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Research paper

# Bipolar Fuzzy Soft Hyperideals and Homomorphism of Gamma-Hypersemigroups

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## Abstract

In this paper, we introduce the concept of bipolar fuzzy soft gamma hyperideals in gamma hyper semigroups. We define bipolar fuzzy soft hyper ideals, bi-ideals and interior ideals of gamma hyper semigroups and discuss some properties.

***Keywords***: *Soft set, Γ- hyper semigroups, bipolar valued fuzzy set, hyper ideal, homomorphism.*

## 1. Introduction

Zadeh [18] introduced the concept of fuzzy sets in 1965. Algebraic hyper structures represent a natural extension of classical algebraic structures, and they were originally proposed in 1934 by Marty [7]. One of the main reasons which attract researchers towards hyperstructures is its unique property that in hyperstructures composition of two elements is a set, while in classical algebraic structures the composition of two elements is an element. Zhang [19] introduced the notion of bipolar fuzzy sets. Lee [4] used the term bipolar fuzzy sets as applied to algebraic structures. Bipolar fuzzy Γ-hyperideals in Γ-hyper semigroups was studied by Naveed Yaqoob et al [14]. Soft set theory was introduced by Molodtsov [8] in 1999, and its a new mathematical model for dealing with uncertainty from a parameterization point of view. Maji et al [6] studied the some new operations on fuzzy soft sets. Aygunoglu and Aygun [3] introduced the notion of a fuzzy soft group. The concept of bipolar fuzzy soft sets has been introduced by Naz et al [12]. Aslam et al [2] worked on bipolar fuzzy soft sets and their special union and intersection. Bipolar fuzzy soft Γ-semigroups was introduced by Muhammad Akram et al [10]. Γ-semigroups was introduced by Sen and Saha [16]. In this paper, we define a new notion of bipolar fuzzy soft Γ- hyper semigroups and investigate some of its properties with examples.

## 2. Preliminaries

In this section, we list some basic definitions.

**Definition 2.1**[16]
Let $S=\{a,b,c,\dots\}$ and $\{\alpha,\beta,\gamma,\dots\}$ be two non-empty sets. Then S is called a Γ-semigroup if it satisfies the conditions
(i) $a\alpha b\in S$,
(ii)$(a\beta b)\gamma c=a\beta(b\gamma c)\ \forall\ a,b,c\in S$ and $\alpha,\beta,\gamma\in\Gamma$.

**Definition 2.2**
A map $\circ: H\times H\to P^*(H)$ is called a hyper operation or join operation on the set $H$, where $H$ is a non-empty set and $P^*(H)=P(H)\backslash\{\phi\}$ denotes the set of all non-empty subset of $H$. A hypergroupoid is a set $H$ together with a (binary) hyperoperation.

**Definition 2.3**
A hypergroupoid $(H,\circ)$, which is associative, that is $x\circ(y\circ z)=(x\circ y)\circ z$ for all $x,y,z\in H$, is called a hyper semigroup. Let $A$ and $B$ be two non-empty subsets of $H$. Then we define

$$A\circ B=\begin{cases}\bigcup_{a\in A,b\in B} a\circ b, & a\circ B=\{a\}\circ B\\ & A\circ b=A\circ\{b\}\end{cases}$$

**Definition 2.4**[1]
Let $S$ and $\Gamma$ be two non-empty sets. $S$ is called a Γ-hypersemigroup if every $\gamma\in\Gamma$ is a hyperoperation on $S$ that is $x\gamma y\subseteq S$ for every $x,y\in S$, and for every $\alpha,\beta\in\Gamma$ and $x,y,z\in H$ we have $x\alpha(y\beta z)=(x\alpha y)\beta z$. If every $\gamma\in\Gamma$ is a hyper operation, then $S$ is a Γ-semigroup. If $(S,\gamma)$ is a hypergroup for every $\gamma\in\Gamma$, then $S$ is called a Γ-hypergroup. Let $A$ and $B$ be two non-empty subsets of $S$ and $\gamma\in\Gamma$. We define $A\gamma B=\cup\{a\gamma b|a\in A,b\in B\}$. Also $A\Gamma B=\cup\{a\gamma b|a\in A,b\in B \text{ and } \gamma\Gamma\}=\bigcup_{A\gamma B}$. Let $S$ be a Γ-hypersemigroup and let $\gamma\in\Gamma$. A non-empty subset $A$ of $S$ is called a Γ-hypersubsemigroup of $S$ if $a_1\gamma a_2\subseteq A$ for every $a_1,a_2\in A$. A Γ-semihypergroup $S$ is called commutative if for all $x,y\in S$ and $\gamma\in\Gamma$ we have $x\gamma y=y\gamma x$.

**Definition 2.5 [8 ]** Let $U$ be an universel set and $E$ be the set of parameters. $P(U)$ denote the power set of $U$. Let $A$ be a non empty subset of $E$ then the pair $(F,A)$ is called a soft set over $U$, where $F$ is a mapping given by $F: A\to P(U)$.

**Definition 2.6**
[18] Let $X$ be a non-empty set. A fuzzy subset $\mu$ of $X$ is a function from $X$ into the closed unit interval $[0,1]$. The set of all fuzzy subset of $X$ is called the fuzzy power set of $X$ and is denoted by $FP(X)$.

**Definition 2.7**[4]
A bipolar fuzzy set $A$ in a universe $U$ is an object having the form $A = \{\langle x, \mu_A^+(x), \mu_A^-(x)\rangle : x \in X\}$, where $\mu_A^+ : X \to [0,1]$ and $\mu_A^- : X \to [-1,0]$. Here $\mu_A^+(x)$represents the degree of satisfaction of an element $x$ to the property and $A = \{\langle x, \mu_A^+(x), \mu_A^-(x)\rangle : x \in X\}$ and $\mu_A^-(x)$ represents the degree of satisfaction of $x$ to some implict counter property of $A$. For the simplicity the symbol $\langle \mu_A^+, \mu_A^- \rangle$ is used for the bipolar fuzzy set $A = \{\langle x, \mu_A^+(x), \mu_A^-(x)\rangle : x \in X\}$.

**Definition 2.8 [2]**
Let $U$ be the universe set and $E$ be the set of parameter. Let $A \subseteq E$ and $BF^U$ denotes the set of all bipolar fuzzy subsets of $U$. Then a pair $(F, A)$ is called a bipolar fuzzy soft sets over $U$, where $F$ is a mapping given by $F: A \to BF^U$.
It is defined as $(F, A) = \{\langle x, \mu_a^+(x), \mu_a^-(x)\rangle : x \in U \text{and } a \in A\}$ For any

$$a \in A, F(a) = \{\langle x, \mu_{F(a)}^+(x), \mu_{F(a)}^-(x)\rangle : x \in U\} = \langle \mu_{F(a)}^+(x), \mu_{F(a)}^-(x)\rangle.$$

**Definition 2.9 [2]**
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft sets over a common universe $U$, then $(F, A)$ AND $(G, B)$ denoted by $(F, A) \wedge (G, B)$ is defined as $(F, A) \wedge (G, B) = (H, C)$ where $C = A \times B$ and $H(a, b) = F(a) \cap G(b)$, for all $(a, b) \in A \times B$.

