International Journal of Engineering & Technology, 7 (4.10) (2018) 417-421

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Effect of Thermal Radiation on Magneto-Convection of a Micropolar Nanoliquid towards a Non-Linear Stretching Surface with Convective Boundary

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Abstract

The objective of this paper is to analyze the effect of thermal radiation on MHD mixed convection flow of a micropolar nanoliquid towards a non-linear stretching surface with convective boundary condition. The governing equations are converted into non-linear ordinary differential equations by using suitable similarity transformations. The homotopy analysis method is used for solving the non-linear ordinary differential equations. The temperature profiles increase due to increase in thermal radiation parameter. The microrotation profile increases when boundary parameter is increased. Also, the skin friction coefficient and local Nusselt are plotted for various parameters.

Keywords: *Thermal radiation; Micropolar nanoliquid; Mixed convection; MHD; Stretching surface.*

1. Introduction

The importance of MHD convective flow and heat(& mass) transfer of fluid is discussed in [1]-[3]. Jagan et al. [4] discussed about the effect of thermal radiation on 3D unsteady MHD nanofluid flow over a stretching surface in a porous medium. Mishra et al. [5] analyzed about the chemical reaction and Soret effects on hydromagnetic micropolar fluid over a stretching sheet. Abou-zeid [6] analyzed about the effects of thermal-diffusion and viscous dissipation on peristaltic flow of micropolar non-Newtonian nanofluid. Mixed convection flow of a micropolar fluid in a vertical channel with boundary conditions of third kind was studied by Umavathi et al. [7]. Hayat et al. [8] examined about the mixed convection flow of a micropolar fluid in the presence of radiation and chemical reaction. Mahmoud et al. [9] examined about the MHD flow and heat transfer of a micropolar fluid over a stretching surface with heat generation/absorption and slip velocity. In this paper, the effect of thermal radiation on magneto-convection of a micropolar nanoliquid towards a non-linear stretching sheet with convective boundary is analyzed.

2. Mathematical Formulation

The MHD mixed convection flow and heat transfer of micropolar nanoliquid towards a nonlinear stretching sheet at y=0 in the presence of thermal radiation are considered. It is assumed that the stretching velocity of the sheet as $u_w(x)=cx^n$ where c is a positive constants. Here $u_w(x)$ is linear if n=1 and non-linear if $n>1$. A non-uniform magnetic field $B=B_0x^{\frac{n-1}{2}}$ which is normal to the surface is imposed along y-axis. It is also assumed that the radiative heat flux as $q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$ and the convective fluid temperature T_f is higher than the ambient temperature T_∞. The governing boundary layer equations for the analysis can be expressed as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\nu+\frac{k_v}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{k_v}{\rho}\frac{\partial N}{\partial y}+g\beta_T\left(T-T_\infty\right)-\frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\left(\frac{\gamma^*}{\rho j}\right)\frac{\partial^2 N}{\partial y^2}-\left(\frac{k_v}{\rho j}\right)\left(2N+\frac{\partial u}{\partial y}\right), \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\sigma B_0^2}{\rho c_p}u^2 + \left(\frac{\mu+k_v}{\rho c_p}\right)\left(\frac{\partial u}{\partial y}\right)^2+\tau D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y} + \frac{\tau D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2-\frac{1}{\rho c_p}\frac{\partial(qr)}{\partial y}, \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}, \tag{5}$$

subject to the boundary conditions:

$$\begin{gathered} u=u_w(x)=c\,x^n,\ v=0,\ N=-m_0\frac{\partial u}{\partial y},\\ -k\frac{\partial T}{\partial y}=h_f\left(T-T_f\right),\ C=C_w\ at\ y=0,\\ u=0,\ N=0,\ T\to T_\infty,\ C\to C_\infty,\ as\ y\to\infty. \end{gathered} \tag{6}$$

