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Research paper

Dominating Energy of Operations on Intuitionistic Fuzzy Graphs

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Abstract

The concept of energy of an Intuitionistic Fuzzy Graph is extended to dominating Energy in operations on Intuitionistic Fuzzy Graph. In this paper, We have obtained the value of dominating Energy in different operations such as complement, Union, Join, Cartesian product and composition between two intuitionistic Fuzzy graphs. Also we study the relation between the dominating Energy in the operations on two Intuitionistic Fuzzy Graphs.

***Keywords**: Intuitionistic fuzzy Graph, complements, union, Join of two intuitionistic fuzzy Graph..*

1. Introduction

Fuzzy set has emerged as a potential area of interdisciplinary research and fuzzy graph theory is of recent interest. The concept of a fuzzy graph relation was defined by Zadeh[18] and it has found applications in the analysis of cluster patterns. Rosen field [14] considered the fuzzy relations on fuzzy relations on fuzzy sets and developed the structure of fuzzy graphs.
Atanassov [2,3]introduced the concept of intuitionistic fuzzy relation and intuitionistic fuzzy graphs(IFG).Recent on the theory of intuitionistic fuzzy sets (IFS) has been witnessing an exponential growth of mathematics and its applications. This ranges from normal mathematics to computer sciences, information sciences and communication systems. Graph spectrum appears in problems in Statistical physics and in combinatorial optimization problems in mathematics. Spectrum of a graph also plays an important role in pattern recognition, modelling virus propagation in computer networks and in securing personal data in databases. A concept related to the spectrum of a graph is that of energy.
Let d_i be the degree of i^{th} vertex of G, i =1,2,...,n. The spectrum of the graph G, consisting of the numbers $\lambda_1, \lambda_2, \ldots, \lambda_n$ is the spectrum of its adjacency matrix [8].The Laplacian spectrum of the graph G, consisting of the numbers $\mu_1, \mu_2, \ldots, \mu_n$ is the spectrum of its Laplacian matrix.
The ordinary and laplacian graph eigen values obey the following well-known relations

$$\sum_{i=1}^{n} \lambda_i = 0 \,, \qquad \sum_{i=1}^{n} \lambda_i^2 = 2m \,, \qquad \sum_{i=1}^{n} \mu_i = 2m \,,$$
$$\sum_{i=1}^{n} \mu_i^2 = 2m + \sum_{i=1}^{n} d_i^2$$

In 1960, the study of domination in graphs was begun. In 1862, C.F. De Jaenisch [5] attempted to determine the minimum number of queens required to cover a $n \times n$ chess board. Cockayne [4] introduced the independent domination number in graphs. Domination in graphs has applications to several fields. A. Somasundaram and S. Somasundaram [15] introduced domination in fuzzy graphs in terms of effective edges. A. Nagoorgani and V.T. Chandrasekaran [10] introduced domination using strong arcs. R. Parvathi and G. Thamizhendhi [12] was introduced dominating set, domination number, independent set, total dominating and total domination number in intuitionistic fuzzy graphs. Study on domination concepts in intuitionistic fuzzy graphs are more convenient that fuzzy graphs, which is useful in the traffic density and telecommunication systems.
This paper is organized as follows. In section 2, we give all the basic definitions related to Laplacian energy of an intuitionistic fuzzy graph and domination in an intuitionistic fuzzy graph. .In section 3, we defined the dominating laplacian energy of different operations of an intuitionistic fuzzy graphs and in section 4, we give the conclusion

2. Preliminaries

2.1. Intuitionistic Fuzzy Graph

Definition 2.1.1:-

[11] An intuitionistic fuzzy graph is defined as G=(V,E,μ,γ) where V is the set of vertices and E is the set of edges ,μ is a fuzzy membership function defined on V$\times$V and γ is a fuzzy non membership function we define $\mu(v_i, v_j)$ by μ_{ij} and $\gamma(v_i, v_j)$ by γ_{ij} such that (i) $0 \le \mu_{ij} + \gamma_{ij} \le 1$ (ii) $0 \le \mu_{ij}, \gamma_{ij}, \pi_{ij} \le 1$ where $\pi_{ij} = 1 - \mu_{ij} - \gamma_{ij}$ .Hence $(V \times V, \mu, \gamma)$ is an Intuitionistic fuzzy graph.

Example 2.1.2:

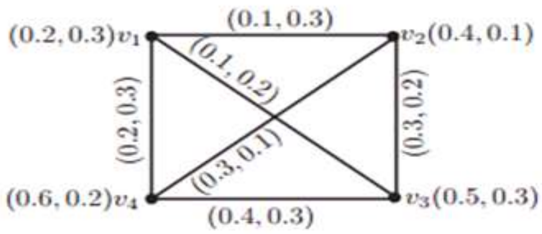

Figure 1 : Intuitionistic Fuzzy Graph G.

2.3: Domination in an Intuitionistic Fuzzy Graph

Definition 2.3.1:-
An intuitionistic Fuzzy graph is of the form G=(V,E) where (i) $V=\{v_1,v_2,...,v_n\}$ such that $\mu : V\to[0,1]$ γ : $V\to[0,1]$ denote the degree of membership and non membership of the element $v\in V$ respectively and $0\le\mu_1(v_i)+\gamma_i(v_i)\le 1$ for every $v_i \in V_i$ (i=1,2,...,n) (ii) $E\subseteq V\times V$ where $\mu_2 : V\times V\to[0,1]$ and $\gamma_2 : V\times V\to[0,1]$ are such that $\mu_2(v_i,v_j)\le\mu_1(v_i)\wedge\mu_1(v_j)$ $\gamma_2(v_i,v_j)\le\gamma_1(v_i)\wedge\gamma_1(v_j)$ and $0\le\mu_2(v_i,v_j)+\gamma_2(v_i,v_j)\le 1$

Definition 2.3.2:-
An arc (v_i,v_j) of an intuitionistic fuzzy graph G is called a strong arc if $\mu_2(v_i,v_j)\le\mu_1(v_i)\wedge\mu_1(v_j)$ and $\gamma_2(v_i,v_j)\le\gamma_1(v_i)\wedge\gamma_1(v_j)$.