**Definition 2.10 1**[2]
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft sets over a common universe $U$, then $(F, A)$ OR $(G, B)$ denoted by $(F, A) \vee (G, B)$ is defined as $(F, A) \vee (G, B) = (H, C)$ where $C = A \times B$ and $H(a, b) = F(a) \cup G(b)$, for all $(a, b) \in A \times B$.

**Definition 2.11 [2]**
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft sets over a common universe $U$ then their extended union is a bipolar fuzzy soft set over $U$ denoted by $(F, A) \cup_\varepsilon (G, B)$ and is defined by $(F, A) \cup_\varepsilon (G, B) = (H, C)$ where $C = A \cup B$, $H: C \to BF^U$ and

$$H(c) = \begin{cases} F(c) & if \ c \in A - B \\ G(c) & if \ c \in B - A \\ F(c) \cup G(c) & if \ c \in A \cap B \end{cases}$$

**Definition 2.12** [2]
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft sets over a common universe $U$ then their extended intersection is a bipolar fuzzy soft set over $U$ denoted by $(F, A) \cap_\varepsilon (G, B)$ and is defined by $(F, A) \cup_\varepsilon (G, B) = (H, C)$ where $C = A \cup B$, $H: C \to BF^U$ and

$$H(c) = \begin{cases} F(c) & if \ c \in A - B \\ G(c) & if \ c \in B - A \\ F(c) \cap G(c) & if \ c \in A \cap B \end{cases}$$

**Definition 2.13**[13 ]
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft sets over a common universe $U$ such that $A \cap B \neq \phi$. The restricted union of $(F, A)$ and $(G, B)$ is defined to be a bipolar fuzzy soft set $(H, C)$ over $U$ where $C = A \cap B$ and $H(c) = F(c) \cup G(c)$, for all $c \in C$. This is denoted by $(H, C) = (F, A) \cup_R (G, B)$.

**Definition 2.14** [11]
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft sets over a common universe $U$ such that $A \cap B \neq \phi$. The restricted intersection of $(F, A)$ and $(G, B)$ is defined to be a bipolar fuzzy soft set $(H, C)$ over $U$ where $C = A \cap B$ and $H(c) = F(c) \cap G(c)$, for all $c \in C$. This is denoted by $(H, C) = (F, A) \cap_R (G, B)$.

**Definition 2.15** [9]
Let $(F, A)$ be a bipolar fuzzy soft set over $U$ for each $t \in [0,1]$ and $s \in [-1,0]$ the set $(F, A)^{(t,s)} = (F^{(t,s)}, A)$ where $(F, A)_a^{(t,s)} = \{x \in U | \mu_{F(a)}^P(x) \geq t, \mu_{F(a)}^N(x) \leq s\}$ for all $a \in A$.

**Definition 2.16**[17]
Let $\phi: H_1 \to H_2$ and $h: E_1 \to E_2$ be two maps, $A \subseteq E_1$ and $B \subseteq E_2$, where $E_1$ and $E_2$ are sets of parameters viewed on $H_1$ and $H_2$, respectively. The pair $(\phi, h)$ is called a fuzzy soft map from $H_1$ to $H_2$. If $\phi$ is a hypergroup homomorphism, then $(\phi, h)$ is called a fuzzy soft homomorphism from $H_1$ to $H_2$.

**Definition 2.17 [3]**
Let $(f, A)$ and $(g, B)$ be two fuzzy soft sets over $H_1$ and $H_2$, respectively, and $(\phi, h)$ be a fuzzy soft function from $H_1$ to $H_2$
(i) The image of $(f, A)$ under the soft function $(\phi, h)$ denoted by $(\phi, h)(f, A)$, is a fuzzy soft set over $H_2$ defined by $(\phi, h)(f, A) = (\phi(f), h(A))$, where for all $b \in h(A)$ and for all $y \in H_2$, then

$$\Phi(f)_b(y) = \begin{cases} \bigvee_{\phi(x)=y} \ \bigvee_{h(a)=b} f_a(x), & \text{if} x \in \phi^{-1}(y) \\ 0 & \text{otherwise} \end{cases}$$

(ii) The inverse image of $(g, B)$ under the fuzzy soft function $(\phi, h)$ denoted by $(\phi, h)^{-1}(g, B)$, is a fuzzy soft set over $B$ defined by $(\phi, h)^{-1}(g, B) = (\phi^{-1}(g), h^{-1}(A))$, where for all $a \in h^{-1}(A)$ and for all $x \in H_1$, $\phi^{-1}(g)_a(x) = g_{h(a)}(\phi(x))$. If $\phi$ and $h$ is injective(surjective), then $(\phi, h)$ is said to be injective (surjective).

**Definition 2.18** [15]
Let $(\phi, \psi)$ be a fuzzy soft $\Gamma$-function from $X$ to $Y$. If $\phi$ is a homomorphism function from $X$ to $Y$, then $(\phi, \psi)$ is said to be fuzzy soft $\Gamma$-homomorphism. If $\phi$ is isomorphism function from $X$ to $Y$ and $\psi$ is one to one mapping from N to M, then $(\phi, \psi)$ is said to be fuzzy soft $\Gamma$-isomorphism.

## 3.Bipolar Fuzzy Soft Γ- Hyper Ideals

In this section, we introduce the notion of bipolar fuzzy soft gamma hyperideals in gamma semigroups and discuss some of its properties $S$ denotes the $\Gamma$- hyper semigroup.

**Definition 3.1**
A bipolar fuzzy soft set $(F, A)$ over a $\Gamma$- hypersemigroups $S$ is called a bipolar fuzzy soft $\Gamma$-subhypersemigroup over $S$ if
(i) $\inf_{x \in y\gamma z} \mu_{F(a)}^+(x) \geq \min\{\mu_{F(a)}^+(y), \mu_{F(a)}^+(z)\}$
(ii) $\sup_{x \in y\gamma z} \mu_{F(a)}^-(x) \leq \max\{\mu_{F(a)}^-(y), \mu_{F(a)}^-(z)\}$ for all $x, y, z \in S$, $\gamma \in \Gamma$ and $a \in A$.

**Definition 3.2**
A bipolar fuzzy soft set $(F, A)$ over a $\Gamma$- hypersemigroups $S$ is called a bipolar fuzzy soft left $\Gamma$-hyperideal over $S$ if
(i) $\inf_{x \in y\gamma z} \mu_{F(a)}^+(x) \geq \mu_{F(a)}^+(z)$
(ii) $\sup_{x \in y\gamma z} \mu_{F(a)}^-(x) \leq \mu_{F(a)}^-(z)$ for all $x, y, z \in S$, $\gamma \in \Gamma$and $a \in A$.