where ν, k_v, ρ, g, σ, B_0, γ^*, j, α, μ, c_p, D_B, D_T, τ, m_0, k and h_f are kinematic viscosity, vortex viscosity, density of fluid, acceleration due to gravity, electric fluid conductivity, magnetic field, spin gradient viscosity, microinertia, thermal diffusivity, dynamic viscosity, specific heat, Brownian motion co-efficient, thermophoresis coefficient, ratio between the effective nano-particle materials and fluid heat capacity, boundary parameter, thermal conductivity and convective heat transfer coefficient.
Using the suitable transformation

$$\begin{gathered} \eta=\sqrt{\frac{c(n+1)}{2\nu}}\,x^{\frac{n-1}{2}}\,y,\quad N=c\sqrt{\frac{c(n+1)}{2\nu}}\,x^{\frac{3n-1}{2}}\,g(\eta),\\ \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_f-C_\infty},\ u=cx^n f'(\eta)\\ v=\sqrt{\frac{c\,\nu(n+1)}{2}}\,x^{\frac{n-1}{2}}\left[f(\eta)+\left(\frac{n-1}{n+1}\right)\eta f'(\eta)\right] \end{gathered} \tag{7}$$

the incompressiblity condition of equation (1) is satisfied identically and the equations (2)-(6) are reduced to

$$(1+K)f'''+f\,f''-\frac{2n}{n+1}f'^2+K\,g'+\frac{2}{n+1}\lambda\theta-\frac{2}{n+1}(Ha)^2 f'=0, \tag{8}$$

$$\left(1+\frac{K}{2}\right)g''+f\,g'-\frac{3n-1}{n+1}f'g-\frac{2K}{n+1}\left[2g+f''\right]=0, \tag{9}$$

$$\left(1+\frac{4}{3}Rd\right)\theta''+\Pr f\,\theta'+(1+K)\Pr Ec\,f''^2+\frac{2}{n+1}\Pr Ec\,Ha^2 f'^2+Nb\Pr\theta'\phi'+Nt\Pr\theta'^2=0, \tag{10}$$

$$\phi''+Sc\,f\,\phi'+\frac{Nt}{Nb}\theta''=0. \tag{11}$$

subject to the boundary condition

$$\begin{gathered} f(0)=0,\ f'(0)=1,\ g(0)=-m_0\,f''(0),\\ \theta'(0)=-\gamma\sqrt{\frac{2}{n+1}}\left(1-\theta(0)\right),\ \phi(0)=1,\\ f'(\eta)=0,\ g(\eta)=0,\ \theta(\eta)=0,\ \phi(\eta)=0\ as\ \eta\to\infty. \end{gathered} \tag{12}$$

where K, n, λ, Ha, Pr, Ec, Nb, Nt, Rd, Sc and γ are micropolar parameter, power law index, mixed convection parameter, Hartmann number, Prandtl number, Eckert number, Brownian motion parameter, thermophoresis parameter, thermal radiation parameter, Schmidt number and thermal Boit number and are defined as

$$Ec=\frac{u_w^2}{c_p\left(T_f-T_\infty\right)},\ \gamma=\frac{h_f}{k}\sqrt{\frac{\nu}{c}}x^{-\left(\frac{n-1}{2}\right)},\ Gr_x=\frac{g\beta_T\left(T_f-T_\infty\right)x^3}{\nu^2},$$

$$Ha^2=\frac{\sigma B_0^2}{c\rho x^{n-1}},\ K=\frac{k_v}{\mu},\ \lambda=\frac{Gr_x}{Re_x^2},\ Nb=\frac{\tau D_B\left(C_f-C_\infty\right)}{\nu},$$

$$Nt=\frac{\tau D_T\left(T_f-T_\infty\right)}{\nu T_\infty},\ Pr=\frac{\mu c_p}{k},\ Rd=\frac{4\sigma^* T_\infty^3}{kk^*},\ Re_x=\frac{u_w x}{\nu},\ Sc=\frac{\nu}{D_B}.$$