Definition 2.3.3:-
Let G=(V,E) an intuitionistic Fuzzy graph. Let $u,v\in V$,we say that u dominates v in G if there exist a strong are between them. A subset $D\subseteq V$ is said to be dominating set In G if for every $v\in V-D$, there exist u in D such that u dominates.

Definition 2.3.4:-
A dominating set D of an intuitionistic fuzzy is said to be minimal dominating set if no proper subset of D is a dominating set. Minimum cardinality among all minimal dominating set is called the intuitionistic fuzzy domination number.

2.4: Dominating Energy in an Intuitionistic Fuzzy Graph

In this section, we consider an intuitionistic fuzzy graph $G=(V,E,\mu,\gamma)$ then we define $(\mu_1,\gamma_1):V\to[0,1]$ and prove that (μ_1,γ_1) is an intuitionistic fuzzy set.

Definition 2.4.1:-
Let $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ be a dominating fuzzy graph .A dominating intuitionistic fuzzy adjacency matrix $D(G)=[d_{ij}]$ where

$$d_{ij}=\begin{cases}(\mu_{ij},\gamma_{ij}) & if\ (v_i,v_j)\in E\\ (1,1) & if\ i=j\ and\ v_i\in D\\ (0,0) & otherwise\end{cases}$$

This dominating intuitionistic fuzzy adjacency matrix D(G) can be written as $D(G)=(\mu_D(G),\gamma_D(G))$
where

$$\mu_D(G)=\begin{cases}\mu_{ij} & if\ (v_i,v_j)\in E\\ 1 & if\ i=j\ and\ v_i\in D\\ 0 & otherwise\end{cases}$$

and

$$\gamma_D(G)=\begin{cases}\gamma_{ij} & if\ (v_i,v_j)\in E\\ 1 & if\ i=j\ and\ v_i\in D\\ 0 & otherwise\end{cases}$$

Let us illustrate these definitions in the following example
In Figure 1 , Consider an IFG, G=(V,E) such that V = $\{v_1,v_2,v_3,v_4\}$,
E= $\{(v_1v_2),(v_2v_3),(v_1v_4),(v_3v_4),(v_2v_4),(v_1v_3)\}$. Consider a dominating intuitionistic fuzzy graph $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ where $V=\{v_1,v_2,v_3,v_4\}$ and μ_1,γ_1 are given by $\mu_1 : V\to[0,1]$ and $\gamma_1 : V\to[0,1]$ where

$$\mu_1(v_1)=\max_{v_j}(\mu(v_1,v_j))$$
$$=\max(\mu(v_1,v_2),\mu(v_1,v_4),\mu(v_1,v_3))$$
$$=\max(0.1,0.2,0.1) = 0.2.$$

Similarly

$$\mu_1(v_2)=\max(0.1,0.3,0.3)=0.3$$
$$\mu_1(v_3)=\max(0.1,0.4,0.3)=0.4$$,
$$\mu_1(v_4)=\max(0.4,0.3,0.2)=0.4$$ and

$$\gamma_1(v_1)=\min_{v_j}(\gamma(v_i,v_j))$$
$$=\min(\gamma(v_1,v_2),\gamma(v_1,v_4),\gamma(v_1,v_3))=\min(0.3,0.3,0.2)=0.2$$

Similarly

$$\gamma_1(v_2)=\min_{v_j}(\gamma(v_i,v_j))=\min(\gamma(v_2,v_1),\gamma(v_2,v_3),\gamma(v_2,v_4))=\min(0.3,0.2,0.1)=0.1$$
,
$$\gamma_1(v_3)=\min_{v_j}(\gamma(v_i,v_j))=\min(\gamma(v_3,v_1),\gamma(v_3,v_2),\gamma(v_3,v_4))=\min(0.2,0.2,0.3)=0.2$$
,
$$\gamma_1(v_4)=\min_{v_j}(\gamma(v_i,v_j))=\min(\gamma(v_4,v_1),\gamma(v_4,v_3),\gamma(v_4,v_2))=\min(0.3,0.3,0.1)=0.1$$

Here V_1 dominates V_3 and V_2 dominates V_4 because $\mu(v_1,v_3)\le\mu_1(v_1)\wedge\mu_1(v_3)$, $\gamma(v_1,v_3)\le\gamma_1(v_1)\wedge\gamma_1(v_3)$ and $\mu(v_2,v_4)\le\mu_1(v_2)\wedge\mu_1(v_4)$, $\gamma(v_2,v_4)\le\gamma_1(v_2)\wedge\gamma_1(v_4)$

Here $V=\{v_1,v_2,v_3,v_4\}$ and Consider a dominating intuitionistic fuzzy graph $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ where $D=\{v_1,v_2\}$, $V-D=\{v_3,v_4\}$

Note that $|D|=2$ = sum of diagonal elements .

For the graph in figure-1,

$$A_D(G)=\begin{bmatrix}(1,1) & (0.1,0.3) & (0.1,0.2) & (0.2,0.3)\\(0.1,0.3) & (1,1) & (0.3,0.2) & (0.3,0.1)\\(0.1,0.2) & (0.3,0.2) & (0,0) & (0.4,0.3)\\(0.2,0.3) & (0.3,0.1) & (0.4,0.3) & (0,0)\end{bmatrix}$$

Where $A_D(\mu(G)=\begin{bmatrix}1 & 0.1 & 0.1 & 0.2\\0.1 & 1 & 0.3 & 0.3\\0.1 & 0.3 & 0 & 0.4\\0.2 & 0.3 & 0.4 & 0\end{bmatrix}$ and

$$A_D(\gamma(G))=\begin{bmatrix}1 & 0.3 & 0.2 & 0.3\\0.3 & 1 & 0.2 & 0.1\\0.2 & 0.2 & 0 & 0.3\\0.3 & 0.1 & 0.3 & 0\end{bmatrix}.$$

