$

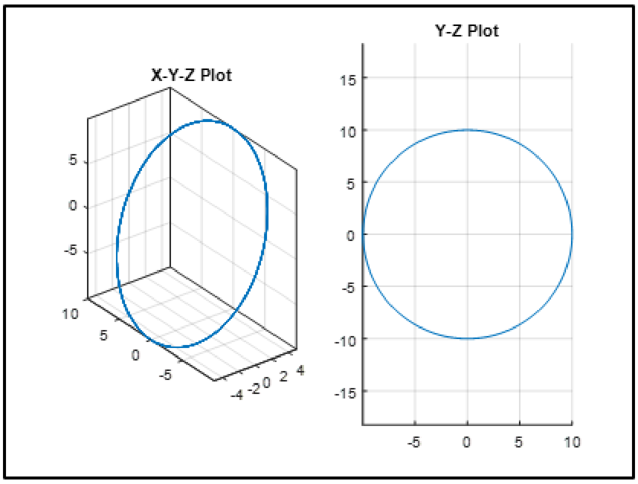


Fig. 2: Projected circular orbit (PCO) formation configuration (ρ = 10 km)

2.3. Satellite performance measure – Ground track coverage (Three Satellite Case)

The coverage of area on the Earth's surface along satellite ground track constitutes one of the critical performance measures being considered during satellite mission analysis and design [9]. For formation flying satellite, characteristics such as an average overlapping area and maximum swath width coverage are among the important ones. Here, we calculate the estimated average overlap area per orbit including its optimum swath width coverage measured at different formation distances and altitudes for three satellites flying in PCO formation. To determine these performance variables, analysis is done using formulations provided by Hughes [5] for the case of multiple boundary overlap configuration. For this configuration, the total overlap area is given by:

$$A_o = A_{outer} + A_{inner} \tag{8}$$

where A_{outer} is obtained by summing the area of each of the lunes or two overlapping circles of any two satellites combination (refer to Figure 3a). In this case, 3 lunes were resulted from three satellites. Calculation of the lune area is obtained using Eqn. 9.

$$A_{lune} = 2\pi R_e^2 - 2R_e^2 \cos\lambda_2 \cos^{-1}\left(\frac{-\cos\lambda_2 \cos\beta}{\sin\lambda_2 \sin\beta}\right) - 2R_e^2 \cos^{-1}\left(\frac{\cos\beta}{\sin\lambda_2}\right) \quad (9)$$

R_e is the radius of Earth approximately valued at 6378 km. λ_2 is the Earth central angle for second satellite overlapped and β is the angle between one sub-satellite point to another. To apply this equation for calculating other lune areas, λ_2 is changed with other value of other satellite Earth central angle (e.g. λ_c for chief and λ_1 for another spacecraft). β can be determined using

$$\beta = \cos^{-1}\left(\frac{\boldsymbol{R_c} \bullet \boldsymbol{R_d}}{R_c R_d}\right) \quad (10)$$

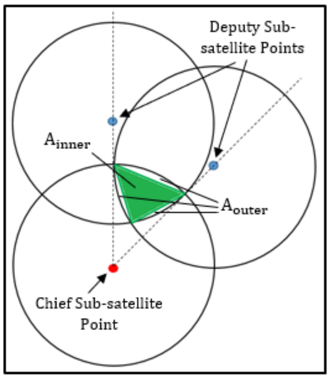
with $\boldsymbol{R_c}$ and $\boldsymbol{R_d}$ denote chief and deputy satellite position vectors in Earth Centered Inertial (ECI) reference frame, respectively, while R_c and R_d are the scalar vectors of chief and deputy satellites orbital radius. Specific condition applies for overlapping to occur is

$$\beta < \lambda_c + \lambda_d \quad (11)$$

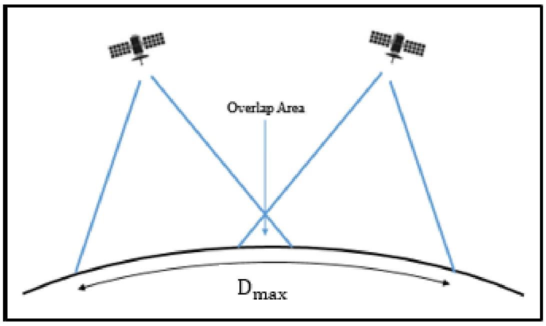
Formulation for A_{inner} is given by

$$A_{inner} = 2\pi R_e \left(1 - \left(n_i - 1\right)\right)\theta \quad (12)$$

In which, n_i is the number of points defining overlap region and θ is the rotation angle of inner overlap area. Throughout this paper, all angles are measured in radian (unless specified) and the unit for total area overlapped, A_0 is in steradian (sr). For analysis simplification purposes, angle θ is set to be 180° due to small distance assumption involved when modelled Earth as a sphere. Figure 3 on the left shows overlapping areas resulting from three satellites flying in PCO formation.