Using the similarity variables (7), the coefficients of skin-friction and local Nusselt number are given by

$$Re_x^{\frac{1}{2}}C_{f_x}=2\sqrt{\frac{n+1}{2}}\left[1+K\left(1-m_0\right)\right]f''(0),$$

$$Re_x^{-\frac{1}{2}}Nu_x=-\sqrt{\frac{n+1}{2}}\left(1+\frac{4}{3}Rd\right)\theta'(0).$$

3. Method of the Solution

The equations (8) to (12) are solved using HAM by choosing the initial approximation and auxiliary linear operators as

$$f_0(\eta)=1-e^{-\eta},\ g_0(\eta)=m_0 e^{-\eta}$$

$$\theta_0(\eta)=\left(\frac{\gamma}{1+\gamma}\right)e^{-\eta},\ \phi_0(\eta)=e^{-\eta}.$$

$$L_f(f)=\frac{d^3 f}{d\eta^3}-\frac{df}{d\eta},\ L_g(g)=\frac{d^2 g}{d\eta^2}-g,$$

$$L_\theta(\theta)=\frac{d^2\theta}{d\eta^2}-\theta,\ L_\phi(\phi)=\frac{d^2\phi}{d\eta^2}-\phi.$$

which satisfies the property

$$L_f\left[A_1+A_2 e^{-\eta}+A_3 e^{\eta}\right]=0,\ L_g\left[A_4 e^{-\eta}+A_5 e^{\eta}\right]=0,$$

$$L_\theta\left[A_6 e^{-\eta}+A_7 e^{\eta}\right]=0,\ L_\phi\left[A_8 e^{-\eta}+A_9 e^{\eta}\right]=0.$$

where $A_1, A_2, ..., A_9$ are the arbitrary constants.
The h-curve is plotted for K=0.2, λ=0.2, Ha=0.1, γ=0.6, n=1.5, Pr=1, Rd=0.3, Sc=1.5, m_0=0.5, Ec=0.3, Nt=0.2 and Nb=0.2 at 25^{th} order of approximation(see Fig 1) and observed that the ad-

missible values of h_f, h_g, h_θ and h_ϕ are $-1.5 \le h_f \le -0.2$, $-1.5 \le h_g \le -0.2$, $-1.5 \le h_\theta \le -0.2$ and $-1.4 \le h_\phi \le -0.2$. The convergence of $-f''(0), -g'(0), -\theta'(0)$ and $-\phi'(0)$ are tabulated in Table 1 for different order of approximations.

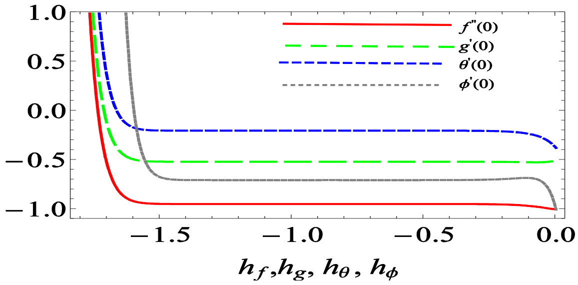

Fig 1: h-curve for $f''(0)$, $g'(0)$, $\theta'(0)$ and $\phi'(0)$

Table 1: Convergence of the series $f''(0)$, $g'(0)$, $\theta'(0)$ and $\phi'(0)$ for *K*=0.2, λ=0.2, *Ha*=0.1, γ=0.6, *n*=1.5, *Pr*=1, *Rd*=0.3, *Sc*=1.5, m_0=0.5, *Ec*=0.3, *Nt*=0.2 and *Nb*=0.2.

m^{th} order approximation	$-f''(0)$	$-g'(0)$	$-\theta'(0)$	$\phi'(0)$
1	0.98880	0.54440	0.22615	0.47500
5	0.94649	0.51667	0.20022	0.69623
10	0.94553	0.51605	0.20025	0.70325
15	0.94569	0.51607	0.20033	0.70264
20	0.94566	0.51607	0.20031	0.70276
25	0.94566	0.51607	0.20031	0.70276