**Definition 3.3**A bipolar fuzzy soft set $(F, A)$ over a $\Gamma$-hypersemigroups $S$ is called a bipolar fuzzy soft right $\Gamma$-hyperideal over $S$ if
(i) $\inf_{x \in y\gamma z} \mu_{F(a)}^+(x) \geq \mu_{F(a)}^+(y)$
(ii) $\sup_{x \in y\gamma z} \mu_{F(a)}^-(x) \leq \mu_{F(a)}^-(y)$ for all $x, y, z \in S$, $\gamma \in \Gamma$ and $a \in A$.

**Definition 3.4**
A bipolar fuzzy soft set $(F, A)$ over a $\Gamma$- hypersemigroups $S$ is called a bipolar fuzzy soft $\Gamma$-hyperideal of $S$ if
(i) $\inf_{x \in y\gamma z} \mu_{F(a)}^+(x) \geq \max\{\mu_{F(a)}^+(y), \mu_{F(a)}^+(z)\}$

(ii) $\sup_{x\in y\gamma z} \mu^-_{F(a)}(x) \le \min\{\mu^-_{F(a)}(y), \mu^-_{F(a)}(z)\}$ for all $x, y, z \in S$, $\gamma \in \Gamma$ and $a \in A$.

**Definition 3.5**
A bipolar fuzzy soft set $(F, A)$ over a $\Gamma$- hypersemigroups $S$ is called a bipolar fuzzy soft $\Gamma$- hyperbi-ideal over $S$ if
(i) $\inf_{p\in x\alpha y\beta z} \mu^+_{F(a)}(p) \ge \min\{\mu^+_{F(a)}(x), \mu^+_{F(a)}(z)\}$
(ii) $\sup_{p\in x\alpha y\beta z} \mu^-_{F(a)}(p) \le \max\{\mu^-_{F(a)}(x), \mu^-_{F(a)}(z)\}$ for all $x, y, z \in S$, $\alpha, \beta \in \Gamma$ and $a \in A$.

**Definition 3.6**
A bipolar fuzzy soft set $(F, A)$ over a $\Gamma$- hypersemigroups $S$ is called a bipolar fuzzy soft $\Gamma$-hyperinterior ideal over $S$ if
(i) $\inf_{p\in x\alpha y\beta z} \mu^+_{F(a)}(p) \ge \mu^+_{F(a)}(y)$
(ii) $\sup_{p\in x\alpha y\beta z} \mu^-_{F(a)}(p) \le \mu^-_{F(a)}(y)$ for all $x, y, z \in S$, $\alpha, \beta \in \Gamma$ and $a \in A$.

**Theorem 3.7**
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft $\Gamma$-hypersubsemigroups over $S$, then $(F, A) \wedge (G, B)$ and $(F, A) \vee (G, B)$ are bipolar fuzzy soft $\Gamma$- hypersubsemigroup of $S$.
Proof. Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft $\Gamma$-hypersubsemigroups over $S$ defined as $(F, A) \wedge (G, B)$ where $C = A \times B$ and $H(a, b) = F(a) \cap G(b)$, for all $(a, b) \in C = A \times B$, $x, y, z \in S \gamma \in \Gamma$

$$\begin{aligned}
\inf_{z\in x\gamma y}\{\mu^+_{H(a,b)}(z)\} &= \inf_{z\in x\gamma y}\{\min\{\mu^+_{F(a)}(z), \mu^+_{G(b)}(z)\}\} \\
&= \min\{\inf_{z\in x\gamma y}\mu^+_{F(a)}(z), \inf_{z\in x\gamma y}\mu^+_{G(b)}(z)\} \\
&\ge \min\{\min\{\mu^+_{F(a)}(x), \mu^+_{F(a)}(y)\}, \min\{\mu^+_{G(b)}(x), \mu^+_{G(b)}(y)\}\} \\
&= \min\{\{\min\{\mu^+_{F(a)}(x), \mu^+_{G(b)}(x)\}, \min\{\mu^+_{F(a)}(q), \mu^+_{G(b)}(q)\}\} \\
&= \min\{(\mu^+_{F(a)} \cap \mu^+_{G(b)})(x), (\mu^+_{F(a)} \cap \mu^+_{G(b)})(y)\} \\
&= \min\{\mu^+_{H(a,b)}(x), \mu^+_{H(a,b)}(y)\}.
\end{aligned}$$

and

$$\begin{aligned}
\sup_{z\in x\gamma y}\{\mu^-_{H(a,b)}(z)\} &= \sup_{z\in x\gamma y}\{\max\{\mu^-_{F(a)}(z), \mu^-_{G(b)}(z)\}\} \\
&= \max\{\sup_{z\in x\gamma y}\mu^+_{F(a)}(z), \inf_{z\in x\gamma y}\mu^+_{G(b)}(z)\} \\
&\ge \max\{\max\{\mu^-_{F(a)}(x), \mu^-_{F(a)}(y)\}, \min\{\mu^-_{G(b)}(x), \mu^-_{G(b)}(y)\}\} \\
&= \max\{\{\max\{\mu^+_{F(a)}(x), \mu^+_{G(b)}(x)\}, \max\{\mu^-_{F(a)}(y), \mu^-_{G(b)}(y)\}\} \\
&= \max\{(\mu^-_{F(a)} \cup \mu^-_{G(b)})(x), (\mu^-_{F(a)} \cup \mu^-_{G(b)})(y)\} \\
&= \max\{\mu^-_{H(a,b)}(x), \mu^-_{H(a,b)}(y)\}.
\end{aligned}$$

Hence $(F, A) \wedge (G, B)$ is a bipolar fuzzy soft $\Gamma$-hypersubsemigroup over $S$.Similarly it can be shown that $(F, A) \vee (G, B)$ are bipolar fuzzy soft $\Gamma$-hypersub semigroup over S.

**Theorem 3.8**
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft $\Gamma$-hyperleft (resp.right) ideals over $S$, then $(F, A) \wedge (G, B)$ and $(F, A) \vee (G, B)$ are bipolar fuzzy soft $\Gamma$-hyperleft (resp.right) ideals of $S$.
Proof. Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft $\Gamma$ -hyperleftideals over $S$ defined as $(F, A) \wedge (G, B)$ where $C = A \times B$ and $H(a, b) = F(a) \cap G(b)$, for all $(a, b) \in C = A \times B$, $x, y, z \in S$ and $\gamma \in \Gamma$.

$$\begin{aligned}
\inf_{z\in x\gamma y}\{\mu^+_{H(a,b)}(z)\} &= \inf_{z\in x\gamma y}\{\min\{\mu^+_{F(a)}(z), \mu^+_{G(b)}(z)\}\} \\
&= \min\{\inf_{z\in x\gamma y}\mu^+_{F(a)}(z), \inf_{z\in x\gamma y}\mu^+_{G(b)}(z)\} \\
&= \min\{\mu^+_{F(a)}(y), \mu^+_{G(b)}(y)\} \\
&= \mu^+_{H(a,b)}(y)\}.
\end{aligned}$$

and

$$\begin{aligned}
\sup_{z\in x\gamma y}\{\mu^-_{H(a,b)}(z)\} &= \sup_{z\in x\gamma y}\{\max\{\mu^-_{F(a)}(z), \mu^-_{G(b)}(z)\}\} \\
&= \max\{\sup_{z\in x\gamma y}\mu^-_{F(a)}(z), \inf_{z\in x\gamma y}\mu^-_{G(b)}(z)\} \\
&\le \max\{\mu^-_{F(a)}(y), \mu^-_{G(b)}(y)\} \\
&= \mu^-_{H(a,b)}(y)
\end{aligned}$$

Hence $(F, A) \wedge (G, B)$ are bipolar fuzzy soft $\Gamma$-left (resp.right) hyperideals over $S$.
Similar proof shows that $(F, A) \vee (G, B)$ is a bipolar fuzzy soft $\Gamma$-left (resp.right) hyperideals over $S$.