Definition 2.4.2:-
The eigen values of dominating intuitionistic fuzzy adjacency matrix D(G) is defined as (X,Y) where X is the set of eigen values of $\mu_D(G)$ and γ is the set of eigen values of $\gamma_D(G)$ .The energy of a dominating intuitionistic fuzzy graph $G=(V,E,\mu,\gamma,\mu_1,\gamma_1)$ is defined $\left(\sum_{\lambda_i\in X}|\lambda_i|,\sum_{\delta_i\in Y}|\delta_i|\right)$ Where $\sum_{\lambda_i\in X}|\lambda_i|$ is the sum of the absolute values of the eigen values of $\mu_D(G)$ and it is denoted by the energy of the membership matrix $E(\mu_D(G))$ and $\sum_{\delta_i\in Y}|\delta_i|$ is the sum of the absolute values of the eigen values of $\gamma_D(G)$ and it is denoted by the energy of the membership matrix $E(\gamma_D(G))$.

3. Dominating Energy in Operations on Intuitionistic Fuzzy Graphs

3.1. Dominating Energy in Complement of an Intuitionistic Fuzzy Graph

Definitions 3.1.1: -The complement of an intuitionistic fuzzy Graph G= (V, E) is an intuitionistic fuzzy graph, $\bar{G}=\langle\bar{V},\bar{E}\rangle$ where

$\bar{\mu}_{1i}=\bar{\mu}_{1i}$ and $\bar{\gamma}_{1i}=\gamma_{1i}$, for all i=1, 2,….,n

$\bar{\mu}_{2ij}=\mu_{1i}\cdot\mu_{1j}-\mu_{2ij}$ and $\bar{\gamma}_{2ij}=\gamma_{1i}\cdot\gamma_{1j}-\gamma_{2ij}$ $for\ all\, i,j=1,2,...,n$.

Example 3.1.2:-
Let V = $\{v_1,v_2,v_3,v_4\}$

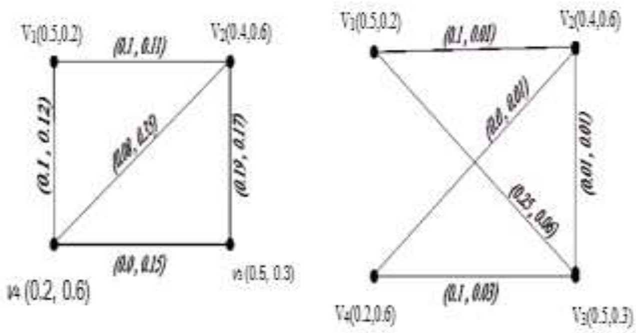

FIG 2(a) FIG 2 (b)

First we find the dominating Energy of Intuitionistic fuzzy Graph G(V,E):-

Consider a dominating intuitionistic fuzzy GraphG=(V,,E,$\boldsymbol{\mu}$, γ ,μ_1 , γ_1) where v={v_1 ,v_2 ,v_3 ,v_4 }
and
$\bar{\gamma}_1$ *are given by* $\mu_1 : V\to[0,1]$ *and* $\gamma_1 : V\to[0,1]$*where*

$$\mu_1(v_1)=\max[\mu(v_1v_2),\mu(v_1v_4)]=\max[0.1,0.1]=0.1.$$

$\mu_1(v_2)=\max\,[\mu(v_2\,v_1),\mu(v_2\,v_3)]=\max[0,1,0.19]=0.19.$
$\mu_1(v_3)=\max[\mu(v_3\,v_2),\mu(v_3\,v_4)]=\max[0.19,0.0]=0.19.$

$$\mu_1(v_4)=\max[\mu(v_4v_3)\mu(v_4v_1)]=\max[0.0,0.1]=0.1.$$

$\gamma_1(v_1)$ = min [γ (v_i v_j)]= min[γ (v_1v_2), γ (v_1v_4)] = min[0.11,0.12] = 0.11.
γ_1 (v_2) =min[γ (v_2v_1), γ (v_2v_3)] = min[0.11,0.17] = 0.11.
γ_1 (v_3) =min [γ (v_3v_2), γ (v_3v_4)] = min[0.17,0.15] = 0.15.

$$\gamma_1(v_4)=\min[\gamma(v_4v_3),\gamma(v_4v_1)]=\min[0.15,0.12]=0.12.$$

Here v_1 dominates v_2 because

$$\mu(v_1v_2)\le\mu_1(v_1)\wedge\mu_1(v_2)$$
$$0.1\le0.1\wedge0.19$$
$$\gamma(v_1v_2)\le\gamma_1(v_1)\wedge\gamma_1(v_2)$$
$$0.11\le0.11\wedge0.11$$

Here v_3 dominates v_4 because

$$\mu(v_3v_4)\le\mu_1(v_3)\wedge\mu_1(v_4)$$
$$0.0\le0.19\wedge0.1$$
$$\bar{V}=V \qquad \gamma(v_3v_4)\le\gamma_1(v_3)\wedge\gamma_1(v_4)$$
$$0.15\le0.15\wedge0.12$$

Here V={v_1 ,v_2 ,v_3 ,v_4 } and D={v_1 ,v_3 } , V-D={v_2 ,v_4 } |D| =2= sum of dominating elements.