4. Results and Discussion

The discussions are made for different combinations of the pertinent parameters involved in the study. It is clear from Fig 2-4 that the velocity profile enlarges with rise in the micropolar (*K*) and mixed convection (λ) parameters whereas it diminishes with rise in *Ha*. In Fig 5-7, the microrotation profile increases with rise in the Hartmann number (*Ha*) and boundary parameter (m_0) parameters whereas it decreases with rise in λ. In Fig 8-11, temperature profile expands while rising thermal radiation (*Rd*) parameter, Eckert number (*Ec*), *Ha* and thermal Boit number (γ). The skin-friction co-efficient decreases while rising *Ha* whereas it increases while increasing *Rd* and m_0 (see Fig 12-14). The local Nusselt number decreases with rise in *Ec*, but, it increases with increase in γ and *Rd* (see Fig 15-17).

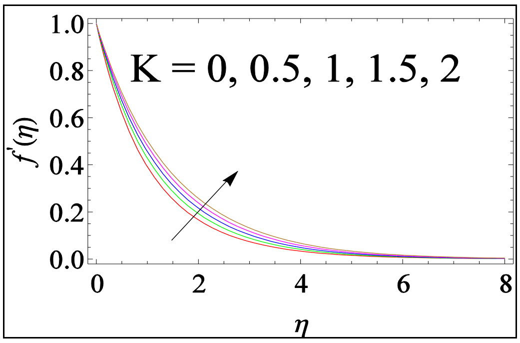

Fig 2: Influence of K on $f'(\eta)$

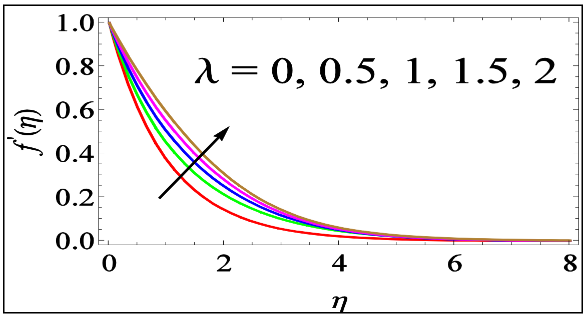

