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Research paper

Diametral Paths in Total Graphs of Paths, Cycles and Stars

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Abstract

The diametral path of a graph is the shortest path between two vertices which has length equal to diameter of that graph. Total graph of a graph is a graph that has vertices representing all vertices and edges of the original graph and edges representing every vertex-vertex adjacency, edge-edge adjacency and edge-vertex incidence. In this paper, the number of diametral paths is determined for the paths, cycles and stars and their total graphs.

Keywords: *Diametral path, Path, Cycle, Star, Diameter, Total graph*

1. Introduction

Outliers provide very interesting and sometimes vital information in any system. In a graph, peripheral vertices are some types of outliers associated with its structure itself. Peripheral vertices are vital in identifying the diametral paths of a graph. Deogun and Kratsch introduced and discussed the concept of diametral paths. [3]

Recently, this authors have initiated the studies on diametral paths in total graphs.[4,5] Total graph of a graph is a graph that has vertices representing all vertices and edges of the original graph and edges representing every vertex-vertex adjacency, edge-edge adjacency and edge-vertex incidence. Hence, the number of edges in the total graph T(G) of a connected graph G is

$\frac{1}{2}\sum_{v\in V(G)} d(v)(d(v)+2)$.[4]

It is proved that for a graph G, the diameter of T(G), denoted by diam(T(G)) is equal to diam(G) or diam(G)+1.[4] The diametral paths in total graphs of Complete Graphs, Complete Bipartite Graphs and Wheels are studied in [5].

We now discuss on the number of diametral paths in paths, cycles and stars, and their total graphs. For more about graphs and various concepts in graphs that are not explained explicitly in this paper, refer to [1] and [6].

2. Paths

A path graph is a connected graph with uninterrupted sequence of distinct vertices and edges. A path with n vertices is denoted by P_n. Since diam(P_n) = n-1, there is only one diametral path in P_n which is the path itself with peripheral vertices as its endvertices. It can be noted that diam(T(P_n)) = n-1. There are four peripheral vertices in T(P_n) which are represented by the endvertices and pendant edges of P_n. We now have the number of diametral paths of the total graph of a path in the next result.

Theorem 1 The number of diametral paths of T(P_n) is 2n-1.

Proof Consider a P_n with vertices $v_1, v_2, \dots, v_n$ and edges $e_1, e_2 \dots e_{n-1}$ such that each e_i is incident on v_i and v_{i+1} for $1 \le i \le$ n-1. Since diam(T(P_n)) is n - 1, any diametral path in T(P_n) will have one of the following pairs of vertices as end vertices.

(1) v_1 and v_n

(2) v_1 and e_{n-1}

(3) e_1 and v_n

In case (1), there is exactly one diametral path between v_1 and v_n in T(P_n).
In case (2), there are n-1 diametral paths which are $v_1v_2\dots v_ke_k\dots e_{n-1}$ where $1 \le k \le$ n-1.
In case (3), there are n-1 diametral paths which are $e_1e_2\dots e_kv_{k+1}v_n$ where $1 \le k \le$ n-1.
Hence total number of diametral paths in T(P_n)=1+ n-1+n-1=2n-1.
We now move to the case of cycle graphs.

3. Cycles

A cycle graph is nothing but a path graph with the same initial and terminal vertices.

It can be noted that diam(C_n)=d=$\lfloor n/2 \rfloor$ and diam(T(C_n))=$\lceil n/2 \rceil$. The number of diametral paths is determined in cycles and their total graphs.

Lemma 1 A cycle C_n has n diametral paths.

Proof Diameter of a cycle C_n is given by diam(C_n)=d= $\lfloor n/2 \rfloor$. Let $v_1,v_2 \dots v_n$ be the n vertices of C_n. Considering the diametral paths from every vertex v_i through v_{i+1}, $1 \le i \le$n-1 and one diametral path from v_n through v_1, there are n diametral paths. Hence cycle C_n has n diametral paths.

Theorem 2 The number of diametral paths in T(C_n) is n(n+1) when n is odd and n(n+2) when n is even.

Proof We complete the proof by analysing two cases.

A. Consider n to be odd.

Let $v_1,v_2 \dots v_n$ be the vertices and $e_1,e_2 \dots e_n$ be the corresponding edges of C_n such that each e_i is incident on v_i and v_{i+1} for $1\leq i\leq n-1$ and e_n incident on v_n and v_1. Hence $T(C_n)$ has vertices $v_1,v_2 \dots v_n$ and new vertices $e_1,e_2 \dots e_n$. When n is odd, $T(C_n)$ has a diametral path from every v_i to a specific e_j. Since $diam(T(C_n)) = d=\lceil n/2\rceil$, every diametral path has one endvertex as v_i and the other endvertex $e_{(i+d-1)}$ for $1\leq i\leq (n-d+1)$ and $e_{(i-d)}$ for the remaining *i*. The diametral path from v_i to $e_{(i+d-1)}$ or $e_{(i-d)}$ accordingly satisfies one of the following conditions.

[(1)] It can pass through only v_i's.

[(2)] It can pass through only e_i's.

[(3)] It can pass through v_i's to some stage and e_i's from there.