Definition :

Let G=[V,E, μ, γ , μ_1, γ_1] be a dominating Intuitionistic Fuzzy Graph. A dominating Intuitionistic Fuzzy adjacency matrix D(G)=(d_{ij}) where

$$d_{ij}=\begin{cases}(\mu_{ij},\gamma_{ij}) & if\ (v_i,v_j)\in E\\ (1,1) & if\ i=j\ \&\ v_i\in D\\ 0 & otherwise\end{cases}$$

$$D(G)=\begin{bmatrix}(1,1) & (0.1,0.11) & (0,0) & (0.1,0.12)\\ (0.1,0.11) & (0,0) & (0.19,0.17) & (0,0)\\ (0,0) & (0.19,0.17) & (1,1) & (0.0,0.15)\\ (0.1,0.12) & (0,0) & (0.0,0.15) & (0,0)\end{bmatrix}$$

$$\mu_D(G)=\begin{bmatrix}1 & 0.1 & 0 & 0.1\\ 0.1 & 0 & 0.19 & 0\\ 0 & 0.19 & 1 & 0\\ 0.1 & 0 & 0 & 0\end{bmatrix},$$

$$\gamma_D(G)=\begin{bmatrix}1 & 0.11 & 0 & 0.12\\ 0.11 & 0 & 0.17 & 0\\ 0 & 0.17 & 1 & 0.15\\ 0.12 & 0 & 0.15 & 0\end{bmatrix}$$

Eigen values of $\mu_D(G)=\{-0.0465,-0.0074,1.0074,1.0465\}$=Spectrum of $\mu_D(G)$

Eigen values of $\gamma_D(G)$ ={-0.0725,-0.0002,1.0002,1.0725}= Spectrum of $\gamma_D(G)$

Dominating Energy of Intuitionistic fuzzy graph G=(V,E)=

$$\left[\sum_{\lambda_i\in X}|\lambda_i|,\sum_{\delta_i\in Y}|\delta_i|\right]=[2.1078,2.1454].$$

Now we find the Dominating Energy of complement of an Intuitionistic Fuzzy Graph G(V,E):-

Consider a dominating Intuitionistic Fuzzy graph G=V,E,μ, γ ,μ_1, γ_1) where V={v_1,v_2,v_3,v_4} and μ_1 and γ_1 given by μ_1:V→[0.1] and γ_1:V→[0,1] where

$$\mu_1(v_1)=\max[\mu(v_1v_2),\mu(v_1v_3)]$$
$$=\max[0.1,0.25]=0.25$$

$\mu_1(v_2)=\max[\mu(v_2 v_1),\mu(v_2 v_4),\mu(v_2 v_3)]=\max[0.1,0.0,0.01]=0.1$

$\mu_1(v_3)=\max[\mu(v_3 v_4),\mu(v_3 v_1),\mu(v_3 v_2)]=\max[0.1,0.25,0.01]=0.25$

$\mu_1(v_4)=\max[\mu(v_4 v_3)],\mu(v_4 v_2),=\max[0.1,0.0]=0.1$

$\gamma_1(v_1)=\min[\gamma(v_i v_j)]=\min[\gamma(v_1 v_2), \gamma(v_1 v_3)]=\min[0.01,0.06]=0.01$

$\gamma_1(v_2)=\min[\gamma(v_2 v_1), \gamma(v_2 v_4), \gamma(v_2 v_3)]=\min[0.01,0.01,0.01]=0.01$

$\gamma_1(v_3)=\min[\gamma(v_3 v_4)\ \gamma(v_3 v_1), \gamma(v_3 v_2)]=\min[0.03,0.06,0.01]=0.01$

$\gamma_1(v_4)=\min[\gamma(v_4 v_3), \gamma(v_4 v_2)]=\min[0.03,0.01]=0.01$

Here v_1 dominates v_2 because

$$\mu(v_1v_2)\le\mu_1(v_1)\wedge\mu_1(v_2)$$
$$0.1\le 0.25\wedge 0.1$$
$$\gamma(v_1v_2)\le\gamma_1(v_1)\wedge\gamma_1(v_2)$$
$$0.01\le 0.01\wedge 0.01$$

Here v_2 dominates v_3 because

$$\mu(v_2v_3)\le\mu_1(v_2)\wedge\mu_1(v_3)$$
$$0.01\le 0.1\wedge 0.25$$
$$\gamma(v_2v_3)\le\gamma_1(v_2)\wedge\gamma_1(v_3)$$
$$0.01\le 0.01\wedge 0.01$$

Here V={v_1 ,v_2 ,v_3 ,v_4 } and D={v_1 ,v_2 } V-D={v_3 ,v_4 }

|D| =2=sum of dominating elements

$$D(G)=\begin{bmatrix}(1,1) & (0.1,0.01) & (0.25,0.06) & (0,0)\\ (0.1,0.01) & (1,1) & (0.01,0.01) & (0,0.01)\\ (0.25,0.06) & (0.01,0.01) & (0,0) & (0.1,0.03)\\ (0,0) & (0,0.01) & (0.1,0.03) & (0,0)\end{bmatrix}$$

$$\mu_D(G)=\begin{bmatrix}1 & 0.1 & 0.25 & 0\\ 0.1 & 1 & 0.01 & 0\\ 0.25 & 0.01 & 0 & 0.1\\ 0 & 0 & 0.1 & 0\end{bmatrix},$$

$$\gamma_D(G)=\begin{bmatrix}1 & 0.01 & 0.06 & 0\\ 0.01 & 1 & 0.01 & 0.01\\ 0.06 & 0.01 & 0 & 0.03\\ 0 & 0.01 & 0.03 & 0\end{bmatrix}$$

Eigen values of μ_D(G)={-0.1314,0.0717,0.9260,1.1337}=2.2628

Eigen values of γ_D (G)={-0.0318,0.0280,0.9912,1.0126}=2.0636

Dominating Energy of Complement of Intuitionistic Fuzzy graph G =(V,E)

$$=\left[\sum_{\lambda_i\in X}|\lambda_i|,\sum_{\delta_i\in Y}|\delta_i|\right]=[2.2628,2.0636].$$

Comparison between Dominating Energy of Intuitionist Fuzzy Graph and its complement:- we observe here that laplacian energy of membership function of an intuitionistic fuzzy graph is less than its complement and laplacian energy of non membership function of an intuitionistic fuzzy graph is greater than its complement.

