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Research paper

The Medium Domination Number of Lexico Product of Two Paths P_2 and P_n

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Abstract

Graph theory is a vibrant area in both applications and hypothetical. Graphs can be utilized as a demonstrating device for many problems of realistic consequence. It can be given out as mathematical models to identify a proper graph-theoretic problem. Domination is a hasty sprouting area of research in graph theory, and its various applications are distributed computing and societal networks. Duygu Vargor and Pinar Dundar [1] computed the idea of medium domination number exploited to scrutinize the pair of vertices. This paper study as explored the medium domination number of lexico product of two paths P_2 and P_n.

Keywords: *Medium domination number, Lexico product.*

1. Introduction

Graph theoretical ideas is initiated in earlier 1730's, back Leonhard Euler appear his cardboard on botheration of seven bridges of Konigsberg. Graphs are actual appropriate accoutrement for apery the relationships amid objects, which are represented by vertices. Product graphs accept consistently been an above address to assemble huge graphs from baby ones appropriately it additionally has abounding applications in the adjusting of alternation networks. There are abounding means to allocate product of two graphs, the best abundantly acclimated one may be the Cartesian product, aboriginal alien by Sabidussi [2]. Hausdorff has authentic the lexicographic product. Many theoretical invariants of lexico graphic product of graphs has been identified and given solution to the issues.

2. Definitions and Observations

Definition 2.1:
The lexicographic product $G_1 \bullet G_2$ has $V_1 \bullet V_2$ as its vertex-set and two vertices x_1x_2 and y_1y_2 are adjacent if only if either $x_1y_1 \in E_1$ or $x_1 = y_1$ and $x_2y_2 \in E_2$.

Definition 2.2:
A walk is called a path if no vertex in it is visited more than once.

Definition 2.3:
Let G be any simple connected graph. Then,

$$MD(G) = \frac{TDV(G)}{nC_2}.$$

Observation 2.1:
The Medium domination number of path p_n is $MD(P_n) = \frac{2n-3}{nC_2}$.

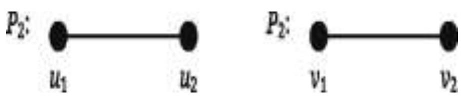
Theorem 2.1:
Let G be a graph of n vertices and q edges and $\deg(v_i) \geq 2$;

$$TDV(G) = q + \sum_{v_i \in V} (\deg v_i C_2) \cdot$$

3. Results

Lemma 3.1:
Let P_2 be the path with two vertices. Then the medium domination number of lexico product of P_2 and P_2 is $MD(P_2 \bullet P_2) = \frac{18}{6}$.

Proof
The diagrammatic representation of the graph is given below



By the definition of lexico product, we draw the graph $P_2 \bullet P_2$ and it is shown below

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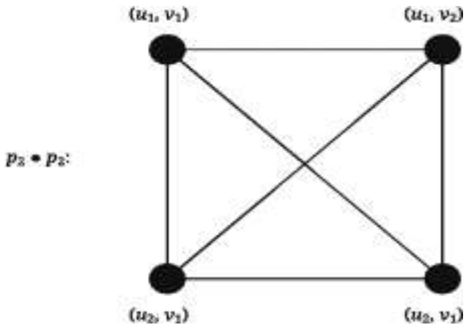


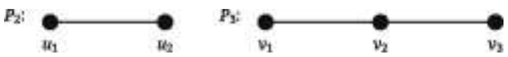
In the above graph, there are four vertices and six edges connecting these four vertices. To find the medium domination number, we have to find $TDV(P_2 \bullet P_2)$.

By Theorem 2.1,

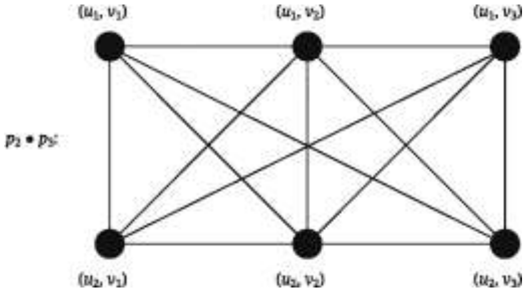
$$TDV(P_2 \bullet P_2) = q + \sum_{v_i \in V} (\deg v_i C_2)$$
$$= q + \sum_{oddv_i} (\deg v_i C_2) + \sum_{evenv_i} (\deg v_i C_2)$$
$$= 6 + 4(3C_2) \qquad \left(\because \sum_{evenv_i} (\deg v_i C_2) = 0 \right)$$
$$= 6 + 12$$
$$TDV(P_2 \bullet P_2) = 18$$
$$MD(P_2 \bullet P_2) = \frac{TDV(P_2 \bullet P_2)}{nC_2}$$
$$= \frac{18}{4C_2}$$
$$= \frac{18}{6}$$

Lemma 3.2:
Let P_2 and P_3 be the path with two and three vertices. Then the medium domination number of lexico product of P_2 and P_3 is $MD(P_2 \bullet P_3) = \frac{57}{15}$.