Fig 3: Influence of λ on $f'(\eta)$

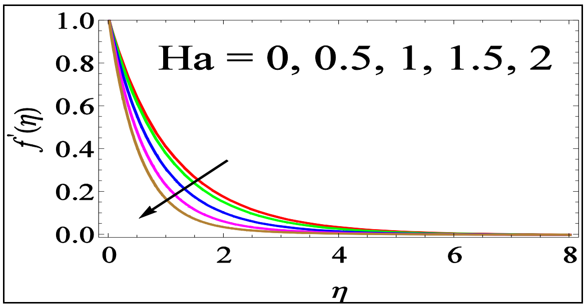

Fig 4: Influence of Ha on $f'(\eta)$

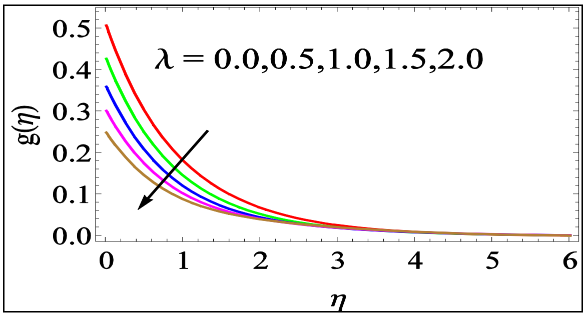

Fig 5: Influence of λ on $g(\eta)$

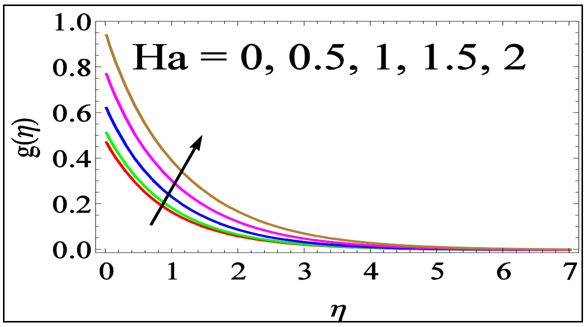

Fig 6: Influence of Ha on $g(\eta)$

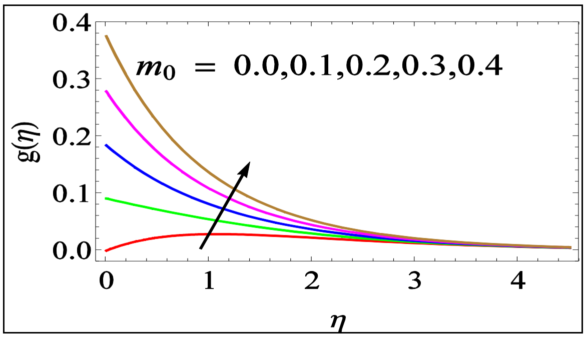

Fig 7: Influence of m_0 on $g(\eta)$

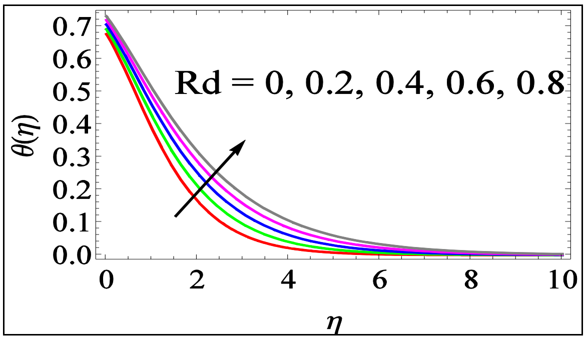

Fig 8: Influence of Rd on $\theta(\eta)$

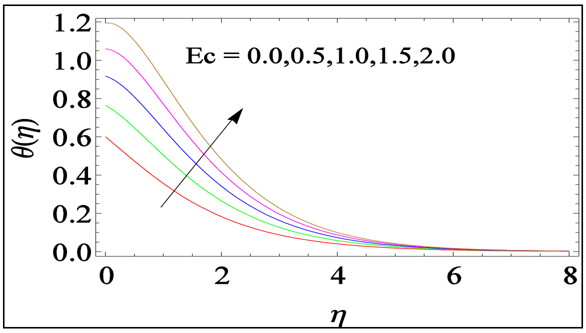

Fig 9: Influence of Ec on $\theta(\eta)$

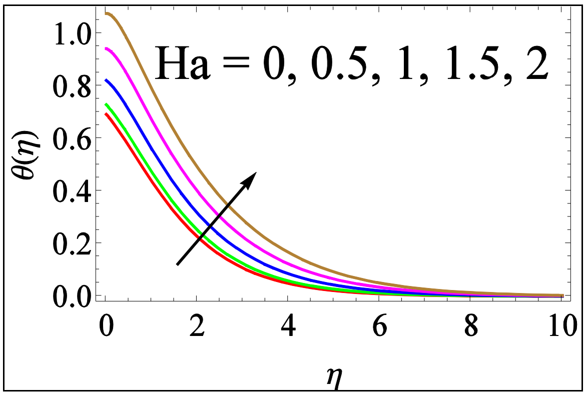

Fig 10: Influence of Ha on $\theta(\eta)$

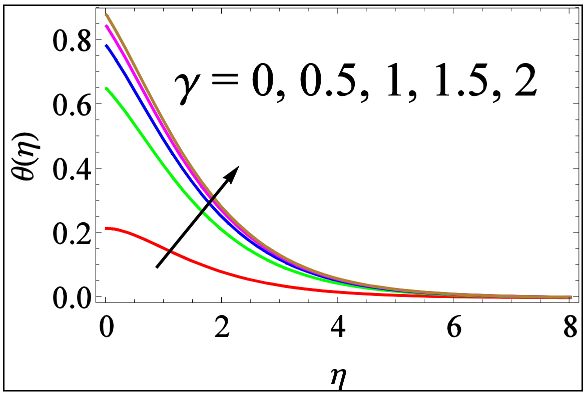

Fig 11: Influence of γ on $\theta(\eta)$

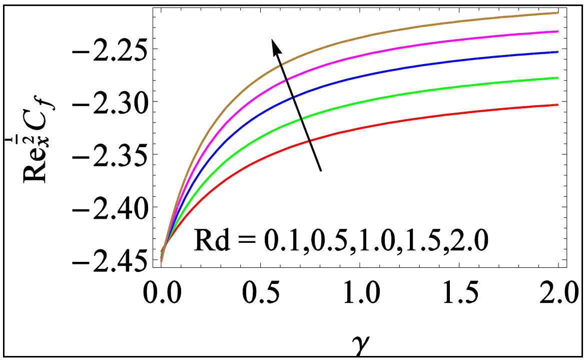

Fig 12: Influence of Rd on $Re_x^{\frac{1}{2}} C_{f_x}$

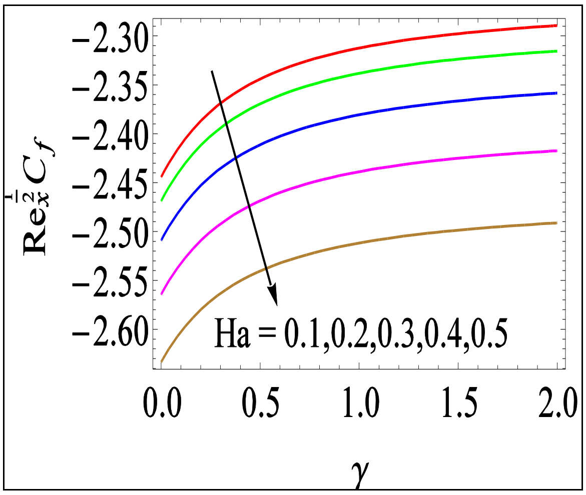

Fig 13: Influence of Ha on $Re_x^{\frac{1}{2}} C_{f_x}$

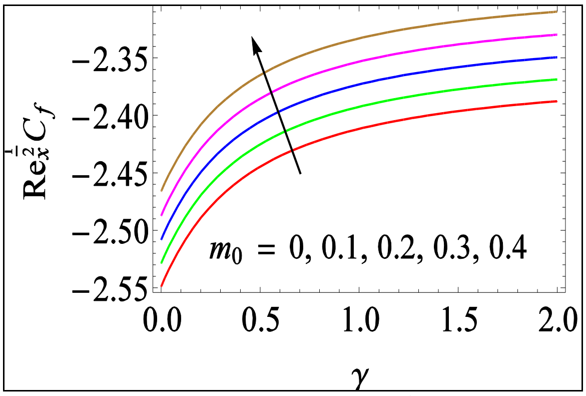