3.2. Dominating Energy in Union of Intuitionistic Fuzzy Graph

Definition 3.2.1:-

Let G_1 = < V_1, E_1> and G_2= <V_2, E_2> be two Intuitionistic Fuzzy Graphs with $V_1 \cap V_2=\phi$ and $G=G_1\cup G_2$=<$V_1\cup V_2$, $E_1\cup E_2$> be the Union of G_1 and G_2. Then the Union of Intuitionistic fuzzy graphs G_1 and G_2 is an intuitionistic fuzzy Graph defined by

$$(\mu_1\cup\mu_1')(v)=\begin{cases}\mu_1(v) & if\ \ v\in v_1-v_2\\ \mu_1'(v) & if\ \ v\in v_2-v_1\end{cases}$$

$$(\gamma_1\cup\gamma_1')(v)=\begin{cases}\gamma_1(v) & if\ \ v\in v_1-v_2\\ \gamma_1'(v) & if\ \ v\in v_2-v_1\end{cases}$$ and

$$(\mu_2 \cup \mu_2')(v_i, v_j) = \begin{cases} \mu_2(e_{ij}) & if\ e_{ij} \in E_1 - E_2 \\ \mu_2'(e_{ij}) & if\ e_{ij} \in E_2 - E_1 \end{cases}$$

Where (μ_1, γ_1) and (μ_1', γ_1') refer the vertex membership and non-membership of G_1 and G_2 respectively; (μ_2, γ_2) and (μ_2', γ_2') refer the edge membership and non-membership of G_1 and G_2 respectively.

Example 3.2.2:-

Let V_1 = {v_2, v_3, v_4 } and V_2 ={ μ_1, μ_2 } such that

$$v_1 \cap v_2 = \phi$$

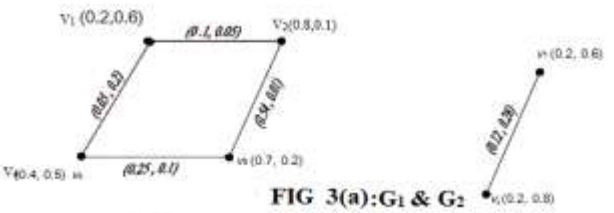

FIG 3(a):G₁ & G₂

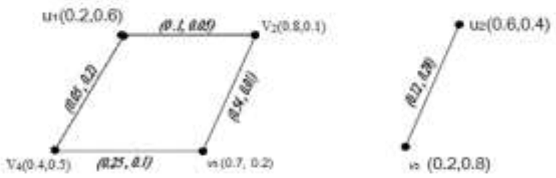

FIG 3(b) :G₁ UG₂

First we find the dominating Energy of Intuitionistic Fuzzy Graph G_1(V,E):-

Consider a dominating intuitionistic fuzzy Graph

G_1=(V,,E,μ,ϒ,μ_1 ,ϒ$_1$) where v={v_1 ,v_2 ,v_3 ,v_4 } and μ_1 and

γ_1 *are given by* $\mu_1: V \to [0,1]$ and $\gamma_1: V \to [0,1]$ *where*

$$\mu_1(v_1) = \max[\mu(v_1v_2), \mu(v_1v_4)]$$
$$= \max[0.1, 0.05] = 0.1$$
$$\mu_1(v_2) = \max[\mu(v_2v_1), \mu(v_2v_3)]$$
$$= \max[0.1, 0.54] = 0.54.$$
$$\mu_1(v_3) = \max[\mu(v_3v_2), \mu(v_3v_4)]$$
$$= \max[0.54, 0.25] = 0.54.$$
$$\mu_1(v_4) = \max[\mu(v_4v_3), \mu(v_4v_1)] = \max[0.25, 0.05] = 0.25$$

γ_1(v_1) = min [γ (v_iv_j)]=

min[γ (v_1v_2), γ (v_1v_4)] = min[0.05,0.2] = 0.05

γ_1 (v_2) =min [γ (v_2v_1), γ (v_2v_3)] = min[0.05,0.01] = 0.01.

γ_1 (v_3) =min [γ (v_3v_2), γ (v_3v_4)] = min[0.01,0.1] = 0.01

γ_1 (v_4) =min [γ (v_4v_3), γ (v_4v_1)] = min[0.1,0.2] = 0.1.

Here v_1 dominates v_2 because

$$\mu(v_1v_2) \le \mu_1(v_1) \wedge \mu_1(v_2)$$
$$0.1 \le 0.1 \wedge 0.54$$
$$\gamma(v_1v_2) \le \gamma_1(v_1) \wedge \gamma_1(v_2)$$
$$0.05 \le 0.05 \wedge 0.01$$

Here v_2 dominates v_3 because

$$\mu(v_2v_3) \le \mu_1(v_2) \wedge \mu_1(v_3)$$
$$0.54 \le 0.54 \wedge 0.54$$
$$\gamma(v_2v_3) \le \gamma_1(v_2) \wedge \gamma_1(v_3)$$
$$0.01 \le 0.01 \wedge 0.01$$

Here v_3 dominates v_4 because

$$\mu(v_3v_4) \le \mu_1(v_3) \wedge \mu_1(v_4)$$
$$0.25 \le 0.54 \wedge 0.25$$
$$\gamma(v_3v_4) \le \gamma_1(v_3) \wedge \gamma_1(v_4)$$
$$0.1 \le 0.01 \wedge 0.1$$

Here V={v_1 ,v_2 ,v_3 ,v_4 } and D={v_1 ,v_2 ,v_3 } V-D={ v_4 }

|D| =3= sum of dominating elements.

$$D(G_1) = \begin{bmatrix} (1,1) & (0.1, 0.05) & (0,0) & (0.05, 0.2) \\ (0.1, 0.05) & (1,1) & (0.54, 0.01) & (0,0) \\ (0,0) & (0.54, 0.01) & (1,1) & (0.25, 0.1) \\ (0.05, 0.2) & (0,0) & (0.25, 0.1) & (0,0) \end{bmatrix}$$

$$\mu_D(G_1) = \begin{bmatrix} 1 & 0.1 & 0 & 0.05 \\ 0.1 & 1 & 0.54 & 0 \\ 0 & 0.54 & 1 & 0.25 \\ 0.05 & 0 & 0.25 & 0 \end{bmatrix}$$

$$\gamma_D(G_1) = \begin{bmatrix} 1 & 0.05 & 0 & 0.2 \\ 0.05 & 1 & 0.01 & 0 \\ 0 & 0.01 & 1 & 0.1 \\ 0.2 & 0 & 0.1 & 0 \end{bmatrix}$$

Eigen Values ofμ_D(G_1)=[-0.0810,0.5108,1.0000,1.5703]= 3.1621.

