

A simple and efficient wavelet approach for evaluating surface integral over curved domain

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Abstract

This paper presents, a simple and efficient wavelet approach for computing the surface integrals over irregular or curved domain, the limit of the integrals are nonlinear function are transformed to standard two square by using finite element basis function, Haar wavelet based integration technique is applied to evaluation of surface integral over curved domain, the computational efficiency of the method is illustrated with several numerical examples.

Keywords: Numerical Integration; Haar Wavelet Method; Curved Domain

1. Introduction

Numerical integration appears in many fields of science and engineering, to finding surface area, volume, length, momentum, mass, stiffness matrix, mass matrix, body force vector etc., In recent years wavelet bases integration approach is more popular in the field of numerical approximation of surface and volume integral. From the literature review we may realize that a lot of works have been done in this area in terms of quadrature rule of numerical integration, A number of polynomial based Gauss Legendre quadrature rule, product of polynomial and logarithmic based Generalized Gaussian quadrature method over various region have been discussed, such as triangle region [1-2], quadrilateral region [3], parabolic region [4], square region [5], circle [6], etc., numerical integration of arbitrary function over convex and non-convex region are evaluated numerically by cubic spline element method [7], convex and non convex region is divided into 4-node, 8-node quadrilateral elements are approximated Numerically in [8-9], very limited work is attempted in curved domain, recently numerical integration of arbitrary function over parabolic edges by Generalized Gaussian Where and Using the orthogonal basis of the Haar wavelet function can be expressed by Haar series function of infinite terms as Where For double integral-divided into 4-node, 8-node quadrilateral elements are approximated numerically in [8,9], very limited work is attempted in curved domain, recently numerical integration of arbitrary function over parabolic edges by Generalized Gaussian quadrature rule are carryout by [10], In this paper, we use Haar wavelet method to approximate the surface integral over curved domain or irregular domain.

method

2.2. Surface integral over XY – plane

In this section is devoted to the numerical integration of arbitrary function over curved domain, Curved or irregular region is expressed as Where and are nonlinear function and a and b are constants are plotted in figure. 1

2. Mathematical preliminaries

2.1. Haar Wavelet method

The explicit form of the function is defined as We shall be using this formula to evaluate the surface integral by Haar wavelet

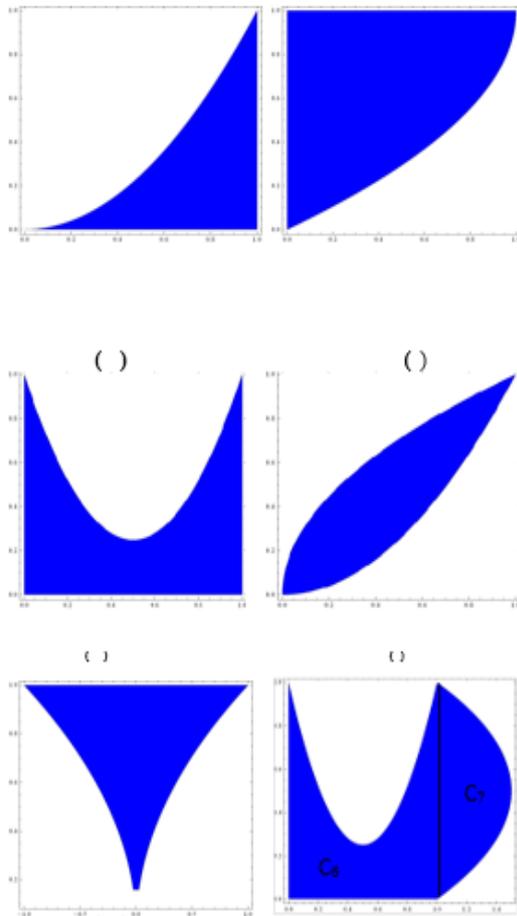


Fig. 1: Curved Boundary Regions.

3. Numerical integration over two dimensional curved region

3.1. Consider the surface integral of arbitrary function+

Over the curved region The surface integral of eq. (3) can be transformed to square The Jacobian of the transformation is hence, Eq. (3) becomes

3.2. Consider the surface integral of arbitrary function

Over the curved region is given by the surface integral of eq. (5) can be transformed to square Hence, eq. (7) becomes.

3.3. Consider the surface integral of arbitrary function

Over the curved region is given by the surface integral of eq. (9) can be transformed to square and hence, eq. (9) becomes.

3.4. Consider the surface integral of arbitrary function

Over the curved region is given by The surface integral of eq. (11) can be transformed to square and, hence, eq. (11) becomes

3.5. Consider the surface integral of arbitrary function

Over the curved region and The surface integral of eq. (13) can be transformed to square We approximate the integral Eqs. (4), (6), (8), (10), (12), (14) by Applying eq. (2) of .Haar wavelet method is described in the.

Table.1: Compare the Obtained Results with Exact Value

Exact value	Order N	Computed value	Error
= 0.0696617011	N=10	0.07088172435	0.00122
	N=100	0.06967379875	1.2E-05
	N=200	0.06966468331	3E-06
= 0.3354005176	N=10	0.33506553638	0.000335
	N=100	0.33539716912	3.35E-06
	N=200	0.33539967764	8.4E-07
= 0.3133093585	N=10	0.31357010110	0.00026
	N=100	0.31331888372	9.5E-06
	N=200	0.31331278756	3.4E-06
= 0.7292185744	N=10	0.73018819262	0.00097
	N=100	0.72924985935	3.1E-05
	N=200	0.72922965894	1.1E-05
= 0.1028703090	N=10	0.10206865780	0.000802
	N=100	0.10286225153	8.06E-06
	N=200	0.10286829387	2.02E-06
= 0.6374363872	N=10	0.63509969592	0.002337
	N=100	0.63736328505	7.31E-05
	N=200	0.63741051888	2.59E-05
= 0.30480315720	N=10	0.30480315720	0.00044
	N=100	0.30436590362	5.8E-06

The surface integral of eq.(5) can be transformed to square

3.6. Consider the surface integral of arbitrary function

Over the curved region is given by The surface integral of eq.(7) can be transformed to square

= 0.3043600979	N=200	0.30436170471	1.6E-06
= 0.05357142857	N=10	0.05372722301	0.00016
	N=100	0.05357299091	1.6E-06
	N=200	0.05357181919	3.9E-07
= 0.93432004929	N=10	0.93448217360	0.00016
	N=100	0.93432167178	1.6E-06
	N=200	0.93432045875	4.1E-07
= 0.5529797990	N=10	0.55263815130	0.000342
	N=100	0.55297668185	3.12E-06
	N=200	0.55297902951	7.69E-07
= 0.8333333333	N=10	0.83312500000	0.000208
	N=100	0.83333125000	2.08E-06
	N=200	0.83333281250	5.21E-07
= 0.3023809524	N=10	0.30216827585	0.000213
	N=100	0.30237880968	2.14E-06
	N=200	0.30238041666	5.36E-07

4. Conclusions

In this paper, Wavelet based integration method is applied for numerical integration of arbitrary function over curved or irregular domain, it converges to the exact value of the surface integral, sufficient value of order N for this proposed method.

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