**Theorem 3.9**
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft $\Gamma$-hyperbi-ideals over $S$, then $(F, A) \wedge (G, B)$and $(F, A) \vee (G, B)$ are bipolar fuzzy soft $\Gamma$-hyperbi-ideals of $S$.
Proof. Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft $\Gamma$ -hypersemigroups over $S$ defined as $(F, A) \wedge (G, B)$ where $C = A \times B$, $H(a, b) = F(a) \cap G(b)$, for all $(a, b) \in C = A \times B$, $x, y, z \in S$ and $\gamma \in \Gamma$.

$$\begin{aligned}
\inf_{z\in x\alpha y\beta z}\{\mu^+_{H(a,b)}(z)\} &= \inf_{z\in x\gamma y}\{\min\{\mu^+_{F(a)}(z), \mu^+_{G(b)}(z)\}\} \\
&= \min\{\inf_{z\in x\gamma y}\mu^+_{F(a)}(z), \inf_{z\in x\gamma y}\mu^+_{G(b)}(z)\} \\
&\ge \min\{\min\{\mu^+_{F(a)}(x), \mu^+_{F(a)}(z)\}, \min\{\mu^+_{G(b)}(x), \mu^+_{G(b)}(z)\}\} \\
&= \min\{\{\min\{\mu^+_{F(a)}(x), \mu^+_{G(b)}(x)\}, \min\{\mu^+_{F(a)}(z), \mu^+_{G(b)}(z)\}\} \\
&= \min\{(\mu^+_{F(a)} \cap \mu^+_{G(b)})(x), (\mu^+_{F(a)} \cap \mu^+_{G(b)})(z)\} \\
&= \min\{\mu^+_{H(a,b)}(x), \mu^+_{H(a,b)}(z)\}.
\end{aligned}$$

and

$$\begin{aligned}
\sup_{z\in x\alpha y\beta z}\{\mu^-_{H(a,b)}(z)\} &= \sup_{z\in x\gamma y}\{\max\{\mu^-_{F(a)}(z), \mu^-_{G(b)}(z)\}\} \\
&= \max\{\sup_{z\in x\gamma y}\mu^+_{F(a)}(z), \inf_{z\in x\gamma y}\mu^+_{G(b)}(z)\} \\
&\le \max\{\max\{\mu^-_{F(a)}(x), \mu^-_{F(a)}(z)\}, \min\{\mu^-_{G(b)}(x), \mu^-_{G(b)}(z)\}\} \\
&= \max\{\{\max\{\mu^-_{F(a)}(x), \mu^-_{G(b)}(x)\}, \max\{\mu^-_{F(a)}(z), \mu^-_{G(b)}(z)\}\} \\
&= \max\{(\mu^-_{F(a)} \cup \mu^-_{G(b)})(x), (\mu^-_{F(a)} \cup \mu^-_{G(b)})(z)\} \\
&= \max\{\mu^-_{H(a,b)}(x), \mu^-_{H(a,b)}(z)\}.
\end{aligned}$$

Hence $(F, A) \wedge (G, B)$ is a bipolar fuzzy soft $\Gamma$-hyperbi-ideal over S.
It can be similarly proved that $(F, A) \vee (G, B)$ is a bipolar fuzzy soft $\Gamma$-hyperbi-ideal over $S$.

**Theorem 3.10**
Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft $\Gamma$-hypersubsemigroups over $S$, then $(F, A) \cap_\varepsilon (G, B)$ is a bipolar fuzzy soft $\Gamma$-hypersubsemigroups of $S$.
Proof. Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft $\Gamma$-hypersubsemigroups over $S$ as defined
$(F, A) \cap_\varepsilon (G, B) = (H, C)$ where $C = A \cup B$
$H(c) = (F(c)$

$$H(c) = \begin{cases} F(c) & if \quad c \in A \setminus B \\ G(c) & if \quad c \in B \setminus A \\ F(c) \cap G(c) & if \quad c \in A \cap B \end{cases}$$

Case(i) $c \in A\backslash B$ and $\gamma \in \Gamma$

$$\begin{aligned}
\inf_{z\in x\gamma y}\{\mu^+_{H(c)}(z)\} &= \inf_{z\in x\gamma y}\mu^+_{F(c)}(z) \\
&\ge \min\{\mu^+_{F(c)}(x), \mu^+_{F(c)}(y)\} \\
&= \min\{\mu^+_{H(c)}(x), \mu^+_{H(c)}(y)\}
\end{aligned}$$

and

$$\begin{aligned}
\sup_{z\in x\gamma y}\{\mu^-_{H(c)}(z)\} &= \sup_{z\in x\gamma y}\mu^-_{F(c)}(z) \\
&\le \max\{\mu^-_{F(c)}(x), \mu^-_{F(c)}(y)\} \\
&= \max\{\mu^-_{H(c)}(x), \mu^-_{H(c)}(y)\}
\end{aligned}$$

Case(ii) $c \in B\backslash A$ and $\gamma \in \Gamma$.

$\inf_{z\in x\gamma y}\{\mu^+_{H(c)}(z)\} = \inf_{z\in x\gamma y}\mu^+_{G(c)}(z)$
$\geq \min\{\mu^+_{G(c)}(x),\mu^+_{G(c)}(y)\}$
$= \min\{\mu^+_{H(c)}(x),\mu^+_{H(c)}(y)\}$
and
$\sup_{z\in x\gamma y}\{\mu^-_{H(c)}(z)\} = \sup_{z\in x\gamma y}\mu^-_{G(c)}(z)$
$\leq \max\{\mu^-_{G(c)}(x),\mu^-_{G(c)}(y)\}$
$= \max\{\mu^-_{H(c)}(x),\mu^-_{H(c)}(y)\}$
Case (iii) $C\in A\cap B$ and $\gamma\in\Gamma$ then $H(c)=F(c)\cap G(c)$ then by theorem 3.7,
$\inf_{z\in x\gamma y}\{\mu^+_{H(c)}(z)\}\geq \inf_{z\in x\gamma y}\{\mu^+_{H(c)}(x),\mu^+_{H(c)}(y)\}$
$=\min\{\mu^+_{H(c)}(x),\mu^+_{H(c)}(y)\}$,
and
$\sup_{z\in x\gamma y}\{\mu^-_{H(c)}(z)\}\leq \sup_{z\in x\gamma y}\{\mu^-_{H(c)}(x),\mu^-_{H(c)}(y)\}$
$=\max\{\mu^-_{H(c)}(x),\mu^-_{H(c)}(y)\}$.
Hence $(F,A)\cap_\varepsilon(G,B)$ is a bipolar fuzzy soft $\Gamma$-hypersubsemigroup over $S$.

