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Research paper

The Energy Graph for Minimum Majority Domination

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Abstract

In this article we have introduced Minimum majority domination energy graph. A set $S \subseteq V$ is called a majority dominating set if at least half of the vertices either in S or adjacent to the vertices S. That is $|N[S]| \leq \left\lceil \frac{|V(G)|}{2} \right\rceil$, $|N[S]| \geq \left\lceil \frac{p}{2} \right\rceil$. The minimum cardinality of a majority dominating set is called majority domination number $\gamma_M(G)$. We defined majority dominating matrix and its energy values for some classes of graphs. Also some boundaries of Energy value of graph G are obtained.

Keywords*: Majority Domination matrix; Minimum majority domination Energy value.*

1. Introduction

In this paper $G = (V, E)$ means finite simple graph with p vertices and q edges. A set D of vertices of a graph G is dominating set if every vertex in $V - D$ is adjacent to some vertex in D. The domination number γ_M of G is the minimum cardinality of all dominating sets of G. A set $S \subseteq V$ is called a majority dominating set if at least half of the vertices either in S or adjacent to the vertices S. That is $|N[S]| \leq \left\lceil \frac{|V(G)|}{2} \right\rceil$, $|N[S]| \geq \left\lceil \frac{p}{2} \right\rceil$. The minimum cardinality of a majority dominating set is called majority domination number $\gamma_M(G)$. This concept was introduced by Jose-line Monora Swaminathan [6]. The concept of energy graph was introduced Ivan Gutman [3]. Let G be the graph with P vertices and q edges and $A = (a_{ij})$ be the adjacency matrix of the graph. λ_i are the Eigen values of the graph G. The energy $E(G)$ of G is defined as $E(G) = \sum_{i=1}^{n} \lambda_i$

2. Minimum Majority Dominating Energy

2.1 Definition

Let 'G' be a simple graph with vertex set $V = \{v_1, v_2, v_3 \ldots v_n\}$ and the edge set $E = \{e_1, e_2, e_3 \ldots e_n\}$ and D is the minimum majority dominating set it is denoted by γ_M - set. The minimum majority dominating matrix G is denoted by MMD (G) is the $n \times n$ matrix defined as follows

2.2 Definition

For any graph 'G' the energy the characteristic polynomial defined by $MD(G, \lambda) = (\det \lambda I - MMD(G))$ and

$$MMD(G) = mmd(i, j) = \begin{cases} 1 & if\ v_i v_j \in E \\ 1 & if\ i = j\ and\ v_i \in D, \\ 0 & otherwise \end{cases}$$

$E_{MMD}(G) = \sum_{i=1}^{n} |\lambda_i|$ where $\lambda_1, \lambda_2, \ldots .\lambda_n$ are the Eigen values of the matrix.

Example:
For the graph G = $C_{7,}$
With the vertex set $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$, then the γ_M - set are $D_1 = \{v_1, v_3\}$ and $D_2 = \{v_2, v_4\}$ $\gamma_M(G) = 2$ .Therefore MMD Matrix of D_1 is

$$mmd(i,j)=\begin{bmatrix}1&1&0&0&0&0&1\\1&0&1&0&0&0&0\\0&1&1&1&0&0&0\\0&0&1&0&1&0&0\\0&0&0&1&0&1&0\\0&0&0&0&1&0&1\\1&0&0&0&0&1&0\end{bmatrix}$$

The characteristics equation is

$$\lambda^7-2\lambda^6-6\lambda^5+10\lambda^4+11\lambda^3-12\lambda^2-6\lambda=0$$

The Eigen value of the matrix MMD of D_1 are

$$\lambda_1=0,\lambda_2=-\sqrt{2}+1,\lambda_3=\sqrt{2}+1,\lambda_4=\sqrt{2},\lambda_5=-\sqrt{2},\lambda_6=\sqrt{3},\lambda_7=-\sqrt{3}$$

Energy value of D_1 is

$$E_{MMD}(G)=\sum_{i=1}^{n}|\lambda_i|=\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7\approx 9.1210$$

2.3. Theorem

For the Graph $G=K_n, n\geq 2$ is complete graph,

Then $E_{MMD}(G)=(n-2)+\sqrt{n^2-2n+5}$

Proof:

Let G complete graph with the $n\geq 2$ and the vertex set

$$V(G)=\{v_1,v_2,v_3\ ...v_n\}$$

$$\deg(v_i)=\Delta(G)$$

Therefore $D=\{v_i\}\Rightarrow \gamma_M(G)=1$.

The minimum majority dominating energy value is

$$E_{MMD}(G)=(n-2)+\sqrt{n^2-2n+5}.$$

Since $A_D(K_n)=MMD(K_n)$.