(a)

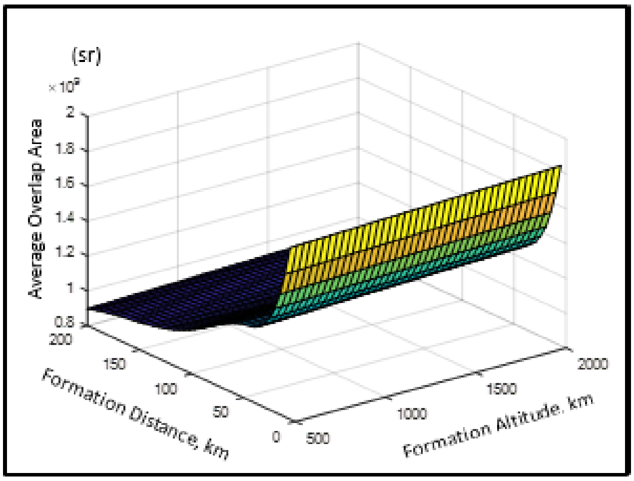


(b)

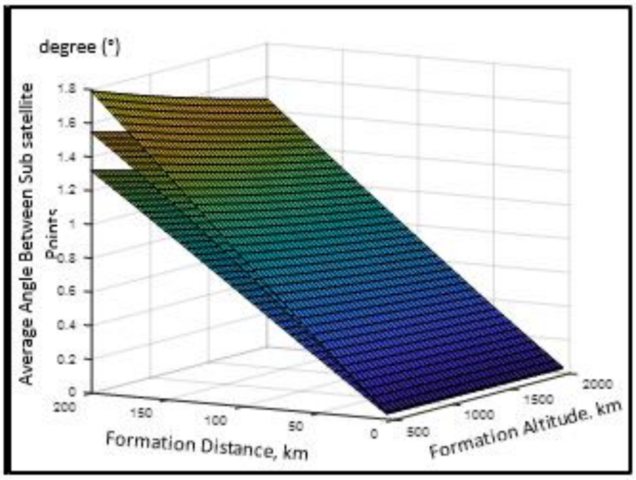
Fig. 3: Formation overlap area and optimum swath width coverage. Figure 3(a), inner and outer area overlapped are defined while Figure 3(b) indicates optimum distance coverage on Earth's surface denoted by D_{max}

3. Performance measure analysis

Results of satellite formation coverage performances are presented. Figure 4a shows 3-dimensional graph of the average total overlapped coverage area for one orbital period plotted at varying on-orbit formation distance and formation altitude. PCO formation distance stretch from 5 km to 200 km apart between chief and both deputy satellites whereas formation height is set to range within 500 km altitude up to 2000 km above Earth's surface. Constraint on formation distance is due to the linearized model factor of Clohessy-Wiltshire equation application which requires $(\rho/r_c) \leq 0$ and can be neglected while altitude limitation imposed is due to LEO segment case study purpose. From the graph, it can be observed that there is a significant trend of variations in total average overlap area with increasing formation distance. For near distance formation, typically less than 50 km apart, the maximum total overlap coverage area is recorded. However, as distance increase, the value gradually decreases to a minimum. This trend suggests that study on the formation optimum overlap area is mostly applicable for close distance formation (less than 50 km apart). In addition, there is very little or less area variations occur when altitude rise to the upper limit of LEO envelope. The probable reason behind insignificant changes in coverage area overlapped when formation height varies is because of altitude restriction imposed as already mentioned earlier.



(a)



(b)

Fig. 4: (a) 3D graph showing total average overlapped coverage area for one orbital period (measured in steradian); (b) Average angle between sub-satellite points (in degree)

Figure 4(b), on the other hand, illustrates the surface plot resulting from the measure of average angle between sub-satellite points on the surface of Earth in unit degree (°). As can be seen, there are 3 folds appear on the graph. The upper fold represents angle variation between chief and second deputy satellite sub-satellite points.

The middle fold is the angle result for chief and first deputy sub-satellite point while the lower fold constitutes sub-satellite point angle variation between first deputy and second deputy satellites. All these folds are derived due to changes in formation distance and formation altitude variables. Similar to the previous case on the overlap area, significant angle change majorly results from altering their formation distance instead of their height. In such study concerned on the relationship characteristics, distance of formation is found to be directly proportional to the average sub-points angular difference. When altitude increases, there exist variations in angular difference between sub-satellite points, but the occurrence is very much less. Therefore, based on parametric studies conducted so far, it is suggested that distance factor have dominant influence over determination of total overlap coverage area along with their associated sub-satellite point angular difference compared to height changes effect. For both cases, more than 80% differences generated are contributed by alteration of formation distance parameter.