**Theorem3.11**
Let $(F,A)$ and $(G,B)$ be two bipolar fuzzy soft $\Gamma$-hypersubsemigroup over $S$, then $(F,A)\cup_\varepsilon(G,B)$ is a bipolar fuzzy soft $\Gamma$-hypersubsemigroup of $S$.
Proof. Straight forward.

**Theorem 3.12**
Let $(F,A)$ and $(G,B)$ be two bipolar fuzzy soft $\Gamma$-hyperbi(interior) ideal over $S$, then $(F,A)\cap_\varepsilon(G,B)$ is a bipolar fuzzy soft $\Gamma$-hyperbi(interior) ideal of $S$.
Proof. Straight forward.

**Theorem 3.13**
Let $(F,A)$ and $(G,B)$ be two bipolar fuzzy soft-$\Gamma$-hyper bi(interior) ideal over $S$, then $(F,A)\cup_\varepsilon(G,B)$ is a bipolar fuzzy soft $\Gamma$- hyperbi(interior) ideal of $S$.
Proof. Straight forward.
**Theorem 3.14**
Let $(F,A)$ and $(G,B)$ be two bipolar fuzzy soft $\Gamma$-hypersubsemigroup over $S$, then $(F,A)\cap_R(G,B)$ is a bipolar fuzzy soft $\Gamma$- hypersubsemigroup of $S$.
Proof. Let $(F,A)$ and $(G,B)$ be two bipolar fuzzy soft $\Gamma$-hypersubsemigroup over S, then $(F,A)\cap_R(G,B)=(H,C)$ where $C=A\cap B$ and $H(c)=F(c)\cap G(c)$ for all $c\in C$.
$\inf_{z\in x\gamma y}\mu^+_{H(c)}(z) = \inf_{z\in x\gamma y}\{\min\{\mu^+_{F(c)}(z),\mu^+_{G(c)}(z)\}\}$
$= \min\{\inf_{z\in x\gamma y}\mu^+_{F(c)}(z),\inf_{z\in x\gamma y}\mu^+_{G(c)}(z)\}$
$\geq \min\{\min\{\mu^+_{F(c)}(x),\mu^+_{F(c)}(y)\},\min\{\mu^+_{G(c)}(x),\mu^+_{G(c)}(y)\}\}$
$= \min\{\min\{\mu^+_{F(c)}(x),\mu^+_{G(c)}(x)\},\min\{\mu^+_{F(c)}(x),\mu^+_{G(c)}(x)\}\}$
$= \min\{(\mu^+_{F(c)}\cap\mu^+_{G(c)})(x),(\mu^+_{F(c)}\cap\mu^+_{G(c)})(y)\}$
$= \min\{\mu^+_{H(c)}(x),\mu^+_{H(c)}(y)\}$.
and
$\sup_{z\in x\gamma y}\mu^-_{H(c)}(z) = \sup_{z\in x\gamma y}\{\max\{\mu^-_{F(c)}(z),\mu^-_{G(c)}(z)\}\}$
$= \max\{\sup_{z\in x\gamma y}\mu^-_{F(c)}(z),\sup_{z\in x\gamma y}\mu^-_{G(c)}(z)\}$
$\leq \max\{\max\{\mu^-_{F(c)}(x),\mu^-_{F(c)}(y)\},\max\{\mu^-_{G(c)}(x),\mu^-_{G(c)}(y)\}\}$
$= \max\{\max\{\mu^-_{F(c)}(x),\mu^-_{G(c)}(x)\},\max\{\mu^-_{F(c)}(y),\mu^-_{G(c)}(y)\}\}$
$= \max\{(\mu^-_{F(c)}\cap\mu^-_{G(c)})(x),(\mu^-_{F(c)}\cap\mu^-_{G(c)})(y)\}$
$= \max\{\mu^-_{H(c)}(x),\mu^-_{H(c)}(y)\}$.
Hence $(F,A)\cap_R(G,B)$ is a bipolar fuzzy soft $\Gamma$- hypersubsemigroup of $S$.

**Theorem 3.15**
Let $(F,A)$ and $(G,B)$ be two bipolar fuzzy soft $\Gamma$-hypersubsemigroup over $S$, then $(F,A)\cup_R(G,B)$ is a bipolar fuzzy soft –hypersub semigroup of $S$.
Proof. Straight forward.

**Theorem 3.16**
Let $(F,A)$ and $(G,B)$ be two bipolar fuzzy soft $\Gamma$-hyperbi(interior)ideal over $S$, then $(F,A)\cap_R(G,B)$ is a bipolar fuzzy soft $\Gamma$-hyper bi(interior)ideal of $S$.
Proof. Straight forward.

**Theorem 3.17**
Let $(F,A)$ and $(G,B)$ be two bipolar fuzzy soft $\Gamma$-hyperbi(interior)ideal over $S$, then $(F,A)\cup_R(G,B)$ is a bipolar fuzzy soft $\Gamma$- hyperbi(interior)ideal of $S$.
Proof. Straight forward.

**Example 3.18**
Every bipolar fuzzy soft $\Gamma$-hyper ideal is bipolar valued fuzzy soft $\Gamma$ –hypersubsemigroups but converse is not true.
Let $S=\{a,b,c,d,e\}$ and $\Gamma=\{\gamma\}$ then $S$ is $\Gamma$-semihypergroup

| γ | a | b | c | d | e |
|---|---|---|---|---|---|
| a | {a, b} | {b, e} | c | {c, d } | e |
| b | {b, e } | e | c | {c, d } | e |
| c | c | c | c | c | c |
| d | {c, d } | {c,d } | c | d | {c, d} |
| e | e | e | c | {c, d } | e |

Let $E=\{u,v,w,x,y\}$ and $A=\{u,v,y\}$. Define the bipolar fuzzy soft set $(F,A)$ as
$(F,A)=\{F(u),F(v),F(y)\}$, where
$F(u)=\{(a,0.6,-0.5),(b,0.7,-0.6),(c,0.4,-0.2),(d,0.3,-0.1),(e,0.9,-0.8)\}$
$F(v)=\{(a,0.8,-0.4),(b,0.9,-0.7),(c,0.6,-0.3),(d,0.2,-0.1),(e,1,-0.9)\}$
$F(y)=\{(a,0.7,-0.8),(b,0.8,-0.9),(c,0.5,-0.4),(d,0.2,-0.3),(e,1,-0.9)\}$
Hence $(F,A)$ is a bipolar fuzzy soft sub $\Gamma$- hypersemigroups but not bipolar valued fuzzy hyperideal. Sinc $\inf_{a\in a\gamma c}\mu^+_{F(a)}(a)\geq \max\{\mu^+_{F(a)}(a),\mu^+_{F(a)}(c)\}$
$=0.4\not\geq 0.6$