Eigen values of γ $_D$(G_1)=[-0.0478,0.9663,1.0033,1.0782] =3.0956.

Dominating Energy of UNION of IFG $\bar{G}_1 = (\bar{V}, \bar{E})$ is =

$$\left[\sum_{\lambda_i \in X} |\bar{\lambda}_i|, \sum_{\delta_i \in Y} |\bar{\delta}_i| \right]$$

= (3.1621, 3.0956)

Also we find the dominating Energy of Intuitionistic Fuzzy Graph G_2(V,E):-

Let V={v_1 , v_2 }

$$\mu_1(v_1) = \max[\mu(v_1v_2)] = \max[0.12] = 0.12.$$
$$\mu_1(v_2) = \max[\mu(v_2v_1)] = \max[0.12] = 0.12.$$

γ_1 (v_1) =min [γ (v_1 v_2)] = min[0.28] = 0.28

γ_1 (v_2) =min [γ (v_2v_1)] = min[0.28] = 0.28

Here v_1 dominates v_2 because

$$\mu(v_1v_2) \le \mu_1(v_1) \wedge \mu_1(v_2)$$
$$0.12 \le 0.12 \wedge 0.12$$
$$\gamma(v_1v_2) \le \gamma_1(v_1) \wedge \gamma_1(v_2)$$
$$0.28 \le 0.28 \wedge 0.28$$

Here V={v_1 ,v_2} and D = {v_1};$V - D$ = {V_2}

D|=1= sum of dominating elements.

$$D(G_2)=\begin{bmatrix}(1,1) & (0.12,0.28)\\(0.12,0.28) & (0,0)\end{bmatrix}$$

$$\mu_D(G_2)=\begin{bmatrix}1 & 0.12\\0.12 & 0\end{bmatrix}$$

$$\gamma_D(G_2)=\begin{bmatrix}1 & 0.28\\0.28 & 0\end{bmatrix}$$

Eigen values of μ_D (G_2) = [-0.0142, 1.0142] =1.0284.
Eigen values of γ_D (G_2) = [-0.0731, 1.0731] = 1.1462.

Dominating laplacian energy of Intuitionistic fuzzy graph G_2 i.e $\bar{G}_2=(\bar{V},\bar{E})$ is = $\left[\sum_{\lambda_i\in X}\left|\bar{\lambda}_i\right|,\sum_{\delta_i\in Y}\left|\bar{\delta}_i\right|\right]$

= (1.0284,1.1462)

Now we find the dominating energy of union of two Intuitionistic fuzzy graphs $G_1\cup G_2$

$$\mu_1(v_1)=max[\mu(v_1v_2),\mu(v_1v_4)]$$
$$=\max[0.1,0.05]=0.1$$
$$\mu_1(v_2)=\max[\mu(v_2v_1),\mu(v_2v_3)]$$
$$=\max[0.1,0.54]=0.54$$
$$\mu_1(v_3)=\max[\mu(v_3v_2),\mu(v_3v_4)]$$
$$=\max[0.54,0.25]=0.54$$
$$\mu_1(v_4)=\max[\mu(v_4v_3),\mu(v_4v_1)]=\max[0.25,0.05]=0.25$$
$$\mu_1(u_2)=\max[\mu(u_2u_1)]=\max[0.12]=0.12$$
$$\gamma_1(v_1)=\min[\gamma(v_1v_2),\gamma(v_1v_4)]=\min[0.05,0.2]=0.05$$
$$\mu_1(u_1)=\max[\mu(u_1u_2)]=\max[0.12]=0.12.$$
$$\gamma_1(v_2)=\min[\gamma(v_2v_1),\gamma(v_2v_3)]=\min[0.05,0.01]=0.01$$
$$\gamma_1(v_3)=\min[\gamma(v_3v_2),\gamma(v_3v_4)]=\min[0.01,0.1]=0.01$$
$$\gamma_1(u_1)=\min[\gamma(u_1u_2)]=\min[0.28]=0.28$$
$$\gamma_1(u_2)=\min[\gamma(u_2u_1)]=\min[0.28]=0.28$$

Here v_1 dominates v_2 because

$$\mu(v_1v_2)\le\mu_1(v_1)\wedge\mu_1(v_2)$$
$$0.1\le 0.1\wedge 0.54$$

Here v_2 dominates v_3 because

$$\mu(v_2v_3)\le\mu_1(v_2)\wedge\mu_1(v_3)$$
$$\gamma(v_2v_3)\le\gamma_1(v_2)\wedge\gamma_1(v_3)$$
$$0.54\le 0.54\wedge 0.54$$
$$0.01\le 0.01\wedge 0.01$$

Here v_3 dominates v_4 because

$$\mu(v_3v_4)\le\mu_1(v_3)\wedge\mu_1(v_4)$$
$$\gamma(v_3v_4)\le\gamma_1(v_3)\wedge\gamma_1(v_4)$$
$$0.25\le 0.54\wedge 0.25$$
$$0.1\le 0.01\wedge 0.1$$

Here v_4 not dominates v_1 because

$$\mu(v_4v_1)\le\mu_1(v_4)\wedge\mu_1(v_1)$$
$$\gamma(v_4v_1)\le\gamma_1(v_4)\wedge\gamma_1(v_1)$$
$$0.05\le 0.25\wedge 0.1$$
$$0.2\le 0.1\wedge 0.05$$

Here u_1 dominates u_2 because

$$\mu_1(u_1u_2)\le\mu_1(u_1)\wedge\mu_1(u_2)$$
$$\gamma(u_1u_2)\le\gamma_1(u_1)\wedge\gamma_1(u_2)$$
$$0.12\le 0.12\wedge 0.12$$
$$0.28\le 0.28\wedge 0.28$$