2.4. Theorem

For the star graph $K_{1,n-1}$, $E_{MMD}(K_{1,n-1})=\sqrt{4n-3}$

2.5. Theorem

If the graph G is a complete bipartite with $m,n\geq 2$ then the minimum majority dominating energy value is

$$E_{MMD}(G)=\sqrt{4(mn)+1}+\frac{(m-n)}{n}.$$

Proof:

Let G be the graph with $m,n\geq 2$ and $m\leq n$ and the vertex set are $V(G)=\{u_1,\mathrm{u}_2,\mathrm{u}_3\ ...\mathrm{u}_m\ v_1,v_2,v_3\ ...v_n\}$ u_i covers n vertices.

$$\therefore d(u_i)=n\geq\left\lceil\frac{m+n}{2}\right\rceil$$

The minimum majority domination set $\{u_i\}$.

$$mmd(i,j)=\begin{bmatrix}1&0&0&\cdots&0&1&\cdots&1\\0&0&0&\cdots&0&1&\cdots&1\\0&0&0&\cdots&0&1&\cdots&1\\\vdots&\vdots&\vdots&\cdots&\vdots&\vdots&\cdots&\cdots\\0&0&0&\cdots&0&1&\cdots&1\\1&1&1&\cdots&1&0&\cdots&0\\\vdots&\vdots&\vdots&\cdots&\vdots&\vdots&\cdots&\vdots\\1&1&1&\cdots&1&0&\cdots&0\end{bmatrix}$$

$$MD(G)=\begin{vmatrix}\lambda-1&0&0&\cdots&0&1&\cdots&1\\0&\lambda&0&\cdots&0&1&\cdots&1\\0&0&\lambda&\cdots&0&1&\cdots&1\\\vdots&\vdots&\vdots&\cdots&\vdots&\vdots&\cdots&\cdots\\0&0&0&\cdots&\lambda&1&\cdots&1\\1&1&1&\cdots&1&\lambda&\cdots&0\\\vdots&\vdots&\vdots&\cdots&\vdots&\vdots&\cdots&\vdots\\1&1&1&\cdots&1&0&\cdots&\lambda\end{vmatrix}$$

The characteristics equation of $K_{\mathrm{m},n}$ is

$$\lambda^{m+n-3}\left[\lambda^3-\lambda^2+mn\lambda+(m-1)n\right]=0$$

$$\lambda=0(n-3)\ times, \lambda=\sqrt{4(mn)+1}+\frac{(n-m)}{n}$$

$$\therefore E_{MMD}(G)=\sqrt{4(mn)+1}+\frac{(n-m)}{n}$$

2.6. Theorem

For the friendship graph F_n with $n>2$, the minimum majority dominating energy graph $E_{MMD}(F_n)=(n-2)+2\sqrt{n-1}$.

Proof:

Let $G=F_n$ friendship graph with $n>2$ and the vertex set $V(G)=\{u,v_1,v_2,v_3\ ...v_{n-1}\}$, u has $\Delta(G)$ and u forms a γ_M - set. Hence MMD matrix is

$$mmd(i,j)=\begin{bmatrix}1&1&1&1&1&\cdots&1&1\\1&0&1&0&0&\cdots&0&0\\1&1&0&0&0&\cdots&0&0\\1&0&0&0&1&\cdots&0&0\\1&0&0&1&0&\cdots&0&0\\\vdots&\vdots&\vdots&\vdots&\vdots&\cdots&\vdots&\vdots\\1&0&0&0&0&\cdots&0&1\\1&0&0&0&0&\cdots&1&0\end{bmatrix}$$

Characteristic polynomial is

$$\begin{vmatrix} \lambda-1 & -1 & -1 & -1 & -1 & \cdots & -1 & -1 \\ -1 & \lambda & -1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & -1 & \lambda & 0 & 0 & \cdots & 0 & 0 \\ -1 & 0 & 0 & \lambda & -1 & \cdots & 0 & 0 \\ -1 & 0 & 0 & -1 & \lambda & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ -1 & 0 & 0 & 0 & 0 & \cdots & \lambda & -1 \\ -1 & 0 & 0 & 0 & 0 & \cdots & -1 & \lambda \end{vmatrix}$$

The characteristics equation is

$$(\lambda+1)^{n-4}(\lambda-1)^{n-5}\left[\lambda^2-2\lambda-(n-2)\right]=0$$

The minimum majority dominating Eigen values are

$\lambda=-1(n-4)\, times, \lambda=1(n-5)\, times, \lambda=1\pm\sqrt{n-1}$

$\therefore E_{MMD}(G)=(n-2)+2\sqrt{n-1}$.

2.7. Theorem

If a graph G be a crown graph with $n>2$, then

$$E_{MMD}(G)=(n-4)+2\sqrt{n(n-1)+1}+1$$

Proof:

Let G be a crown graph S_n^0 with vertex

$V(G)=\{u_1,u_2,u_3\ ...u_n\ v_1,v_2,v_3\ ...v_n\}$

The minimum majority dominating set is $D=\{u_1\}$

$$MMD(G)=\begin{pmatrix} 1 & 0 & 0 & ... & 0 & 0 & 1 & 1 & ... & 1 \\ 0 & 0 & 0 & ... & 0 & 1 & 0 & 1 & ... & 1 \\ 0 & 0 & 0 & ... & 0 & 1 & 1 & 0 & ... & 1 \\ 0 & 0 & 0 & ... & 0 & 1 & 1 & 1 & ... & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & ... & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & ... & 0 & 1 & 1 & 1 & ... & 0 \\ 0 & 1 & 1 & ... & 1 & 0 & 0 & 0 & ... & 0 \\ 1 & 0 & 1 & ... & 1 & 0 & 0 & 0 & ... & 0 \\ 1 & 1 & 0 & ... & 1 & 0 & 0 & 0 & ... & 0 \\ 1 & 1 & 0 & ... & 1 & 0 & 0 & 0 & ... & 0 \\ \vdots & \vdots & \vdots & ... & \vdots & \vdots & \vdots & \vdots & ... & \vdots \\ 1 & 1 & 1 & ... & 0 & 0 & 0 & 0 & ... & 0 \end{pmatrix}$$

$$MMD(G)=\begin{vmatrix} 1-\lambda & 0 & 0 & ... & 0 & 0 & 1 & 1 & ... & 1 \\ 0 & 0-\lambda & 0 & ... & 0 & 1 & 0 & 1 & ... & 1 \\ 0 & 0 & 0-\lambda & ... & 0 & 1 & 1 & 0 & ... & 1 \\ 0 & 0 & 0 & ... & 0 & 1 & 1 & 1 & ... & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & ... & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & ... & 0-\lambda & 1 & 1 & 1 & ... & 0 \\ 0 & 1 & 1 & ... & 1 & 0-\lambda & 0 & 0 & ... & 0 \\ 1 & 0 & 1 & ... & 1 & 0 & 0-\lambda & 0 & ... & 0 \\ 1 & 1 & 0 & ... & 1 & 0 & 0 & 0-\lambda & ... & 0 \\ 1 & 1 & 0 & ... & 1 & 0 & 0 & 0 & ... & 0 \\ \vdots & \vdots & \vdots & ... & \vdots & \vdots & \vdots & \vdots & ... & \vdots \\ 1 & 1 & 1 & ... & 0 & 0 & 0 & 0 & ... & 0-\lambda \end{vmatrix}$$

The characteristics equation is

$$(\lambda+1)^{n-2}(\lambda-1)^{n-2}\left[\lambda^4-\lambda^3-\left(n^2-2(n-1)+(n-1)(n-2)+1\right)+(n-1)^2\right]=0$$

The minimum majority dominating Eigen values are

$\lambda=1[(n-2)\, times], \lambda=-1[(n-2)\, times], \left(\lambda=2\sqrt{n(n-1)+1}+1\right)$

$\therefore E_{MMD}(G)=(n-4)+2\sqrt{n(n-1)+1}+1$

2.8. Theorem

Let G be a graph with order n, size m, majority domination number $\gamma_M(G)$.

If $MD(G,\lambda)=a_0\lambda^n+a_1\lambda^{n-1}+a_2\lambda^{n-2}+.....+a_n$ be the Characteristic polynomial of minimum majority of dominating matrix of G then $(i)\ a_0=1 \quad (ii)\ a_1=-\lambda_M(G)$

2.9. Theorem

For any graph G with vertex set $V(G)=\{v_1,v_2,v_3\ ...v_n\}$, edge set E and λ_M -set $D=\{u_1,u_2,u_3\ ...u_k\}$ .If $\lambda_1,\lambda_2,.....,\lambda_n$ are the Eigen values of the matrix $MMD(G)$ then

(i) $\sum_{i=1}^{n}\lambda_i=|D|$

(ii) $\sum_{i=1}^{n}\lambda_i^2=2|E|+|D|$

2.10. Theorem

For any graph G of order n, size m then $\sqrt{2m+\gamma_M(G)}\le E_{MMD}(G)\le\sqrt{n(2m+\gamma_M(G))}$

Proof:

By Cauchy Schwartz inequality

$$\left(\sum_{i=1}^{n}a_ib_i\right)^2\le\left(\sum_{i=1}^{n}(a_i)^2\right)\left(\sum_{i=1}^{n}(b_i)^2\right)$$

Let $a_i=1$ *and* $b_i=|\lambda_i|$,

$$\begin{aligned}(E_{MMD}(G))^2&=\left(|\lambda_i|^2\right)\\ &\le\left(\sum_{i=1}^{n}1\right)\left(\sum_{i=1}^{n}\lambda_i^2\right)\\ &\le n\left(2m+|D|\right)\\ &\le n\left(2m+\gamma_M(G)\right)\end{aligned}$$

Also

$$\begin{aligned}\left(\sum_{i=1}^{n}\lambda_i\right)^2&\ge\left(\sum_{i=1}^{n}\lambda_i^2\right)\\ &\ge\left(\sum_{i=1}^{n}\lambda_i^2\right)\\ &=2m+|D|\end{aligned}$$

$$\geq 2m + \gamma_M(G)$$
$$\geq \sqrt{2m + \gamma_M(G)}$$

3. Conclusion

In this article we have introduced the concept of minimum majority dominating energy graph and its energy value. We have obtained minimum majority dominating energy value for some classes of graph. Also some bounds of the energy value are achieved.

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