**Example 3.19**
Every bipolar fuzzy soft $\Gamma$-hyperideal is bipolar valued fuzzy soft $\Gamma$ hyper bi-ideals but converse is not true.
Let $S=\{a,b,c,d,e\}$ and $\Gamma=\{\alpha,\beta\}$ then $S$ is $\Gamma$-hypersemigroup

| α | a | b | c | d | e |
|---|---|---|---|---|---|
| a | {a, b} | {b, e} | c | {c, d } | e |
| b | {b, e} | e | c | {c, d } | e |
| c | c | c | c | c | c |
| d | {c, d} | {c,d} | c | d | {c, d} |
| e | e | e | c | {c, d } | e |

| β | a | b | c | d | e |
|---|---|---|---|---|---|
| a | {b, e} | e | c | {c, d} | e |
| b | e | e | c | {c, d} | e |
| c | c | c | c | c | c |
| d | {c, d} | {c,d} | c | d | {c, d} |
| e | e | e | c | {c, d} | e |

Let $E=\{u,v,w,x,y\}$ and $A=\{w,x,y\}$. Define the bipolar fuzzy soft set $(F,A)$ as
$(F,A)=\{F(w),F(x),F(y)\}$, where
$F(w)=\{(a,0.2,-0.1),(b,0.4,-0.3),(c,1,-0.9),(d,0.6,-0.7),(e,0.7,-0.8)\}$
$F(x)=\{(a,0.1,-0.2),(b,0.2,-0.3),(c,0.7,-0.8),(d,0.4,-0.5),(e,0.5,-0.6)\}$

$F(y) = \{(a, 0.3, -0.1), (b, 0.4, -0.2), (c, 0.9, -0.7),$
$(d, 0.6, -0.3), (e, 0.8, -0.5)\}$

Hence $(F, A)$ is a bipolar fuzzy soft $\Gamma$-hyperbi-ideal but not bipolar valued fuzzy hyper ideal,

Since $\inf_{a\in d\alpha e} \mu^{+}_{F(a)}(a) \geq \max\{\mu^{+}_{F(a)}(d), \mu^{+}_{F(a)}(e)\}$
$= 0.6 \ngeq 0.7.$

**Example 3.20**

Every bipolar fuzzy soft $\Gamma$-hyperideal is bipolar valued fuzzy soft $\Gamma -$ hyper intrior-ideal but converse is not true.

For the example 3.19, define the bipolar fuzzy soft set $(F, A)$ as $(F, A) = \{F(w), F(x), F(y)\}$, where

$F(w) = \{(a, 0.3, -0.2), (b, 0.6, -0.5), (c, 0.9, -0.8),$
$(d, 0.2, -0.1), (e, 0.8, -0.7)\}$

$F(x) = \{(a, 0.4, -0.3), (b, 0.5, -0.4), (c, 0.8, -0.7),$
$(d, 0.3, -0.1), (e, 0.6, -0.5)\}$

$F(y) = \{(a, 0.3, -0.2), (b, 0.4, -0.5), (c, 0.7, -0.9),$
$(d, 0.2, -0.1), (e, 0.5, -0.8)\}$

Hence $(F, A)$ is a bipolar fuzzy soft $\Gamma$-hyperinteriorideal but not bipolar valued fuzzy soft $\Gamma$-hyperideal, as $\inf_{a\in b\alpha d} \mu^{+}_{F(a)}(a) \geq \max\{\mu^{+}_{F(a)}(b), \mu^{+}_{F(a)}(d)\} = 0.2 \ngeq 0.6.$

**Theorem 3.21**

Let $(F, A)$ be a bipolar fuzzy soft set over $S$. $(F, A)$ is a bipolar fuzzy soft $\Gamma$-hypersemigroup if and only if $(F, A)^{(t,s)}$ is a soft $\Gamma$-hypersemigroup of $S$ for each $t \in [0,1]$ and $s \in [-1,0]$.

Proof. Assume that $(F, A)^{(t,s)}$ is a bipolar soft $\Gamma$-hypersemigroup over S for each $t \in [0,1]$ and $s \in [-1,0]$.For each $x_1, x_2 \in S$ and $a \in A$, let $t = \min\{\mu^{+}_{F(a)}(x_1), \mu^{+}_{F(a)}(x_2$ and $s = \max\{\mu^{-}_{F(a)}(x_1), \mu^{-}_{F(a)}(x_2)\}$, then $x_1, x_2 \in \mu^{(t,s)}_{F(a)}$. Since $\mu^{(t,s)}_{F(a)}$ is a $\Gamma$- hypersubsemigroup of S, then $x_1, x_2 \in \mu^{(t,s)}_{F(a)}$. That is $\mu^{+}_{F(a)}(x_1\gamma x_2) \geq t = \min\{\mu^{+}_{F(a)}(x_1), \mu^{+}_{F(a)}(x_2)\}$ and $\mu^{-}_{F(a)}(x_1\gamma x_2) \leq s = \max\{\mu^{-}_{F(a)}(x_1), \mu^{-}_{F(a)}(x_2)\}$. This shows that $\mu_{F(a)}$ is bipolar fuzzy $\Gamma$-hypersubsemigroup over S. Thus $(F, A)$ is a bipolar fuzzy soft $\Gamma$-hypersemigroup over $S$.

Conversely, assume that $(F, A)$ is a bipolar fuzzy soft $\Gamma$-hypersemigroup. For each $a \in A, t \in [0,1]$ and $s \in [-1,0]$ and $x_1, x_2 \in \mu^{(t,s)}_{F(a)}$.

we have $\mu^{+}_{F(a)}(x_1) \geq t, \mu^{+}_{F(a)}(x_2) \geq t$ and $\mu^{-}_{F(a)}(x_1) \leq s$, $\mu^{-}_{F(a)}(x_2) \leq s$. Therefore $\mu_{F(a)}$ is a bipolar fuzzy $\Gamma$-hypersubsemigroup of $S$. Thus $\gamma \in \Gamma$ there exists $z \in x_1\gamma x_2$ such that

$\inf_{z\in x_1\gamma x_2}(z) \geq \min\{\mu^{+}_{F(a)}(x_1), \mu^{+}_{F(a)}(x_2)\} \geq t$ and $\sup_{z\in x_1\gamma x_2}(z) \leq \max\{\mu^{-}_{F(a)}(x_1), \mu^{-}_{F(a)}(x_2)\} \leq s$. Therefore for all $z \in x_1\gamma x_2$ we have $z \in \mu^{(t,s)}_{F(a)}$, this implies that $x_1\gamma x_2 \in \mu^{(t,s)}_{F(a)}$, that is $\mu^{(t,s)}_{F(a)}$ is hyper $\Gamma$-subsemigroup of $S$. Therefore $(F, A)^{(t,s)}$ is a soft $\Gamma$-hypersemigroup of $S$ for each $t \in [0,1]$ and $s \in [-1,0]$.