In case (1), since each vertex v_i has two neighbors in $v_1,v_2 \dots v_n$, there are two diametral paths through only v_j's, from each v_i.
In case (2), since each vertex v_i has two neighbors in $e_1,e_2 \dots e_n$, there are two diametral paths through only e_i's, from each v_i.
In case (3), there are n-3 v_j's left through which the diametral path can pass when the initial vertex v_j and neighbors of the corresponding e_j are excluded. Hence there are n-3 diametral paths in this case from each v_i.

Since there are n vertices, the number of diametral paths=n(2+2+n-3)=n(n+1).

B. Consider n to be even.

Let $v_1,v_2 \dots v_n$ be the vertices and $e_1,e_2 \dots e_n$ be the corresponding edges of C_n such that each e_i is incident on v_i and v_{i+1} for $1\leq i\leq n-1$ and e_n incident on v_n and v_1. Hence $T(C_n)$ has vertices $v_1,v_2 \dots v_n$ and new vertices $e_1,e_2 \dots e_n$. Since $diam(T(C_n)) = \lceil n/2\rceil$, $T(C_n)$has diametral paths from every v_i to a specific v_j, e_j and $e_{(j-1)}$ and diametral paths with endvertices e_i's. Since every diametral path with one endvertex as v_i has the other endvertex from one of the following categories.

[(a)] v_{i+d} or $e_{(i+d-1)}$ or e_{i+d} for $1\leq i \leq n-d$

[(b)] v_1 or e_1 or e_n for i = d+1

[(c)] v_{i-d} or e_{i-d} or $e_{(i-d-1)}$ for the remaining i.

We consider diametral paths in three cases.

[(1)] The endvertices are v_i's .

[(2)] One endvertex is v_i and other is a specific e_j.

[(3)] The endvertices are e_i's.

In case (1), there is a diametral path from each v_i only through vertices v_j's to a specific v_j. Since there are n v_i's, there are n diametral paths in this case.

In case (2), there are two diametral paths from each v_i to corresponding e_j through vertices v_k's. Also there are two diametral paths from each v_i to corresponding e_j through e_k's. There are diametral paths which pass through v_k's to some stage and e_k's from there. Since there are n-4 vertices left of the v_i's when the initial vertex v_i and neighbours of the corresponding e_j's are excluded, there are n-4 diametral paths from each v_i. Since there are n v_i's, the number of diametral paths is $n(2+2+n-4)= n^2$.

In case (3), from each e_i, there is a diametral path only through vertices e_j's to a specific e_j.

Since there are n e_i's, there are n diametral paths in this case.

Hence the total number of diametral paths = $n+n^2+n = n(n+2)$.

We now proceed with the discussions on stars.

4. Stars

A star graph is a connected graph with exactly one vertex is adjacent to every other vertex and no two of the other vertices are adjacent to each other. It can be noted that $diam(T(K_{1,n})) = 2$ for $n \geq 2$. The number of diametral paths is determined in stars and their total graphs.

Lemma 2 A star $K_{1,n}$, $(n \geq 2)$ has ${}^nC_2$ diametral paths.

Proof It is a known fact that $Diam(K_{1,n}) = 2$. Since there are n peripheral vertices, there are ${}^nC_2$ diametral paths through the central vertex.

Hence the number of diametral paths = ${}^nC_2$.

Theorem 3 The number of diametral paths in $T(K_{1,n})$ is $((5n^2)-5n)/2$ for $n \geq 2$.

Let $v_1,v_2 \dots v_{n+1}$ be the vertices and $e_1,e_2 \dots e_n$ be the corresponding edges of $K_{1,n}$ such that each e_i is incident on v_i and v_{n+1} for $1 \leq i \leq n$. Hence $T(K_{1,n})$ has vertices $v_1,v_2 \dots v_{n+1}$ and new vertices $e_1,e_2 \dots e_n$.

Since $diam(T(K_{1,n})) = 2$, the peripheral vertices are $v_1,v_2 \dots v_n$, $e_1,e_2 \dots e_n$. Hence $T(K_{1,n})$ has diametral paths with one endvertex to be v_i and the other endvertex to be v_j or e_j $(i\neq j)$. Since there are n peripheral vertices $v_1,v_2 \dots v_n$, the number of diametral paths through v_{n+1} with endvertices v_i and v_j $(i\neq j)$ is ${}^nC_2$. Also a diametral path with endvertices v_i and e_{j} $(i\neq j)$ passes through v_{n+1} or e_i. Since diametral path from each v_i can have n-1 possible endvertices $e_1,e_2 \dots e_{i-1},e_{i+1} \dots e_n$, there are n-1 diametral paths through v_{n+1} and n-1 diametral paths through e_i.

Since there are n such v_i's, the number of diametral paths is n(n-1+n-1) = n(2n-2).

Hence total number of diametral paths = ${}^nC_2$+ n(2n-2) = $((5n^2)-5n)/2$.

5. Conclusion

The number of diametral paths is determined for paths, stars and cycles and their total graphs are computed here. The focus of further research would be the decomposition or packing of total graphs of paths, cycles and starts into diametral paths.

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