Here V={v_1,v_2,v_3,v_4,u_1,u_2} and D={v_1,v_2,v_3,u_1} V-D={v_4,u_2}

$|D|$ =4= Sum of dominating elements.

$$D(G_1\cup G_2)=\begin{bmatrix}(1,1) & (0.1,0.05) & (0,0) & (0.05,0.2) & (0,0) & (0,0)\\(0.1,0.05) & (1,1) & (0.54,0.01) & (0,0) & (0,0) & (0,0)\\(0,0) & (0.54,0.01) & (1,1) & (0.25,0.1) & (0,0) & (0,0)\\(0.05,0.2) & (0,0) & (0.25,0.1) & (0,0) & (0,0) & (0,0)\\(0,0) & (0,0) & (0,0) & (0,0) & (1,1) & (0.12,0.28)\\(0,0) & (0,0) & (0,0) & (0,0) & (0.12,0.28) & (0,0)\end{bmatrix}$$

$$\mu_D(G_1\cup G_2)=\begin{bmatrix}1 & 0.1 & 0 & 0.05 & 0 & 0\\0.1 & 1 & 0.54 & 0 & 0 & 0\\0 & 0.54 & 1 & 0.25 & 0 & 0\\0.05 & 0 & 0.25 & 0 & 0 & 0\\0 & 0 & 0 & 0 & 1 & 0.12\\0 & 0 & 0 & 0 & 0.12 & 0\end{bmatrix}$$

$$\gamma_D(G_1\cup G_2)=\begin{bmatrix}1 & 0.05 & 0 & 0.2 & 0 & 0\\0.05 & 1 & 0.01 & 0 & 0 & 0\\0 & 0.01 & 1 & 0.1 & 0 & 0\\0.2 & 0 & 0.1 & 0 & 0 & 0\\0 & 0 & 0 & 0 & 1 & 0.28\\0 & 0 & 0 & 0 & 0.28 & 0\end{bmatrix}$$

Eigen values of μ_D ($G_1\cup G_2$) = {-0.0810,-0.0142,0.5108,1.0000,1.0142,1.5703}
=4.1905.

3.3. Dominating Energy in Join of Intutionistic Fuzzy Graphs:-

Definations 3.3.1:-

The join of two Intutionistic fuzzy graph G=G_1 + G_2 =<v_1 ∪ v_2 , E_1 ∪ E_2 ∪ E^l> defined by

$$(\mu_1+\mu_1')(v)=(\mu_1\cup\mu_1')(v)\quad if\;\; v\in V_1\cup V_2$$

(γ + $\gamma_1{}^l$) (v) = (γ_1 ∪ $\gamma_1{}^l$) (v) If VE v_1 ∪ v_2

(μ_2 + $\mu_2{}^l$) (v_i v_J) = (μ_1 ∪ $\mu_2{}^l$) (v_i v_J) If v_i v_J ϵ E_1 ∪ E_2

=(μ (v_i) $\mu_1{}^l$(v_J) If v_i v_J ϵ E_1

Example 3.3.2:-

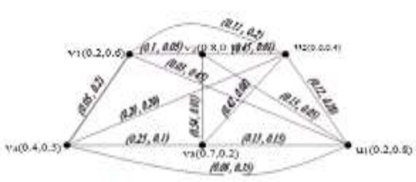

FIG 4: G=G_1 + G_2

Now we find the dominating Energy of join of IFG G(V,E):-
Consider a dominating Intutionistic fuzzy Graph G=(V,,E, μ, γ, μ_1, γ_1) where v={v_1, v_2, v_3, v_4 } and
and γ_1 are given by $\mu : V \to [0,1]$ and $\gamma\ V \to [0,1]$ where

$$\mu_1(v_1) = \max[\mu(v_1v_2), \mu(v_1u_1), \mu(v_1u_2), \mu(v_1v_4)] = \max[0.1, 0.03, 0.11, 0.05] = 0.11.$$
$$\mu_1(v_2) = \max[\mu(v_2u_2), \mu(v_2u_1), \mu(v_2v_3), \mu(v_2v_1)] = \max[0.45, 0.15, 0.54, 0.1] = 0.54.$$
$$\mu_1(v_3) = \max[\mu(v_3u_1), \mu(v_3u_2), \mu(v_3v_2), \mu(v_3v_4)] = \max[0.13, 0.42, 0.54, 0.25] = 0.54.$$
$$\mu_1(v_4) = \max[\mu(v_4v_1), \mu(v_4u_2), \mu(v_4v_3), \mu(v_4u_1)] = \max[0.05, 0.20, 0.25, 0.08] = 0.25.$$
$$\mu_1(u_1) = \max[\mu(u_1u_2), \mu(u_1v_2), \mu(u_1v_1), \mu(u_1v_3), \mu(u_1v_4)] = \max[0.12, 0.15, 0.03, 0.13, 0.08] = 0.15.$$
$$\mu_1(u_2) = \max[\mu(u_2v_1), \mu(u_2v_2), \mu(u_2v_4), \mu(u_2v_3), \mu(u_2v_1)] = \max[0.11, 0.45, 0.20, 0.42, 0.12] = 0.45$$
$$\gamma_1(v_1) = \min[\gamma(v_1v_2), \gamma(v_1u_1), \gamma(v_1u_2), \gamma(v_1v_4)] = \min[0.05, 0.45, 0.2, 0.2] = 0.05.$$
$$\gamma_1(v_2) = \min[\gamma(v_2u_2), \gamma(v_2u_1), \gamma(v_2v_3), \gamma(v_2v_1)] = \min[0.01, 0.05, 0.01, 0.05] = 0.01.$$
$$\gamma_1(v_3) = \min[\gamma(v_3u_1), \gamma(v_3u_2), \gamma(v_3v_2), \gamma(v_3v_4)] = \min[0.15, 0.08, 0.01, 0.1] = 0.01.$$
$$\gamma_1(v_4) = \min[\gamma(v_4v_1), \gamma(v_4u_2), \gamma(v_4v_3), \gamma(v_4u_1)] = \min[0.2, 0.20, 0.1, 0.35] = 0.1.$$
$$\gamma_1(u_1) = \min[\gamma(u_1u_2), \gamma(u_1v_2), \gamma(u_1v_1), \gamma(u_1v_3), \gamma(u_1v_4)] = \min[0.28, 0.05, 0.45, 0.15, 0.35] = 0.05.$$
$$\gamma_1(u_2) = \min[\gamma(u_2v_1), \gamma(u_2v_2), \gamma(u_2v_4), \gamma(u_2v_3), \gamma(u_2u_1)] = \min[0.2, 0.01, 0.20, 0.08, 0.28] = 0.01.$$