**Theorem 3.22**

Let $(F, A)$ be a bipolar fuzzy soft set over $S$. $(F, A)$ is a bipolar fuzzy soft $\Gamma$-hyperleft(right)ideal if and only if $(F, A)^{(t,s)}$ is a soft $\Gamma$-hyper left(right) ideal of $S$ for each $t \in [0,1]$ and $s \in [-1,0]$.

Proof. Suppose that $(F, A)^{(t,s)}$ is a bipolar soft $\Gamma$-hyperleftideal of S for each $t \in [0,1]$, $s \in [-1,0]$ and $a \in A, \gamma \in \Gamma$. For each $x_1 \in S$, let $t = \mu^{+}_{F(a)}(x_1)$, then $x_1 \in \mu^{(t,s)}_{F(a)}$. Since $\mu^{(t,s)}_{F(a)}$ is a $\Gamma$-hyper left ideal of S, then $x\gamma x_1 \in \mu^{(t,s)}_{F(a)}$, for each $x \in S$. Hence $\mu^{+}_{F(a)}(x\gamma x_1) \geq t = \mu^{+}_{F(a)}(x_1)$ and $\mu^{-}_{F(a)}(x\gamma x_1) \leq s = \mu^{-}_{F(a)}(x_1)$. This shows that $\mu_{F(a)}$ is bipolar fuzzy $\Gamma$-hyperleftideal of S. By definition 3.2,$(F, A)$ is a bipolar fuzzy soft $\Gamma$-hyperleftideal of S.

Conversely, assume that $(F, A)$ is a bipolar fuzzy soft $\Gamma$-hyper left ideal of $S$. For each $a \in A, t \in [0,1]$ and $s \in [-1,0]$ and $x_1 \in \mu^{(t,s)}_{F(a)}$ we have $\mu^{+}_{F(a)}(x_1) \geq t$, and $\mu^{-}_{F(a)}(x_1) \leq s$ and by definition 3.2, $\mu^{+}_{F(a)}$ and $\mu^{-}_{F(a)}$ is a bipolar fuzzy $\Gamma$-hyper left ideal of $S$. Thus for $\gamma \in \Gamma$ there exists $z \in x\gamma x_1$ such that $\inf_{z\in x\gamma x_1}(z) \geq \mu^{+}_{F(a)}(x_1) \geq t$ and $\sup_{z\in x\gamma x_1}(z) \leq \mu^{-}_{F(a)}(x_1) \leq s$. Therefore for all $z \in x\gamma x_1$ we have $z \in \mu^{(t,s)}_{F(a)}$, that is $\mu^{(t,s)}_{F(a)}$ is hyper $\Gamma$-left ideal of S. Therefore $(F, A)^{(t,s)}$ is a soft $\Gamma$-hyper left ideal of S for each $t \in [0,1]$ and $s \in [-1,0]$. Similar proof holds for right ideal also.

**Theorem3.23**

Let $(F, A)$ be a bipolar fuzzy soft set over $S$, $(F, A)$ is a bipolar fuzzy soft $\Gamma$-hyperideal if and only if $(F, A)^{(t,s)}$ is a soft $\Gamma$-hyperideal of $S$ for each $t \in [0,1]$ and $s \in [-1,0]$.

Proof. The proof follows from theroem 3.22

## 4.Bipolar Fuzzy Soft Image and Inverse Image of Hyper $\Gamma$-Semigroups

**Definition 4.1**

[9] Let $\eta: H_1 \to H_2$ and $\psi: A \to B$ be two functions, $A$ and $B$ be two parametric sets from the crisp sets $H_1$ and $H_2$,respectively.Then the pair $(\eta, \psi)$ is called a bipolar fuzzy soft function from $H_1$ to $H_1$.

**Definition 4.2**

Let $(F, A)$ and $(G, B)$ be two bipolar fuzzy soft sets over the sets $H_1$ and $H_2$, respectively, and $(\eta, \psi)$ be a bipolar fuzzy soft map from $H_1$ to $H_2$.

(i) The image of $(F, A)$ under $(\eta, \psi)$ denoted by $(\eta, \psi)(F, A)$, is a bipolar fuzzy soft set over $H_2$ defined by $(\eta, \psi)(F, A) = (\eta(F), \psi(A))$, where for all $b \in \psi(A)$ and for all $y \in H_2$,

$$\mu^{+}_{\eta_{F(b)}}(y) = \begin{cases} \sup\limits_{\eta(x)=y} \sup\limits_{\psi(a)=b} \mu^{+}_{F(a)}(x), & \text{if} \eta^{-1}(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

$$\mu^{-}_{\eta_{F(b)}}(y) = \begin{cases} \inf\limits_{\eta(x)=y} \inf\limits_{\psi(a)=b} \mu^{-}_{F(a)}(x), & \text{if} \eta^{-1}(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

(ii) The inverse image of $(G, B)$ under $(\eta, \psi)$ denoted by $(\eta, \psi)^{-1}(G, B)$, is a bipolar fuzzy soft set over $H_1$ defined by $(\eta, \psi)^{-1}(G, B) = (\eta^{-1}(G), \psi^{-1}(B))$, where for all $a \in \psi^{-1}(B)$ and for all $x \in H_1$,$\mu^{+}_{\eta^{-1}_{G(a)}}(y) = \mu^{+}_{G_{\psi(a)}}(\eta(x))$ and $\mu^{-}_{\eta^{-1}_{G(a)}}(y) = \mu^{-}_{G_{\psi(a)}}(\eta(x))$

**Theorem 4.3**

Let $\eta: H_1 \to H_2$ be a homomorphism of S. If $(G, B)$ is a bipolar fuzzy soft $\Gamma$-hypersubsemigroup of $H_2$, then $(\eta, \psi)^{-1}(G, B)$ is a bipolar fuzzy soft $\Gamma$-hypersubsemigroup of $H_1$.