Finally, the plot for maximum formation coverage distance were tabulated as shown in Figure 5. These surface plots were obtained following the results from angular difference between sub-satellite points analyzed previously. Clearly, three folds can be differentiated in this figure in which the order follows the same pattern as Figure 4(b). Upper fold represents optimum width for chief-second deputy satellite combination, middle fold belongs to the chief and first deputy configuration while the lower fold specifies swath width trend between first deputy and second deputy satellite combination. In this case study, optimum coverage width is achieved primarily by maximizing the opening angle between any two sub-satellite points on Earth's surface with a condition that all satellites in formation should have identical Earth central angle. Again, as depicted in Figure 5, on-orbit distance plays important role for alteration of formation coverage distance if compared to modifying their altitudes. Changing height of the formation only slightly lengthen the overall coverage swath width. The formation coverage swath width parameter happens to have direct linear relationship with its corresponding formation distance variation variable. Table 1 provides numerical evidence of coverage swath width evolution related to this case study. The results are derived from calculations of optimum coverage swath width, D_{max} determined particularly for the upper fold surface plot as in Figure 5.

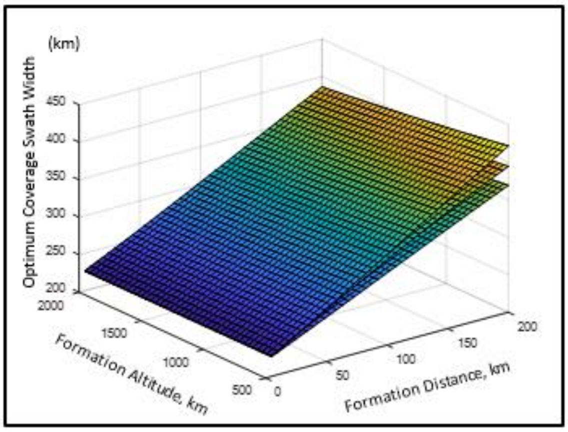


Fig. 5: Three-dimensional plot for maximum coverage distance between three satellites flying in formation

Table 1: Optimum coverage swath width, D_{max} for chief and second deputy satellite measured at varying formation altitude and formation distance (in kilometer, km)

Distance \ Altitude	500 km	800 km	1000 km	1300 km	1500 km	1800 km	2000 km
5 km	227.311	227.115	226.993	226.822	226.715	226.565	226.471
10 km	232.020	231.625	231.380	231.036	230.821	230.520	230.330
50 km	270.709	268.649	267.371	265.580	264.464	262.894	261.911
100 km	320.844	316.608	313.978	310.294	307.996	304.764	302.741
150 km	371.678	365.311	361.351	355.793	352.322	347.433	344.368
200 km	421.935	413.621	408.433	401.132	396.561	390.107	386.052

From Table 1, it is obvious that the formation coverage swath width progress faster with the increase in formation distance. Unfortunately, when formation height increase, a reverse pattern is observed where the width of coverage slowly decreased. These trends are true for all cases of varying formation distance and formation altitude. For instance, at a maximum distance of 200 km on orbit formation with altitude of 2000 km above Earth's surface, coverage swath width recorded steady decrease from approximately 422 km wide at 500 km altitude to 386 km width at the height of 2000 km. This is equivalent to almost 36 km width difference from ranging altitude of 500 to 2000 km which is the highest gap throughout this study. Thus, these findings suggested that for gaining optimal formation coverage swath width, altering formation distance would be far more beneficial in obtaining fast and wider coverage if compared to changing the height of the formation.

4. Conclusion

Changing PCO on-orbit formation distance have significant influence in terms of the total average overlap coverage area per orbit if compared to modifying their altitudes in all cases. The altitude factor insignificancy is mostly due to height restriction imposed to study the effect only in LEO boundary. In this study involving three satellites, it was found that the widest angle between sub-satellite points occur for the chief and second deputy satellite combination which later translated into the optimum coverage swath width in our case. At constant Earth central angle, the sub-satellite points angular difference characteristics largely determine the optimum distance covered on the Earth's surface. While studying the variations in overlap coverage area, it should be highlighted that there exists dynamic relationship between effects of changing formation distance and changing their formation altitude which require further in-depth studies. Linear relationship exists for maximum swath width coverage at varying formation distance and formation altitude. But in all circumstances, formation distance largely influenced coverage width compared to the formation altitude factor which only recorded minor width changes.

Acknowledgement

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