Fig 14: Influence of m_0 on $Re_x^{\frac{1}{2}} C_{f_x}$

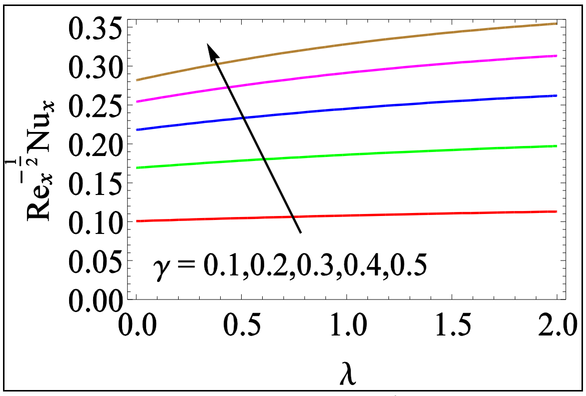

Fig 15: Influence of γ on $Re_x^{\frac{1}{2}} C_{f_x}$

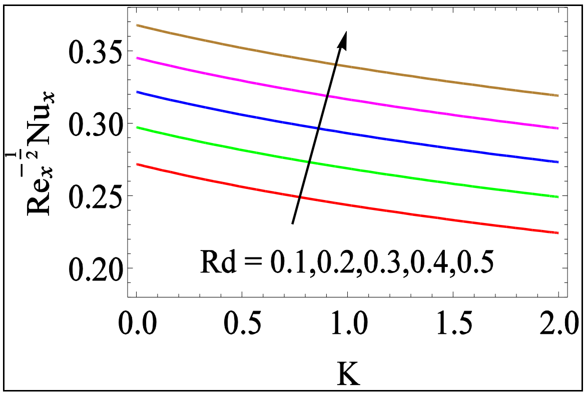

Fig 16: Influence of Rd on $Re_x^{-\frac{1}{2}} Nu_x$

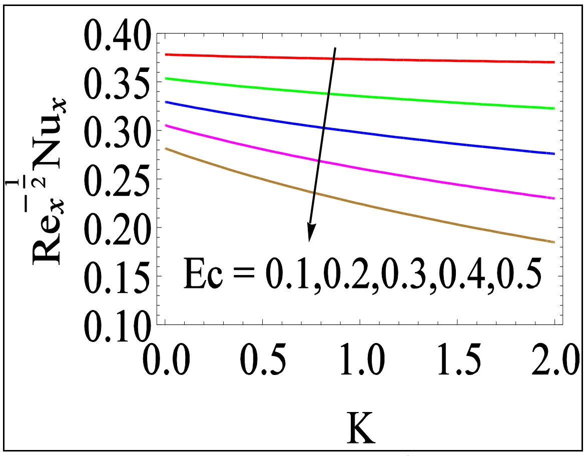

Fig 17: Influence of Ec on $Re_x^{-\frac{1}{2}} Nu_x$

The regression equation of skin friction, Nusselt number and Sherwood number correlated with thermal radiation parameter (Rd), micropolar parameter (K), mixed convection parameter (λ), Biot number (γ) and Hartmann number (Ha) are

$$Re_z^{\frac{1}{2}} C_f = -2.3572 + 0.0664 * Rd - 0.5536 * K + 0.7428 * \lambda + 0.0849 * \gamma - 0.7200 * Ha,$$

$$Re_z^{-\frac{1}{2}} Nu = 0.0840 + 0.2381 * Rd - 0.0136 * K + 0.0873 * \lambda + 0.2494 * \gamma - 0.1079 * Ha,$$

$$Re_z^{-\frac{1}{2}} Sh = 0.8036 + 0.0257 * Rd - 0.0742 * K + 0.0591 * \lambda + 0.0844 * \gamma - 0.0067 * Ha.$$

where Rd, K & λ varies from 0 to 0.5 and γ & Ha varies from 0 to 1.

5. Conclusion

In this paper, the effect of thermal radiation on magneto-hydrodynamic mixed convection flow of a micropolar nanoliquid towards a nonlinear stretching surface with convective boundary condition is presented. The thermal boundary layer thickness, skin friction coefficient and local Nusselt number increase if thermal radiation parameter is increased. Increase in boundary parameter results with increase in microrotation profile and skin friction coefficient. When Hartmann number is increased, the microrotation profile and thermal boundary layer thickness increase where as the momentum boundary layer thickness and skin friction coefficient decrease. The thermal boundary layer thickness increase with increase in Eckert number, but, Nusselt number decrease with increase in Eckert number. Increase in Boit number leads to increase in thermal boundary layer thickness and Nusselt number.

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