Proof. Let $(G, B)$ is a bipolar fuzzy soft $\Gamma$-hypersubsemigroup of $H_2$. Let $x, y, z \in H_1, \gamma \in \Gamma_1$ then we have

$$\begin{aligned}
\inf_{z\in x\gamma y}\left\{\mu^{+}_{\eta^{-1}_{G(a)}}(z)\right\} &= \inf_{z\in x\gamma y}\left\{\mu^{+}_{g_{\psi(a)}}(\eta(z))\right\} \\
&= \inf_{\eta(z)\in\eta(x\gamma y)}\left\{\mu^{+}_{g_{\psi(a)}}(\eta(z))\right\} \\
&= \inf_{\eta(z)\in\eta(x)h(\gamma)\eta(y)}\left\{\mu^{+}_{g_{\psi(a)}}(\eta(z))\right\} \\
&\geq \min\left\{\mu^{+}_{g_{\psi(a)}}\eta(x), \mu^{+}_{g_{\psi(a)}}\eta(y)\right\} \\
&= \min\left\{\mu^{+}_{\eta^{-1}_{G(a)}}(x), \mu^{+}_{\eta^{-1}_{G(a)}}(y)\right\}
\end{aligned}$$

and

$$\begin{aligned}
\sup_{z\in x\gamma y}\left\{\mu^{-}_{\eta^{-1}_{G(a)}}(z)\right\} &= \sup_{z\in x\gamma y}\left\{\mu^{-}_{g_{\psi(a)}}(\eta(z))\right\} \\
&= \sup_{\eta(z)\in\eta(x\gamma y)}\left\{\mu^{-}_{g_{\psi(a)}}(\eta(z))\right\} \\
&= \sup_{\eta(z)\in\eta(x)h(\gamma)\eta(y)}\left\{\mu^{-}_{g_{\psi(a)}}(\eta(z))\right\} \\
&\leq \max\left\{\mu^{-}_{g_{\psi(a)}}\eta(x), \mu^{-}_{g_{\psi(a)}}\eta(y)\right\} \\
&= \max\left\{\mu^{-}_{\eta^{-1}_{G(a)}}(x), \mu^{-}_{\eta^{-1}_{G(a)}}(y)\right\}
\end{aligned}$$

Therefore $(\eta,\psi)^{-1}(G,B)$ is a bipolar fuzzy soft $\Gamma$-hypersubsemigroup of $H_1$.

**Theorem 4.4**
Let $\eta: H_1 \rightarrow H_2$ be a homomorphism of S. If $(G,B)$ is a bipolar fuzzy soft $\Gamma$-hyperleft(right, bi-ideal, interior) of $H_2$, then $(\eta,\psi)^{-1}(G,B)$ is a bipolar fuzzy soft $\Gamma$-hyperleft(right, bi-ideal, interior)ideal of $H_1$.
Proof. Straightforward.

**Theorem4.5**
Let $\eta: H_1 \rightarrow H_2$ be a homomorphism of S. If $(F,A)$ is a bipolar fuzzy soft $\Gamma$-hypersubsemigroup of $H_1$, then $(\eta,\psi)(F,A)$ is a bipolar fuzzy soft $\Gamma$-hypersubsemigroup of $H_2$.
Proof. Let $(F,A)$ is a bipolar fuzzy soft $\Gamma$-hypersubsemigroup of $H_1$. Let $x_1, y_1 z_1 \in H_2, \gamma \in \Gamma_2$ then we have

$$\begin{aligned}
\inf_{z_1\in x_1\gamma y_1}\{\mu^+_{\eta_{F(b)}}(z_1)\} &= \inf_{z_1\in x_1\gamma y_1}\left\{\sup_{t\in\eta^{-1}(z_1)}\sup_{\psi(a)=b}\mu^+_{F(a)}(t)\right\}\\
&\geq \inf_{z\in x_1\gamma y_1}\left\{\sup_{\psi(a)=b}\mu^+_{F(b)}(z_1)\right\}\\
&= \inf_{\eta(z)\in\eta(x)h(\gamma)\eta(y)}\left\{\sup_{\psi(a)=b}\mu^+_{F(b)}(z)\right\}\\
&= \inf_{\eta(z)\in\eta(x\gamma y)}\left\{\sup_{\psi(a)=b}\mu^+_{F(b)}(z)\right\}\\
&= \inf_{z\in x\gamma y}\left\{\sup_{\psi(a)=b}\mu^+_{F(b)}(z)\right\}\\
&\geq \sup_{\psi(a)=b}\min\{\mu^+_{F(b)}(x),\mu^+_{F(b)}(y)\}\\
&\geq \sup_{x\gamma y\subseteq\eta^{-1}(x_1)h^{-1}(\gamma)\eta^{-1}(y_1)}\left\{\sup_{\psi(a)=b}\min\{\mu^+_{F(b)}(x),\mu^+_{F(b)}(y)\}\right\}\\
&= \min\left\{\sup_{\eta(x)=y}\sup_{\psi(a)=b}\mu^+_{F(a)}(x),\ \sup_{\eta(x)=y}\sup_{\psi(a)=b}\mu^+_{F(a)}(y)\right\}\\
&\geq \min\left\{\mu^+_{\eta_{F(b)}}(x_1),\mu^+_{\eta_{F(b)}}(y_1)\right\}
\end{aligned}$$

and

$$\begin{aligned}
\sup_{z_1\in x_1\gamma y_1}\{\mu^-_{\eta_{F(b)}}(z_1)\} &= \sup_{z_1\in x_1\gamma y_1}\left\{\inf_{t\in\eta^{-1}(z_1)}\inf_{\psi(a)=b}\mu^-_{F(a)}(t)\right\}\\
&\leq \sup_{z\in x_1\gamma y_1}\left\{\inf_{\psi(a)=b}\mu^-_{F(b)}(z_1)\right\}\\
&= \sup_{\eta(z)\in\eta(x)h(\gamma)\eta(y)}\left\{\inf_{\psi(a)=b}\mu^-_{F(b)}(z)\right\}\\
&= \sup_{\eta(z)\in\eta(x\gamma y)}\left\{\inf_{\psi(a)=b}\mu^-_{F(b)}(z)\right\}\\
&= \sup_{z\in x\gamma y}\left\{\inf_{\psi(a)=b}\mu^-_{F(b)}(z)\right\}\\
&\leq \inf_{\psi(a)=b}\max\{\mu^-_{F(b)}(x),\mu^-_{F(b)}(y)\}\\
&\leq \inf_{x\gamma y\subseteq\eta^{-1}(x_1)h^{-1}(\gamma)\eta^{-1}(y_1)}\left\{\inf_{\psi(a)=b}\max\{\mu^-_{F(b)}(x),\mu^-_{F(b)}(y)\}\right\}\\
&= \max\left\{\inf_{\eta(x)=y}\inf_{\psi(a)=b}\mu^-_{F(a)}(x),\ \inf_{\eta(x)=y}\inf_{\psi(a)=b}\mu^-_{F(a)}(y)\right\}\\
&\leq \max\left\{\mu^-_{\eta_{F(b)}}(x_1),\mu^-_{\eta_{F(b)}}(y_1)\right\}
\end{aligned}$$

Therefore $(\eta,\psi)(F,A)$ is a bipolar fuzzy soft $\Gamma$-hyper subsemigroup of $H_2$.

**Theorem 4.6**
Let $\eta: H_1 \rightarrow H_2$ be a homomorphism of S. If $(F,A)$ is a bipolar fuzzy soft $\Gamma$-hyperleft(right, bi-ideal, interior)ideal of $H_1$, then $(\eta,\psi)(F,A)$ is a bipolar fuzzy soft $\Gamma$-hyperleft(right, bi-ideal, interior)ideal of $H_2$.
Proof. Straighforward.

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