Here v_1 dominates v_2 because
$$\mu(v_1v_2) \le \mu_1(v_1) \wedge \mu_1(v_2)$$
$$0.1 \le 0.11 \wedge 0.54$$
.
$$\gamma(v_1v_2) \le \gamma_1(v_1) \wedge \gamma_1(v_2)$$
$$0.05 \le 0.05 \wedge 0.01$$
Here v_2 dominates v_3 because
$\mu\ (v_2\ v_3) \le \mu\ \mu_1\ (v_2) \wedge \mu_1(v_3)$
$\gamma\ (v_2\ v_3) \le \gamma_1\ (v_2) \wedge \gamma_1(v_3)$
$$0.54 \le 0.54 \wedge 0.54$$
$$0.01 \le 0.01 \wedge 0.01$$
Here v_3 dominates v_4 because
$$\mu(v_3v_4) \le \mu_1(v_3) \wedge \mu_1(v_4)$$
$$0.25 \le 0.54 \wedge 0.25$$
$$\gamma(v_3v_4) \le \gamma_1(v_3) \wedge \gamma_1(v_4)$$
$$0.1 \le 0.01 \wedge 0.1$$
Here u_1 dominates v_2 because
$$\mu(u_1v_2) \le \mu_1(u_1) \wedge \mu_1(v_2)$$
$$0.15 \le 0.15 \wedge 0.54$$
$$\gamma(u_1v_2) \le \gamma_1(u_1) \wedge \gamma_1(v_2)$$
$$0.05 \le 0.05 \wedge 0.01$$
Here u_2 dominates v_2 because
$$\mu(u_2v_2) \le \mu_1(u_2) \wedge \mu_1(v_2)$$
$$0.45 \le 0.45 \wedge 0.54$$

$$\gamma(u_2v_2) \le \gamma_1(u_2) \wedge \gamma_1(v_2)$$
$$0.01 \le 0.01 \wedge 0.01$$
Here V={v_1, v_2, v_3, v_4, u_1, u_2} and D={v_1, v_2, v_3, u_1, u_2 } V-D={ v_4 }
|D| =5= sum of dominating elements.

D(G)=
$$\begin{bmatrix} (1,1) & (0.1,0.05) & (0,0) & (0.05,0.2) & (0.03,0.45) & (0.11,0.2) \\ (0.1,0.05) & (1,1) & (0.54,0.01) & (0,0) & (0.15,0.05) & (0.45,0.01) \\ (0,0) & (0.54,0.01) & (1,1) & (0.25,0.1) & (0.13,0.15) & (0.42,0.08) \\ (0.05,0.2) & (0,0) & (0.25,0.1) & (0,0) & (0.08,0.35) & (0.20,0.20) \\ (0.03,0.45) & (0.15,0.05) & (0.13,0.15) & (0.08,0.35) & (1,1) & (0.12,0.28) \\ (0.11,0.2) & (0.45,0.01) & (0.42,0.08) & (0.20,0.20) & (0.12,0.28) & (1,1) \end{bmatrix}$$

$$\mu_D(G) = \begin{bmatrix} 1 & 0.1 & 0 & 0.05 & 0.03 & 0.11 \\ 0.1 & 1 & 0.54 & 0 & 0.15 & 0.45 \\ 0 & 0.54 & 1 & 0.25 & 0.13 & 0.42 \\ 0.05 & 0 & 0.25 & 0 & 0.08 & 0.02 \\ 0.03 & 0.15 & 0.13 & 0.08 & 1 & 0.12 \\ 0.11 & 0.45 & 0.42 & 0.02 & 0.12 & 1 \end{bmatrix}$$
$$\gamma_D(G) = \begin{bmatrix} 1 & 0.05 & 0 & 0.2 & 0.45 & 0.2 \\ 0.05 & 1 & 0.11 & 0 & 0.05 & 0.01 \\ 0 & 0.11 & 1 & 0.1 & 0.15 & 0.08 \\ 0.2 & 0 & 0.1 & 0 & 0.35 & 0.02 \\ 0.45 & 0.05 & 0.15 & 0.35 & 1 & 0.28 \\ 0.2 & 0.01 & 0.08 & 0.02 & 0.28 & 1 \end{bmatrix}$$

Eigen values of μ_D(G) =[-0.1025,0.5084,0.5989,0.9467,1.0014,2.0471]=5.205.
Eigen values of γ_D(G) =[-0.1263,0.5294,0.8161,0.9902,1.0090,1.7816]=5.2526.
Dominating Energy of Join of IFG $\bar{G} = (\bar{V}, \bar{E})$ is =
$$\left[\sum_{\lambda_i \in X} |\lambda_i|, \sum_{\delta_i \in Y} |\delta_i| \right] = (5.205, 5.2526)$$
Eigen values of $\gamma_D(G_1 \cup G_2)$ ={-0.0731,-0.0478,0.9663,1.0033,1.0731,1.0782}=4.2418.

4. Conclusion

In this paper, we have defined the dominating energy in different operations of intuitionistic fuzzy graphs and examined the dominating energy on complement of an Intuitionistic fuzzy graph , union and join of the two intuitionistic fuzzy graphs.

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