Proof
The diagrammatic representation of the graph is given below



By the definition of lexico product, we draw the graph $P_2 \bullet P_3$ and it is shown below



In the above graph, there are six vertices and thirteen edges. When we consider the degree of the vertices there are four vertices having degree four and there are two vertices having degree five.

By Theorem 2.1,

$$TDV(P_2 \bullet P_3) = q + \sum_{v_i \in V} (\deg v_i C_2)$$
$$= q + \sum_{oddv_i} (\deg v_i C_2) + \sum_{evenv_i} (\deg v_i C_2)$$
$$= 13 + 2(5C_2) + 4(4C_2)$$
$$= 13 + 20 + 24$$
$$TDV(P_2 \bullet P_2) = 57$$
$$MD(P_2 \bullet P_3) = \frac{TDV(P_2 \bullet P_3)}{nC_2}$$
$$= \frac{57}{6C_2}$$
$$= \frac{57}{15}$$

Theorem 3.1:
The medium domination number of lexico product of P_2 and P_n is $MD(P_2 \bullet P_n) = \frac{n^3 + 4n^2 - 6}{2nC_2}$.

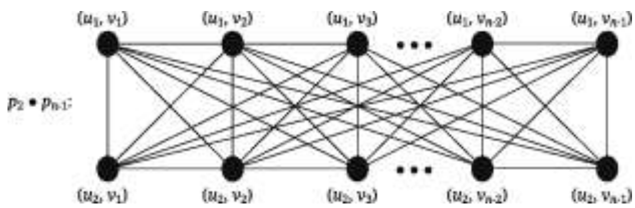
Proof
We prove this by induction on n.

Step 1:
By Lemma 3.1, clearly the result is true for the lexico product of the paths P_2 with P_2.

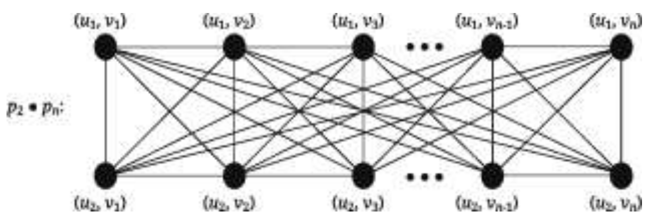
Step 2:
Now we assuming that this is true for p_2 and p_{n-1} and $TDV(P_2 \bullet P_{n-1}) = (n-1)^3 + 4(n-1)^2 - 6$.

Step 3:
To prove the result is true for P_n.
Let us consider the graphs P_2 and P_{n-1}.

Using the definition of the lexico product, we constructed the graph $P_2 \bullet P_{n-1}$



The graph $P_2 \bullet P_{n-1}$ has 2(n – 1) vertices and (n^2 + 2n – 2) edges. In this figure there are 4 vertices having degree n and (2n – 6) vertices having (n + 1). Now consider the graph $P_2 \bullet P_n$



The graph $P_2 \bullet P_n$ has $2n$ vertices and $(n^2 + 4n + 1)$ edges. In this figure there are 4 vertices having degree $(n + 1)$ and $(2n - 4)$ vertices having $(n + 2)$.

By comparing this graph with $P_2 \bullet P_{n-1}$, the graph $P_2 \bullet P_n$ has 2 new vertices and $(2n + 1)$ edges than $P_2 \bullet P_{n-1}$. These extra vertices gives the additional term $3n^2 + 3n - 4$. By the discussion, we observed that from one stage to another stage there is an increase in the number of edges as $(2n + 1)$ and 2 vertices are added.

$$TDV(P_2 \bullet P_n) = TDV(P_2 \bullet P_{n-1}) + (2n+1) + (3n^2 + 3n - 4)$$

$$= (n-1)^3 + 4(n-1)^2 - 6 + 2n + 1 + 3n^2 + 3n - 4$$

$$= (n^3 + 3n - 3n^2 - 1) + 4(n^2 + 1 - 2n) - 6 + 2n + 1 + 3n^2 + 3n - 4$$

$$TDV(P_2 \bullet P_n) = n^3 + 4n^2 - 6$$

$$MD(P_2 \bullet P_n) = \frac{TDV(P_2 \bullet P_n)}{2nC_2}$$

$$= \frac{n^3 + 4n^2 - 6}{2nC_2}$$

$$\therefore MD(P_2 \bullet P_n) = \frac{n^3 + 4n^2 - 6}{2nC_2}$$

4. Conclusion

Many solidity parameters have been studied like domination number and independence number in graph theory concepts. Here we are considering all pairs of vertices and identifying vertices which are dominating both ends. In this paper, we obtained the bound of the medium domination number of lexico product of P_2 and P_n.

References

[1] Duygu Vargör, Pınar Dündar, The Medium domination number of a graph, Int. J. Pure and Applied Mathematics, 70(2011), 297–306

[2] G. Sabidussi, Graph multiplication. Math. Z., 72(1960), 446–457